

Week 11

21-1, 2, 3, 5, 6, 18, 20, 22, 24, 29, 31

21-1 : No, electric field lines cannot cross.
If they did, then the \vec{E} -vector would point in two different directions at the same place

21-2 The particle at point B would experience a greater force than at A, because the E -lines are denser at B

21-3 If the same number of field lines enter the closed surface than leave, the flux is zero but the \vec{E} field is not zero.

Example two equal charges but opposite sign produce such a field

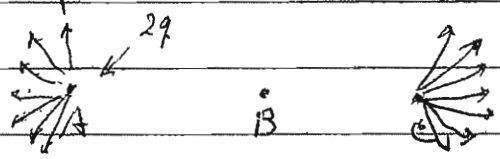
21-5 Flux = $E A$ only when \vec{E} direction is parallel to the ^{vector} normal to the surface A

21-6 : closed surface, 8 field lines emerge.

If a second charge is placed outside the closed surface still 8 field lines emerge, but they will come out of the surface at different places, because the total \vec{E} field will change.

21-18 : The net charge is $2q$

Proof



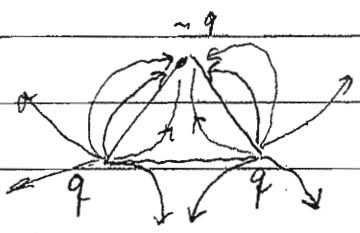
7 lines go out ^{to space} to the left of A, 7 lines leave A ^{to the right} to the right of A, and converge on to B. 7 more lines converge on B from the right. So $q_B = -2q = -14$ ^{lines}

7 lines leave C to the left and converge on B
 7 more lines " " " " right and go out to space

So $q_C = +2q = +14$ lines.

$$\begin{aligned} \text{So, the net charge} &= q_A + q_B + q_C = 2q = 3\mu\text{C} \\ &\quad \uparrow \quad \uparrow \quad \uparrow \\ &\quad 2q \quad -2q \quad +2q \\ &\quad \quad \quad \uparrow \\ &\quad \quad \quad -3\mu\text{C} \end{aligned}$$

21-20

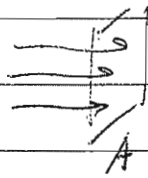


Only half of the field lines (three) that come out of each $+q$ can converge on $-q$, the rest goes out to space.

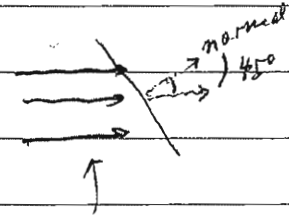
The total charge is q

21-22 :

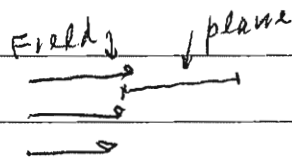
Area A of flat surface = 2.0 m^2
strength of $E = 850 \text{ N/C}$



← this case $\phi = \text{flux} = E \cdot A = 850 \cdot 2 = 1700 \text{ N/C} \cdot \text{m}^2$

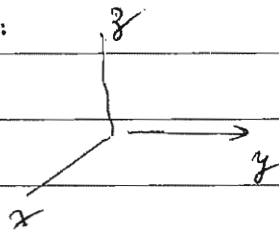


$$\phi = E \cdot A \cdot \cos(45^\circ) = \frac{1700}{\sqrt{2}} \text{ N/C} \cdot \text{m}^2$$



$$\phi = E \cdot A \cdot \cos(90^\circ) = 0$$

21-24 :



Surface $A = 0.14 \text{ m}^2$ lies in xy plane
 $\vec{E} = 5.1 \hat{i} + 2.1 \hat{j} + 3.5 \hat{k}$

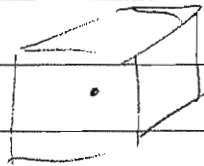
The normal to the plane = \hat{k}

$$\therefore \vec{A} = \text{area} = A \cdot \hat{k}$$

$$\vec{E} \cdot \vec{A} = E_x A_x + E_y A_y + E_z A_z =$$

$$\begin{matrix} \text{zero} & \text{zero} & 3.5 \times 0.14 = 0.49 \\ & & (\text{N/C})\text{m}^2 \end{matrix}$$

21-29 :

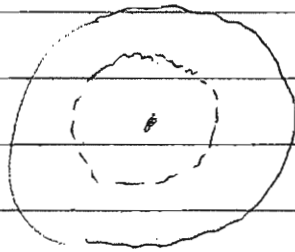


charge q at center of a cube
cube has 6 faces, the flux through
each face is the same = ϕ_s

$$6 \times \phi_s = \text{flux through a sphere surrounding charge} \\ = 4\pi R^2 q = 49 \times 10^3 (\text{N/C})\text{m}^2$$

$$\therefore \phi_s = \text{flux through each side} = \frac{1}{6} \times 4\pi R^2 q \\ = \frac{4}{6} \times 3.1416 \times 9 \times 10^{+9} \times 2.6 \times 10^{-6}$$

21
~~29~~. 31



Sphere, radius 0.25 m; carries
 $Q = 14 \mu\text{C}$; Find \vec{E} at various
 radii 0.15 m, 0.25 m, 0.50 m

$$\text{charge/volume} = \rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{14 \times 10^{-6} \text{ C}}{\frac{4}{3} \times \pi \times (0.25)^3} = 21$$

$$= 2.14 \times 10^{-4} \text{ C/m}^3$$

Charge contained inside a imaginary sphere of radius
 $r = \text{Vol. of sphere} \times \text{charge/vol}$

$$q(r) = \frac{4}{3}\pi r^3 \times \frac{Q}{\frac{4}{3}\pi R^3} = \left(\frac{r}{R}\right)^3 \times Q$$

$$\begin{aligned} \text{Flux through that sphere} &= E \times \text{area of sphere} \\ &= E \times 4\pi r^2 \\ &= 4\pi k \cdot q(r) \end{aligned}$$

$$E = \frac{1}{4\pi r^2} \times (4\pi k) \times \left(\frac{r}{R}\right)^3 \times Q$$

$$\begin{aligned} \text{if } r = 0.15 \quad E &= \frac{1}{8.85 \times 10^{-12}} \times \left(\frac{0.15}{0.25}\right)^3 \times \frac{14 \times 10^{-6} \text{ C}}{4\pi(0.15)^2} \\ &= \cancel{3.42 \times 10^5} \text{ (N/C)} = \boxed{1.21 \times 10^6 \text{ N/C}} \end{aligned}$$

$$\begin{aligned} \text{if } r = 0.25 \quad E &= \frac{1}{8.85 \times 10^{-12}} \times (1)^3 \times \frac{14 \times 10^{-6} \text{ C}}{4\pi(0.25)^2} \\ &= \cancel{1.58 \times 10^6} \text{ (N/C)} = \boxed{2.01 \times 10^6 \text{ N/C}} \end{aligned}$$

$$\begin{aligned} \text{if } r = 0.5 \quad E &= \frac{4\pi k Q}{4\pi r^2} = \frac{9 \times 10^9 \times 14 \times 10^{-6}}{(0.5)^2} = \boxed{5.04 \times 10^5 \text{ N/C}} \end{aligned}$$