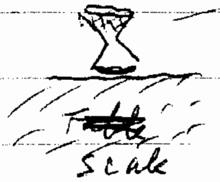
Week 9 Examples 9-2,3,

Probl. 9-4, 5, 6, 15, 16, 74 extra problem

9-4: System of pool balls + cue ball

The momentum of the system changes after one of the balls strikes the edge of the table, because some of the momentum is lost to the table.

9-5  → hour glass placed on a scale, sand on top.

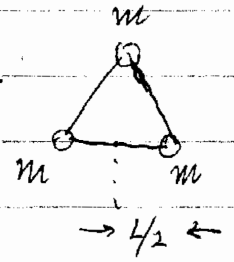
a) before sand hits bottom Scale reads less than the total weight of the hour glass, so as to allow for the amount of sand that is falling through the air

b) while sand is hitting bottom: scale reads more than the total weight, to allow for the momentum required to stop the falling sand

c) when all the sand is on the bottom

9-6: The more time the car ~~__~~ spends in getting crushed, the smaller is the average force of the impact.

9-15

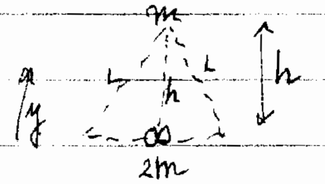


Equilateral triangle

center of mass?

c.of m of the two bottom masses is at L/2

combine that mass of the two bottom masses with the mass at the top



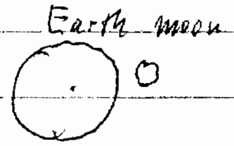
$$2m \times (y=0) + m\ (y=h) = 3m\ Y_{CM}$$

$$Y_{CM} = \frac{m}{3m} \cdot h$$

$$h^2 + \left(\tfrac{L}{2}\right)^2 = L^2 \qquad h^2 = \tfrac{3}{4} L^2 \qquad h = \sqrt{\tfrac{3}{4}}\ L$$

$$Y_{CM} = \frac{m}{3m} \cdot \sqrt{\frac{3}{4}} \cdot L = \boxed{\sqrt{\frac{1}{12}} \cdot L}$$

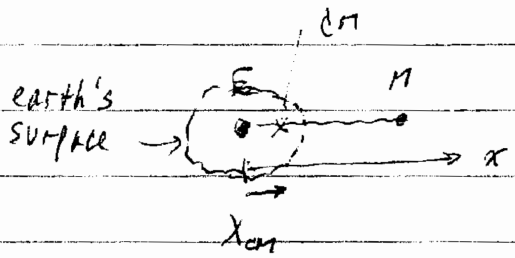
9-16



radius of orbit of moon = 0.385×10^6 km

mass of moon = 0.0735×10^{24} kg

mass of earth = 5.97×10^{24} kg



$$M_E \times (x=0) + M_M \times 0.385 \times 10^6\ \text{km} =$$

$$= (M_E + M_M) \times X_{CM}$$

$$X_{CM} = \frac{M_M}{M_E + M_M} \times 0.385 \times 10^6\ \text{km}$$

$$= \frac{0.0735 \times 10^{24}\ \text{kg}}{(5.97 + 0.07) \times 10^{24}\ \text{kg}} \times 0.385 \times 10^6\ \text{km}$$

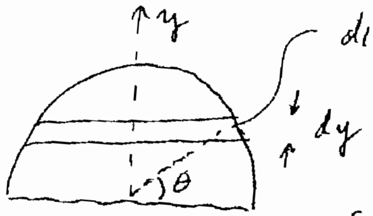
$$= \underbrace{\qquad\qquad}_{0.012}$$

$$\boxed{X_{CM} = 4.68 \times 10^3\ \text{km}}$$

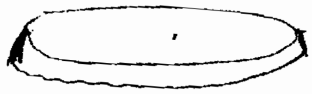
This is a ~~small~~ good fraction of the radius of the earth

6.37×10^6 m $= 6.37 \times 10^3$ km extra Probe

~~9~~ : CM of <u>one</u> half sphere



divide into (disc) slices of thickness dy



d(mass of each disc) = volume × density

letter d means differential or Δ

$\pi r^2 \cdot dy \; * \; \rho$

$r = R\cos\theta$

$y = R\sin\theta$

$dy = R\cos\theta\, d$

$\int y\, dm = Y_{CM} \cdot m$ ← basic def'n of c. of mass

$$\int_{\theta=0 \to \pi/2} R\sin\theta \; \pi \underbrace{(R\cos\theta)^2}_{r^2} \cdot \underbrace{R\cos\theta\, d\theta}_{dy} * \rho =$$

$$= R^4 \cdot \pi \cdot \rho \underbrace{\int_0^{\pi/2} (\cos\theta)^3 \cdot \underbrace{\sin\theta\, d\theta}_{-d(\cos\theta)}}$$

call $z = \cos\theta$

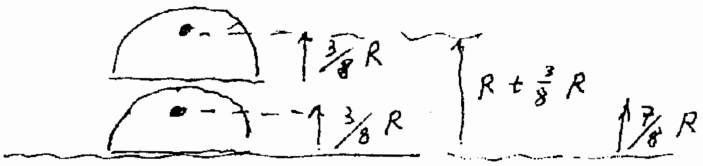
$$-\int_1^0 z^3 dz = \frac{1}{4} z^4 \Big|_0^1 = \frac{1}{4}$$

$\therefore \int y\, dm = R^4 \cdot \pi \cdot \rho \cdot \frac{1}{4}$

now $\int y\, dm = Y_{CM} * m$; $\rho = \frac{m}{vol} = \frac{m}{\frac{1}{2}\frac{4}{3}\pi R^3}$ $\therefore m = \frac{2}{3}\pi R^3 \cdot \rho$

$$Y_{CM} = \frac{\int y\, dm}{m} = \frac{R^4 \cdot \pi \cdot \cancel{\rho} \cdot \frac{1}{4}}{\frac{2}{3}\pi R^3 \cdot \cancel{\rho}} = \boxed{R \cdot \frac{3}{8}}$$

9-3 for both half spheres

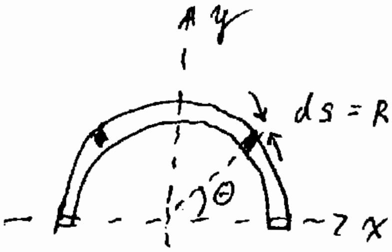


m = mass of each half sphere

$$m \cdot \frac{3}{8} R + m \times \left(R + \frac{3}{8} R\right) = Y_{Total} \times 2m$$

$$Y_{Total} = \frac{\frac{3}{8} R + \left(R + \frac{3}{8} R\right)}{2} = \frac{R + \frac{6}{8} R}{2} = \boxed{\frac{7}{8} R}$$

9-74



$ds = R\,d\theta$ $\quad dm = \lambda \cdot ds$; λ = mass/length

$\lambda = \dfrac{m}{\pi R}$

$y = R \sin\theta$

$\int y\,dm = \int R \sin\theta \cdot \lambda \cdot R\,d\theta$

$\int y\,dm = 2 R^2 \lambda \int_0^{\pi/2} \sin\theta\,d\theta$

to allow for the side to the left of the y axis

$= Y_{cm} * m = 2 \cdot R^2 \cdot (m/\pi R) \cos\theta \Big|_{\pi/2}^{0} = 2 R^2 (m/\pi R)$

$$Y_{cm} = \frac{\int y\,dm}{m} = \boxed{\frac{2}{\pi} R}$$