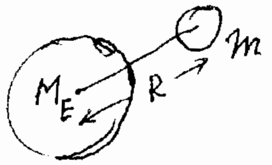
Week 7 8-1, 3, 4, 9, 49 Bonus 50 Bonus 59

8-1 Elliptic orbits are more general than strictly circular orbits. Also, Kepler found (observed) elliptic orbit.

8-3 G is the universal Gravit'l constant, units of $(N/(kg)^2) \times m^2$ while g is the acceleration of gravity at the surface of a planet $g = \frac{GM}{R^2}$

8-4 You are standing on earth. The mass very near you is not much, but there is a lot of mass far away from you. The total vector addition of the forces due to all the mass particulates on the earth is the same as if all mass were concentrated at the center of the earth

8-9 given moon period of the moon's orbit

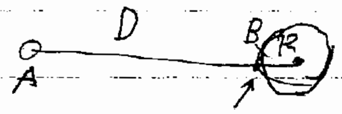


No, cannot obtain the moon's mass, because the moon's mass cancels when $\underbrace{\text{Gravit'l force}}_{\frac{\cancel{m} MG}{R^2}} = \cancel{\text{mass}} \times \text{centripetal Accel}$

8-49:

8-49 Meteor 1, striking the earth. (head-on) ($v_A = 2.1$ km/s

$D = 250{,}000$ km $= 2.5 \times ^8$ m



$(KE)_A + (V)_A = (KE)_B + (V)_B$

$$\frac{1}{2} \cancel{m} v_A^2 + \left(-\frac{\cancel{m} M_E}{D}\right) G = \frac{1}{2} \cancel{m} v_B^2 + \left(-\frac{\cancel{m} M_E}{R_E}\right) G$$

$M_E = 5.97 \times 10^{24}$ kg

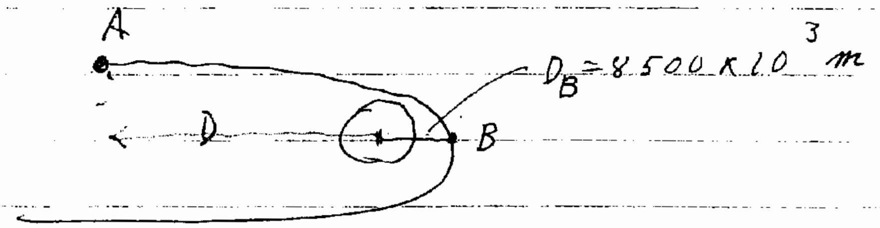
$M_E \times G = 3.98 \times 10^{14}$

$R_E = 6.37 \times 10^6$

$$\frac{1}{2} v_B^2 = \frac{1}{2} v_A^2 + \underbrace{\frac{M_E G}{R_E} - \frac{M_E G}{D}}_{6.09 \times 10^7}$$

$$\boxed{v_B = 1.12 \times 10^4 \text{ m/s}}$$

-49 cont'd
2d meteor



$$\frac{1}{2} v_A^2 - \frac{M_E G}{D} = \frac{1}{2} v_B^2 - \frac{M G}{D_B}$$

$$\frac{1}{2} v_B^2 = \underbrace{\frac{1}{2} v_A^2 + MG \underbrace{\left(\frac{1}{D_B} - \frac{1}{D}\right)}_{1.13 \times 10^{-7}}}_{4.52 \times 10^7}$$

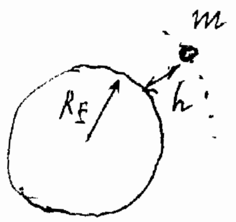
$$v_B^2 = \left\{ v_A^2 + 2 \times 4.52 \times 10^7 \right\}^{1/2}$$

$v_B = 9.51 \times 10^3$ m/s

Bonus 8-50

The energy needed to launch a satellite of mass $\underline{m}$ into circular orbit at an altitude of h

is $\dfrac{GM_E m}{R_E} \cdot \dfrac{R_E + 2h}{2R_E + 2h}$



U_E of satellite sitting on earth's surface

$$U_E = -\frac{GM_E m}{R_E}$$

U of sattellite in orbit $= U_o = -\frac{1}{2}\dfrac{GM_E m}{R_E + h}$

the $\frac{1}{2}$ is due the KE of satellite

$$U_o - U_E = -\frac{1}{2}\frac{GM_E m}{R_E + h} - \left(-\frac{GM_E m}{R_E}\right)$$

$$= GM_E m\left[\frac{-1/2}{R_E + h} + \frac{1}{R_E}\right]$$

$$\underbrace{}$$

$\dfrac{-\frac{1}{2}R_E + R_E + h}{(R_E + h)R_E}$ gives correct result.

Bonus 8-59

show that $\Delta U \cong mgh$

where $\Delta U = U(R_E + h) - U(R_E)$

$$\Delta U = -\frac{GM_E m}{R_E + h} + \frac{GM_E m}{R_E}$$

Binomial approx $(1+x)^p = 1 + px$

$$\Delta U = -\frac{GM_E m}{R_E(1 + h/R_E)} + \frac{GM_E m}{R_E} = \frac{GM_E m}{R_E}\left(\frac{1}{1 + h/R_E} - \frac{1}{R_E}\right)$$

$$\frac{1}{1 + h/R_E} = \frac{1}{1+x} = (1+x)^{-1} \quad \therefore p = -1 \text{ here}$$

where $x = h/R_E$ $\quad = 1 - x + x^2 - x^3 + \dots$