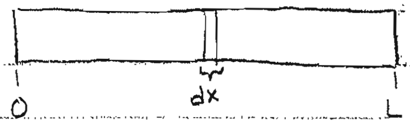
Kristen Basiaga
Quiz 7 - 3/26



A straight, one-dimensional rod lies on the X-axis. Its ends are located at $X=0$ and $X=L=1.5m$. The mass distribution (i.e. mass per unit length) is given as

$$\lambda = \frac{dm}{dx} = cx^{3/2} \qquad c = 0.23 \text{ kg m}^{-5/2}$$

a) Calculate mass M of the rod.

We know that "mass per unit length" implies

$$\lambda = \frac{dm}{dx}.$$

By separation of variables, this can be rearranged as

$$dm = \lambda dx.$$

Integrating both sides

$$\int_0^M dm = \int_0^L \lambda \, dx$$

$$m\Big|_0^M = \int_0^L c x^{3/2} \, dx$$

$$M = (0.23 \text{ kg-m}^{-5/2}) \frac{2}{5} x^{\frac{5}{2}}\Big|_0^L$$

$$= (0.23 \text{ kg-m}^{-5/2}) \cdot \frac{2}{5} (1.5 \text{ m})^{5/2}$$

$$\boxed{M = 0.253 \text{ kg}}$$

b) Calculate the center of mass, x_{cm}, of the rod, where x_{cm} is measured from the left end of the rod.

For a continuous distribution in one-dimensional space, center of mass is an integral

$$x_{cm} = \frac{1}{M} \int x \, dm$$

But from earlier,

$$dm = \lambda dx.$$

Replacing this definition of infinitesimal mass,

$$x_{cm} = \frac{1}{M} \int x \lambda \, dx$$

$$= \frac{c}{M} \int_0^L x \cdot x^{3/2} \, dx$$

$$= \frac{c}{M} \int_0^L x^{5/2} \, dx$$

$$x_{cm} = \frac{c}{M} \cdot \frac{2}{7} x^{7/2} \Big|_0^L$$

Using $M = 0.253$ kg from part (a),

$$x_{cm} = \frac{(0.23\ \text{kg}\cdot\text{m}^{-5/2})}{(0.253\ \text{kg})} \cdot \frac{2}{7} (1.5\ \text{m})^{7/2}$$

$$\boxed{x_{cm} = 1.07\ \text{m}}$$

POINT BREAKDOWN

a. $M = \int \lambda\, dx$ — 15
   correct integration — 5
   correct final answer — 5
   no unit confusion — 5

b. treat as continuous — 10
   $x_{cm} = \frac{1}{M}\int x\, \lambda\, dx$ — 10
   correct integration — 5
   correct final answer — 5
   sensible answer — 5

c. overall correct units — 5

70 + 30 free