Week 10

Example 22-7, 8 → 22-7: Potential due to charged disk; 8: Field, from $E = -\frac{dU}{dr}$

Probl 22-4, 6, 7, 8, 15, 31, 74 ← only part a

Bonus 22-75

22-4: Proton and positron accelerated through same potential

a) Their final energies are the same: $e(\Delta V)$

charge of positron or proton

potential difference.

b) Their final speeds differ because their masses are very different: $\frac{1}{2} m v^2 = e \cdot \Delta V$

positron has smaller mass, hence higher v^2

m (positron) = m (electron) = 9.11×10^{-31} kg

m (proton) = 1.67×10^{-27} kg

ratio = 1,833.

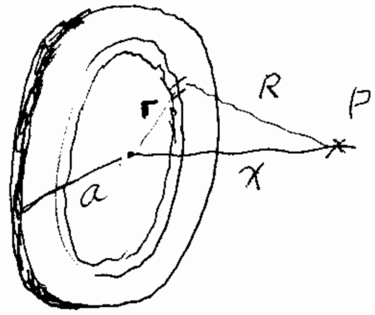
ratio of velocities = $\sqrt{\text{ratio of masses}} = \sqrt{1{,}833} = \boxed{42.8}$

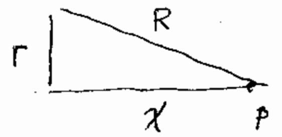
22-6 No, the potential need not be zero where E is zero since $E = -dV/dx$, when E is zero, V is at a extremum

22-7. NO, the E field need not be zero where V is zero. The value of V is arbitrary, since one can always re-scale the potential by adding a constant in all space, without affecting the E-field

22-7

Potential due to disk





$$R = \sqrt{r^2 + x^2}$$

r = radius of small hoop

dr = thickness of " "

$dq = \sigma \cdot d(\text{area})$

$dq = \sigma \cdot (2\pi r \cdot dr)$

$$dV = k \frac{dq}{R}$$

$$dV = k \cdot \frac{\sigma \cdot 2\pi r \cdot dr}{\sqrt{x^2 + r^2}}$$

$$V = \int dV = \int k \frac{\sigma \cdot 2\pi r \cdot dr}{\sqrt{x^2 + r^2}} = k \cdot \sigma \cdot 2\pi \int_0^a \frac{r\,dr}{\sqrt{x^2 + r^2}}$$

but $$\frac{d}{dr}\sqrt{x^2 + r^2} = \frac{1}{2}\frac{1}{\sqrt{x^2 + r^2}} \cdot 2r$$

∴ can integrate

$$V = \int dV = k\sigma \cdot 2\pi \times \sqrt{x^2 + r^2}\Big|_0^a$$

$$\boxed{V = k\sigma \cdot 2\pi \left[\sqrt{x^2 + a^2} - \sqrt{x^2}\right]}$$

as $x \gg a$

expansion of $\sqrt{x^2 + a^2} = x\sqrt{1 + a^2/x^2} = x\left(1 + \frac{1}{2}\frac{a^2}{x^2} + \cdots\right)$

$$V \xrightarrow[x \to \infty]{} k\sigma \cdot 2\pi \left[x\left(\cancel{1} + \frac{1}{2}\frac{a^2}{x^2} + \cdots\right) - \cancel{x}\right] \sim k \underbrace{\sigma \cdot 2\pi \cdot \frac{1}{2} a^2}_{Q} \frac{x}{x^2}$$

But total charge $Q = \text{area} \times \sigma = \pi a^2 \cdot \sigma$

$$\therefore V_{x \to \infty} = k\, Q/x \quad \text{as should be}$$

22-8 : is $E = -\frac{dV}{dx}$?

$$V = k \cdot \sigma \cdot 2\pi \left[\sqrt{x^2+a^2} - x \right]$$

$$\sigma = Q/\pi a^2$$

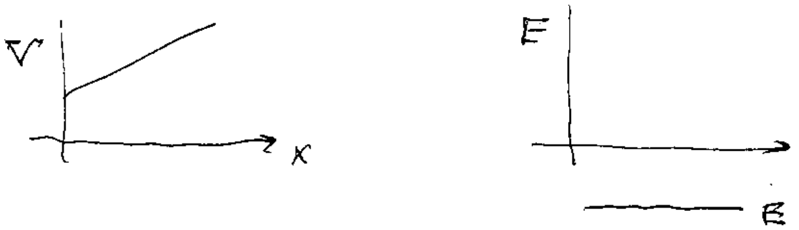
$$\therefore V = k \cdot \frac{2Q}{a^2} \left[\sqrt{x^2+a^2} - x \right]$$

$$\frac{dV}{dx} = \frac{k \cdot 2Q}{a^2} \left[\frac{1}{2} (x^2+a^2)^{-1/2} \cdot 2x \quad - 1 \right]$$

agrees with result of problem 20-69

22-8 : If the potential is a constant throughout a certain volume, then the E-field is zero throughout this volume.

22-15



V increases linearly

E = constant = negative.

22-31 : $V = 2xy - 3xz + 5y^2$

at point P find V $\qquad P = (1, 1, 1)$ coordinates

$V = 2\times1\times1 - 3\times1\times1 + 5\times1^2 = 2-3+5 = 4$ volts

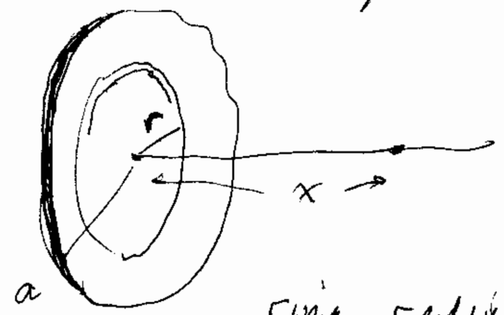
$E_x = -\frac{\partial V}{\partial x} = -2y + 3z = -2\times1 + 3\times1 = +1$ V/m

$E_y = -\frac{\partial V}{\partial y} = -2x - 10y = -2-10 = -12$ V/m

$E_z = -\frac{\partial V}{\partial z} = +3x = +3 = 3$ V/m

22-74 disk with non-uniform surface charge density

$\sigma = \sigma_0 \, r/a \qquad a = \text{radius}$



ring radius r, thickness dr, charge $dq = 2\pi r\, dr \cdot \sigma$

produces a potential $\dfrac{k\,dq}{\sqrt{r^2+x^2}} = \dfrac{k \cdot 2\pi r\,dr \cdot \sigma_0 r/a}{\sqrt{r^2+x^2}}$

total potential is the integral of the above =

$$= \frac{k \cdot 2\pi \cdot \sigma_0}{a} \int_0^R \frac{r^2\,dr}{\sqrt{r^2+x^2}}$$

from table of integrals. $\left[\frac{r}{2}\sqrt{r^2+x^2} - \frac{x^2}{2}\ln\left(r+\sqrt{r^2+x^2}\right)\right.$

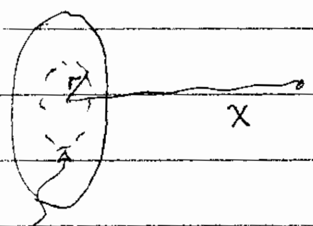
$$= \frac{R}{2}\sqrt{R^2+x^2} - \frac{x^3}{2}\ln\left(R+\sqrt{R^2+x^2}\right)$$

$$- \frac{x^3}{2}\ln(x)$$

23-74 : Disk, non uniform charge density $\sigma = \sigma_0 (r/a)$

a = radius.

a) pot'l on axis



Pot'l of ring: $dV = k \dfrac{\sigma \cdot 2\pi r\, dr}{(r^2 + x^2)^{1/2}}$

Integrate over all rings: $V = \int dV = \dfrac{k \cdot \sigma_0 (r/a) \cdot 2\pi r\, dr}{(r^2+x^2)^{1/2}}$

$$V = k \cdot \sigma_0 \frac{1}{a} \cdot 2\pi \int_0^a \frac{r^2 dr}{(r^2+x^2)^{1/2}}$$

use

$$\int \frac{x^2 dx}{(x^2+a^2)^{1/2}} = \frac{x}{2}(x^2+a^2)^{1/2} - \frac{a^2}{2} \log\left(x + \sqrt{x^2+a^2}\right)$$

$x \to r, \quad a \to x$

$$\left[\frac{r}{2} (r^2+x^2)^{1/2} - \frac{x^2}{2} \log\left(r + \sqrt{r^2+x^2}\right)\right]\Bigg|_{r=0}^{a}$$

$$= \frac{a}{2}(a^2+x^2)^{1/2} - \frac{x^2}{2} \log\left(a + \sqrt{a^2+x^2}\right.$$

$$- \left\{0 \qquad - \frac{x^2}{2} \log(0 + x)\right\}$$

$$\boxed{V = \frac{a}{2}(a^2+x^2)^{1/2} - \frac{x^2}{2} \log\left[\frac{a + \sqrt{a^2+x^2}}{x}\right]} \times k \cdot \sigma_0 \frac{1}{a} \cdot 2\pi$$

b) Electric field on x axis $\qquad E_x = -\dfrac{dV}{dx}$

$$-\frac{dV}{dx} = -\Big\{ \frac{a}{2}\left(a^2+x^2\right)^{-1/2}\cdot x - \frac{2x}{2}\log\left[\frac{a+\sqrt{a^2+x^2}}{x}\right]$$

$$- \frac{x^2}{2}\left[\frac{1}{a+\sqrt{a^2+x^2}}\cdot\frac{x}{\left(a^2+x^2\right)^{1/2}} - \frac{1}{x}\right]\Big\} * k.\sigma.\frac{1}{a}.2\pi$$

c) when $x >> a$

$$\left(a^2+x^2\right)^{1/2} \to x + \frac{1}{2}\frac{a^2}{x}$$

$$\therefore -\frac{dV}{dx} = -\Big\{\frac{a}{2}\,\frac{x}{x} - \frac{2x}{2}\log\Big[\frac{\overset{\to a}{\cancel{a}} + x}{x}\Big]$$

$\log 1 = 0$

$$- \frac{x^2}{2}\underbrace{\left[\frac{1}{\cancel{a}+x}\cdot\frac{x}{x} - \frac{1}{x}\right]}_{\cancel{0}\ -\frac{a}{x^2}}\Big\} * k\,\sigma\,\frac{1}{a}\cdot 2\pi$$

Too complicated

$$= -\left\{-\frac{a}{2} + \frac{a}{2}\right\} \to 0$$

Expand the ~~integral~~ integrand in powers of r/x $\qquad$ & use binomial

$$\int\frac{r^2dr}{\left(r^2+x^2\right)^{1/2}} = \int\frac{r^2dr}{x\left(1+r^2/x^2\right)^{1/2}} = \int\frac{r^2dr}{x}\left(1-\frac{1}{2}\,r^2/x^2\right)$$

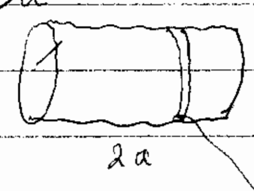
$$= \frac{1}{x}\int_0^a r^2dr - \frac{1}{2}\,\frac{1}{x^3}\int r^4dr =$$

$$V = \left\{\frac{1}{x}\cdot\frac{1}{3}a^3 - \frac{1}{2}\,\frac{1}{x^3}\cdot\frac{1}{5}a^5\right\} * k\cdot\sigma_0\,\frac{1}{a}\cdot 2\pi$$

$$-\frac{dV}{dx} = +\left\{\frac{1}{x^2}\cdot\frac{1}{3}a^3 - \text{neglect}\right\} k\cdot\sigma_0\cdot\frac{1}{a}\cdot 2\pi \simeq k\,\frac{Q}{x^2} \qquad \text{OK}$$

BONUS

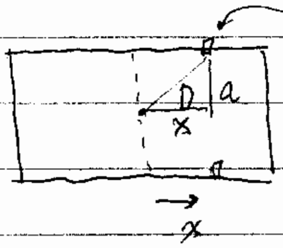
22-75 a



charge q spread uniformly over the surface of the cylinder.

V on the axis at its center?

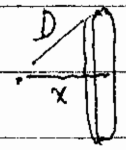
divide into rings



radius a, width dx, area $2\pi a \cdot dx$

contains charge $dq = \sigma \cdot 2\pi a\, dx$

↑ charge/area



Each ring produces a potential

$$dV = k \cdot \frac{dq}{D} \qquad D = \sqrt{x^2 + a^2}$$

$$\text{Total potential} = \int dV = \int_{-a}^{+a} k \frac{dq}{\sqrt{x^2+a^2}} = \int_{-a}^{+a} \cancel{k \cdot \sigma \cdot 2\pi} a$$

$$= k \cdot \sigma \cdot 2\pi a \underbrace{\int_{-a}^{+a} \frac{dx}{\sqrt{x^2+a^2}}} = k\,\sigma \cdot 2\pi a \cdot 2 \int_0^a \frac{dx}{\sqrt{x^2+a^2}}$$

use:

$$2 \log\left(x + \sqrt{x^2+a^2}\right)\Big|_0^a =$$

$$= 2\log\left(a + \sqrt{2a^2}\right) - 2\log a$$

$$= 2\log\left[\frac{a \cdot (1+\sqrt{2})}{a}\right] = 2\log\left(1+\sqrt{2}\right)$$

$$\therefore \boxed{V = k \cdot \sigma \cdot 4\pi a \log\left(1+\sqrt{2}\right)}$$