PHYS 1230-1530, Rawitscher, Spring 2009

Quiz-7 March 26

---

Name, (1230?) (1530?)

A straight rod of length $L = 1.5\ m$ is placed on the $x$-axis. End $A$ is located at $x = 0$, and end $B$ is located at $x = L$. The rod is not uniform; the mass per unit length $\lambda$ varies with $x$ as $\lambda = c\, x^{3/2}$ , where $c = 0.23\ kg\ m^{-5/2}$.

**a)** Calculate the mass $M$ of the rod.

$$M = \int dm = \int_0^L \lambda\, dx = \int_0^L c\, x^{3/2}\, dx = c \frac{1}{5/2} x^{5/2} \Big|_0^L = c \cdot \frac{2}{5} \cdot L^{5/2}$$

$$M = 0.23 \times \frac{2}{5} \times (1.5)^{5/2} = \boxed{0.253\ kg}$$

basic eq.

**b)** Calculate the position $x_{cm}$ of the center of mass of the rod, where $x_{cm}$ is the distance from the left end $A$ of the rod.

$$\int x\, dm = M \cdot X_{cm} \quad \text{basic Eq.}$$

$$\int x\, dm = \int x \cdot \lambda\, dx = \int x \cdot c\, x^{3/2} dx = c \int_0 x^{5/2} dx = c \cdot \frac{1}{7/2} L^{7/2}$$

$$\int x\, dm = c \cdot \frac{2}{7} \cdot L^{7/2} = M \cdot X_{cm}$$

$$X_{cm} = \frac{\int x\, dm}{M} = \frac{c \cdot 2/7 \cdot L^{7/2}}{c \cdot \frac{2}{5} \cdot L^{5/2}} = \frac{5}{7} L = \frac{5 \times 1.5}{7} = \boxed{1.071\ m}$$

basic eq.