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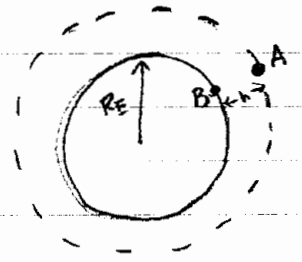
Quiz 6 - 3/19/09



A satellite orbits Earth.

- mass, $m = 350$ kg
- height, $h = 200 \cdot 10^3$ m

Useful constants

- local acceleration due to gravity, $g = 9.8134 \frac{m}{s^2}$
- radius of Earth, $R_E = 6.3700 \times 10^6$ m
- Gravitational constant, $G = 6.6700 \times 10^{-11}$
- Mass of Earth, $M_E = 5.97 \times 10^{24}$ kg

Find the difference in potential energy from point A to B:

a) using $U_A - U_B = mgh$

$$\Delta U = mgh$$
$$= (350\text{kg})\left(9.8134 \tfrac{m}{s^2}\right)(2 \times 10^5 \text{ m})$$
$$\boxed{\Delta U = -6.869 \times 10^8 \text{ J}}$$

b) using the exact expression for U_A, U_B

The exact equation for potential energy due to gravity is

$$U = G \frac{m_1 m_2}{R}$$

At point B, on the Earth's surface,

$$U_B = G \frac{m M_E}{R_E}$$

$$= \frac{(6.67 \times 10^{-11})(350\text{ kg})(5.97 \times 10^{24}\text{ kg})}{6.37 \times 10^6 \text{ m}}$$

$$U_B = 2.1879 \times 10^{10} \text{ J}$$

In orbit, $R = h + R_E$. Thus,

$$U_A = G \frac{m M_E}{R_E + h}$$
$$= \frac{(6.67 \times 10^{-11})(350\text{ kg})(5.97 \times 10^{24}\text{ kg})}{6.37 \times 10^6 \text{ m} + 2 \times 10^5 \text{ m}}$$
$$U_A = 2.1213 \times 10^{10} \text{ J}$$

In this case, $\Delta U = \boxed{U_A - U_B = -6.660 \times 10^8 \text{ J}}$

The negative sign in both results indicates that potential energy is greater on the surface than in orbit; i.e. $U_B > U_A$.

c) Compare results in a) and b) and comment whether the result is reasonable or not.

→ The acceleration due to gravity at Earth's surface is larger than at orbit height. As a result, the approximation $\Delta U = U_A - U_B = mgh$ overestimates the strength of gravity at point B. This explains why $|\Delta U|$ in part a) was larger than the exact calculation in part b).

on surface: $F_s = \left(G\frac{M_E}{R_E^2}\right)m$ $\quad \boxed{g_s = G\frac{M_E}{R_E^2}}$ $\quad > \quad$ in orbit: $F_o = \left(G\frac{M_E}{(R_E+h)^2}\right)m$ $\quad \boxed{g_o = G\frac{M_E}{(R_E+h)^2}}$

POINT BREAKDOWN

a) $\Delta U = mgh$	15
4 ± 1 significant figures	5
b) General potential $U = G\frac{m_1 m_2}{r}$	15
Find U_A	10
Find U_B	10
$\Delta U = U_A - U_B$	10
4 ± 1 significant figures	5
c) comment on g vs. G	5
units (J or N-m or $kg\frac{m^2}{s^2}$)	5
	80 + 20 free