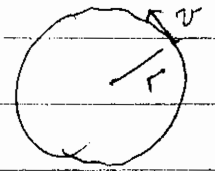
Week 13

26-26, 42, 58, Bonus 68

26-26: electrons circulate at a frequency of 2.4 GHz in a magnetic field B.

a) magnitude of B ?



$q v B = \text{force} = m a = m \, v^2/r$ v & B are perpend. to each other

$q B = m \frac{v}{r}$ Eq(1)

$\text{period} = \frac{2\pi r}{v}$; $\text{freq.} = \frac{1}{\text{period}} = \frac{qB}{m} \cdot \frac{1}{2\pi}$

(note, the radius cancels)

$\therefore B = \frac{2\pi f m}{q} = \frac{2 \times \pi \times 2.4 \times 10^9 \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19}} = \boxed{0.0856\ T}$

b) max diameter 2.5 mm. Max energy of the electron?

From Eq (1) $qB = \frac{mv}{r}$ $\therefore v = \frac{rqB}{m}$

$r < \frac{2.5 \times 10^{-3}\,m}{2}$ ∴

$\therefore v < \frac{1.25 \times 10^{-3}\,m \times 1.6 \times 10^{-19}\,C \times 85.6 \times 10^{-3}\,T}{9.11 \times 10^{-31}\,kg}$

$= 18.8 \times 10^6\ m/s$

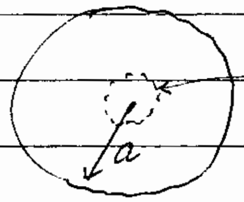
$\frac{1}{2} m v^2 < \frac{1}{2} \times 9.11 \times 10^{-31} \times (18.8 \times 10^6)^2 = \boxed{1.62 \times 10^{-16}\ J}$

or (by dividing by 1.6×10^{-19})

1.01 keV

electron volts.

26-42 wire 1.00 mm diameter, carrying 5 A
B at r = 0.10 mm



Ampere's Law $\int \vec{B}\, d\vec{\ell} = \mu_0 \bar{I}$

$\int d\ell = 2\pi r = 2\pi \times 0.1 \times 10^{-3}$ m

$a = 0.5 \times 10^{-3}$ m
$= \frac{1}{2}$ diameter

$\bar{I}$ is the current through the area of the wire

$\bar{I} = \frac{\text{current}}{\text{Area}} \times \underbrace{\text{area of wire}}_{\pi r^2}$

$\frac{\text{current}}{\text{Area}} = \frac{5\,A}{\pi a^2} = \frac{5}{\pi \times (0.5 \times 10^{-3})^2} = 6.37 \times 10^{6}\ A/m^2$

$\bar{I} = 6.37 \times 10^{6}\ A/m^2 \times \pi \times (0.1 \times 10^{-3} m)^2 = 0.2\ A$

↑ is the current going throw th dashed circle, radius 0.1 mm

$B \times 2\pi r = \mu_0 \bar{I}$ $\qquad B = \frac{\mu_0 \times 0.2 A}{2\pi \times 0.1 \times 10^{-3} m} =$

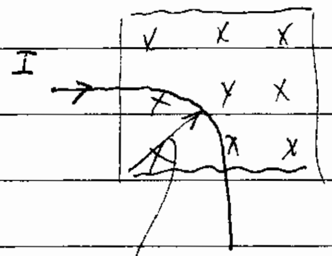
$\frac{4\pi \times 10^{-7} \times 0.2}{2\pi \times 0.1 \times 10^{-3}} = \frac{0.4}{0.1} \times 10^{-4} = \boxed{4 \times 10^{-4}\ T = 4\ \text{Gauss.}}$

b) at the surface of the wire

$B \times 2\pi a = \mu_0 I$

$B = \frac{\mu_0}{2\pi} \times \frac{I}{a} = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{5}{0.5 \times 10^{-3}} = \boxed{20 \times 10^{-4}\ T = 20\ \text{Gauss}}$

26-58:

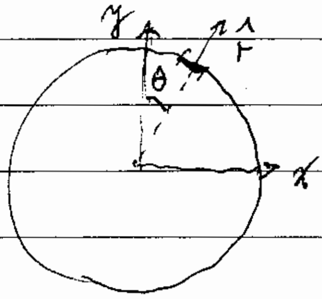


B into the paper, 48×10^{-3} T

$I = 1.5$ A

$\vec{F} = \vec{I} \times \vec{B} \cdot d\ell$

radius = 0.21 m



on segment $d\ell = r\,d\theta$

$d\vec{\ell} \times \vec{B} = d\ell \cdot B \cdot \hat{r}$

The cross product points in the $\hat{r}$ direction

x component of $\hat{r} = \sin\theta$

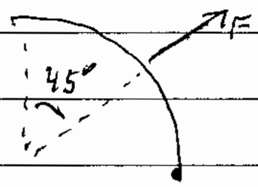
y " " $\hat{r} = \cos\theta$

$$\therefore d(F_x) = I \cdot B \cdot r\,d\theta \cdot \sin\theta$$

$$F_x = \int dF_x = I \cdot B \cdot r \int_0^{\pi/2} \sin\theta\, d\theta = IBr \left[\cos\theta\right]_{\pi/2}^{0} = IBr$$

$$F_y = \int dF_y = IBr \int_0^{\pi/2} \cos\theta\, d\theta = IBr \sin\theta\Big|_0^{\pi/2} = IBr$$

Hence the net force $= \sqrt{(IBr)^2 + (IBr)^2} = IBr\sqrt{2}$



$= 1.5\text{ A} \times 48 \times 10^{-3} \times 0.21 \times \sqrt{2}$

$= 2.14 \times 10^{-2}$ N