

A rock is thrown into the air, with an initial speed of $v_0 = 12 \text{ m/s}$, and an initial angle with the horizontal $\theta_0 = 37^\circ$. If z and x are the vertical and horizontal coordinates of this ballistic trajectory, then the equation of the trajectory, as written down in class, is $z = ax + bx^2$ where $a = \tan(\theta_0)$; $b = -\frac{g}{2v_0^2 \cos^2(\theta_0)}$. For the values of θ_0 and v_0 given above,

$$a = 0.7536; \quad b = -0.0534; \quad \text{Range} = -\frac{a}{b} = 14.125 \text{ m}$$

1. At what times does the rock reach the x -coordinates $x_1 = 3.531 \text{ m}$ and $x_2 = 7.062 \text{ m}$

$$v_x = v_0 \times \cos \theta_0 = 12 \times \cos(37) = 9.58 \text{ m/s}$$

$$t_1 = \frac{x_1}{v_x} = 0.3684 \quad t_2 = \frac{x_2}{v_x} = 0.7369$$

2. What is the average velocity vector for the time interval for which x goes from $x_1 = 3.531 \text{ m}$ to $x_2 = 7.062 \text{ m}$? Note: *vector* means

- i) that you need to give the x and the z components of this average velocity vector, and
ii) give the magnitude and the angle with respect to the horizontal.

$$z_1 = a \cdot x_1 + b x_1^2 = 1.995$$

$$z_2 = a \cdot x_2 + b x_2^2 = 2.659$$

$$\vec{v} = \frac{r_2 - r_1}{t_2 - t_1} = \frac{(z_2 - z_1) \hat{j}}{t_2 - t_1} + \frac{x_2 - x_1 \hat{i}}{t_2 - t_1} = 1.801 \hat{j} + 9.584 \hat{i} \quad \begin{matrix} \text{m/s} \\ \text{m/s} \end{matrix}$$

$$\bar{v} = \sqrt{(1.801)^2 + (9.584)^2} = \cancel{9.571} \quad \boxed{9.751 \text{ m/s}}$$

$$\tan \theta = \frac{\bar{v}_z}{\bar{v}_x} = 0.188$$

$$\boxed{\theta = 10.64^\circ}$$

$$\bar{v}_x = 9.584 \text{ m/s}$$

$$\bar{v}_z = 1.801 \text{ m/s}$$