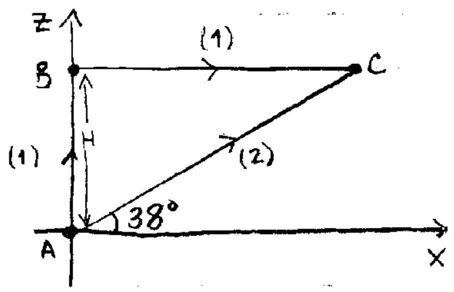
Quiz 5 - 3/4/09
K. Basiaga



Mass m is taken from A to C along path (1), ABC, and along path (2), AC. If gravity acts in the negative $\hat{z}$ direction,

$$\vec{F} = mg\,(-\hat{z}).$$

a) Work done by gravity on path (1)

In general, work is the dot-product of force and path length:

$$W = \vec{F}\cdot\vec{d} = |F||d|\cos\theta$$

Path (1) consists of two lengths: $\overline{AB} = H$ and $\overline{BC}$, which was unnamed. Clearly,

$$\vec{d}_1 = \overline{AB}\,\hat{z} + \overline{BC}\,\hat{x}.$$

Note that the following evaluates the dot product in two, equally-correct ways by unit vector orthogonality and by angle:

$W_1 = \vec{F}\cdot d_1$

$$W = (-mg\,\hat{z})\cdot[\overline{AB}\,\hat{z} + \overline{BC}\,\hat{x}]$$
$$= -mg\,[\overline{AB}\,(\hat{z}\cdot\hat{z}) + \overline{BC}\,(\hat{z}\cdot\hat{x})]$$

but $\hat{z}\cdot\hat{z} = 1$, $\hat{z}\cdot\hat{x} = 0$, and $\overline{AB} = H$

$\therefore$ $\boxed{W = -mgH}$

OR

$W_1 = |\vec{F}||\vec{d}_1|\cos\theta$

$$W = (-mg)\,[\overline{AB}\cos\theta_{zz} + \overline{BC}\cos\theta_{zx}]$$

The angle between $\hat{z}$ and $\hat{z}$, $\theta_{zz} = 0$ and between $\hat{z}$ & $\hat{x}$, $\theta_{zx} = 90° = \pi/2$ rad.

$\therefore$ $W = -mg\,[\overline{AB}\cos 0 + \overline{BC}\cos\frac{\pi}{2}]$

$\boxed{W = -mgH}$

Either method results in a correct answer. //

b) Work done by gravity along path (2)

Let the length from A to C be called 'r', whose length is determined by geometry:

$$\sin 38° = \frac{H}{|r|}$$

$$|r| = \overline{AC} = \frac{H}{\sin 38}$$

Define infinitesimal length along $\overline{AC}$ to be

$$d\vec{r} = \cos 38^\circ \, dr \, \hat{x} + \sin 38^\circ \, dr \, \hat{z}.$$

Work is then an integral over the path length:

$$W_2 = \int_0^r \vec{F} \cdot d\vec{r}$$

With $\vec{F} = -mg\,\hat{z}$,

$$W_2 = \int_0^r (-mg\,\hat{z}) \cdot (\cos 38 \, dr \, \hat{x} + \sin 38 \, dr \, \hat{z})$$

$$= -mg \cos 38^\circ \int_0^r dr \, (\hat{z} \cdot \hat{x}) - mg \sin 38 \int_0^r dr \, (\hat{z} \cdot \hat{z})$$

As before, $\hat{z} \cdot \hat{x} = 0$ and $\hat{z} \cdot \hat{z} = 1$. This leaves

$$W_2 = -mg \sin 38 \int_0^r dr$$

$$W_2 = -mg \sin 38 \, |r|$$

But by definition $|r| = \frac{H}{\sin 38}$,

$$W_2 = -mg \sin 38 \cdot \frac{H}{\sin 38}$$

$$\boxed{W_2 = -mgH}$$

Quiz 5 Grading

a)	Definition of work, $W = \vec{F} \cdot \vec{d}$	15
	Use of dot-product	15
	Work is NOT a vector	10
	$W_1 = -mgH$	10
b)	$W = \int \vec{F} \cdot d\vec{r}$	10
	Use of dot product	10
	Show calculations	10
	$W_2 = -mgH$	10
	Answers $W_1 = W_2$	10