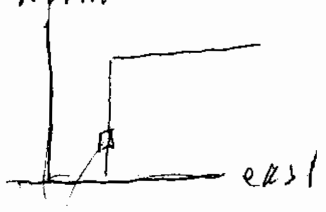
Week 5 6-6, 11, 22, 40, 54, 77 Bonus 71

6-6 The gravitational force of the sun on a planet does do no work on the planet, because the force and the displacement of the planet are perpendicular for a circular orbit. However if the orbit is elliptical, force and displacement of the planet are no longer $\perp$, and yes, the sun does work on the planet (sometimes positive) (sometimes negative)

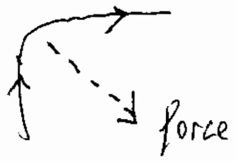
6-11



a) the kinetic energy $\underbrace{}_{}$ $\left(\frac{1}{2}mv^2\right)$ is the same on both the northward part of the trip as well as on the eastward part since the speed is the same ~~$\frac{1}{2}mv^2$~~

b) the magnitude of the momentum $|\vec{p} = m\vec{v}|$ has not changed, but the direction of $\vec{p}$ has changed. Therefore, the vector $\vec{p}$ has changed

c) Yes, a force acted on the truck as it is rounding the curve



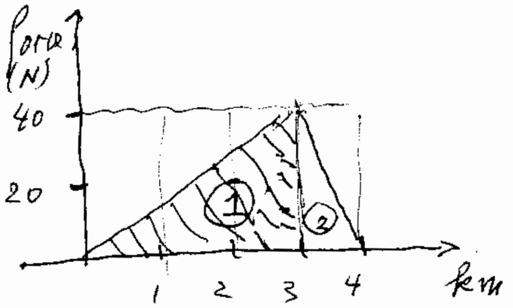
the average value of the force ~~=~~ $\times$ time to round the curve T

= impulse ~~...~~ $\vec{F} \cdot T$

= change in momentum $m\vec{v}_f - m\vec{v}_i$

No work has not been done on the truck

5-22 Total work done by force



work = area = $f \times$ (distance)

= area of $\Delta(1)$ + area of triangle 2

$= \frac{1}{2}(3 \times 40) + \frac{1}{2}(1 \times 40) = \frac{1}{2}(4 \times 40) = \boxed{80 \text{ Nkm}}$

6-53 : $F = a\sqrt{x}$ $a = 9.5\ N/\sqrt{m}$

a) work $x = 0 \to x = 3$ $= \int_0^3 F dx = a \int_0^3 x^{1/2} dx = a \times \frac{2}{3} x^{3/2}$

$= 9.5 \times \frac{2}{3} \times 3^{3/2} = \boxed{32.9 \text{ Nm}}$ Nm = J Joule

b) from 3 → 6 m $a \times \frac{2}{3} \cdot 6^{3/2} - a \times \frac{2}{3} \times 3^{3/2} = 93.1 - 32.9 = \boxed{60.2}$ J

c) answer $\boxed{77.9 \text{ J}}$

6-40 sunshine 1 kw/m²

solar collector Area = 15 m². How long to collect 40 kWh?

1 Gallon of Gasol-

1 kw/m² × Area of 15 m² = 15 kW = 15×10^3 Joules/s.

40 kwh = 40×10^3 Joules/s × 3600 s = 144×10^6 J

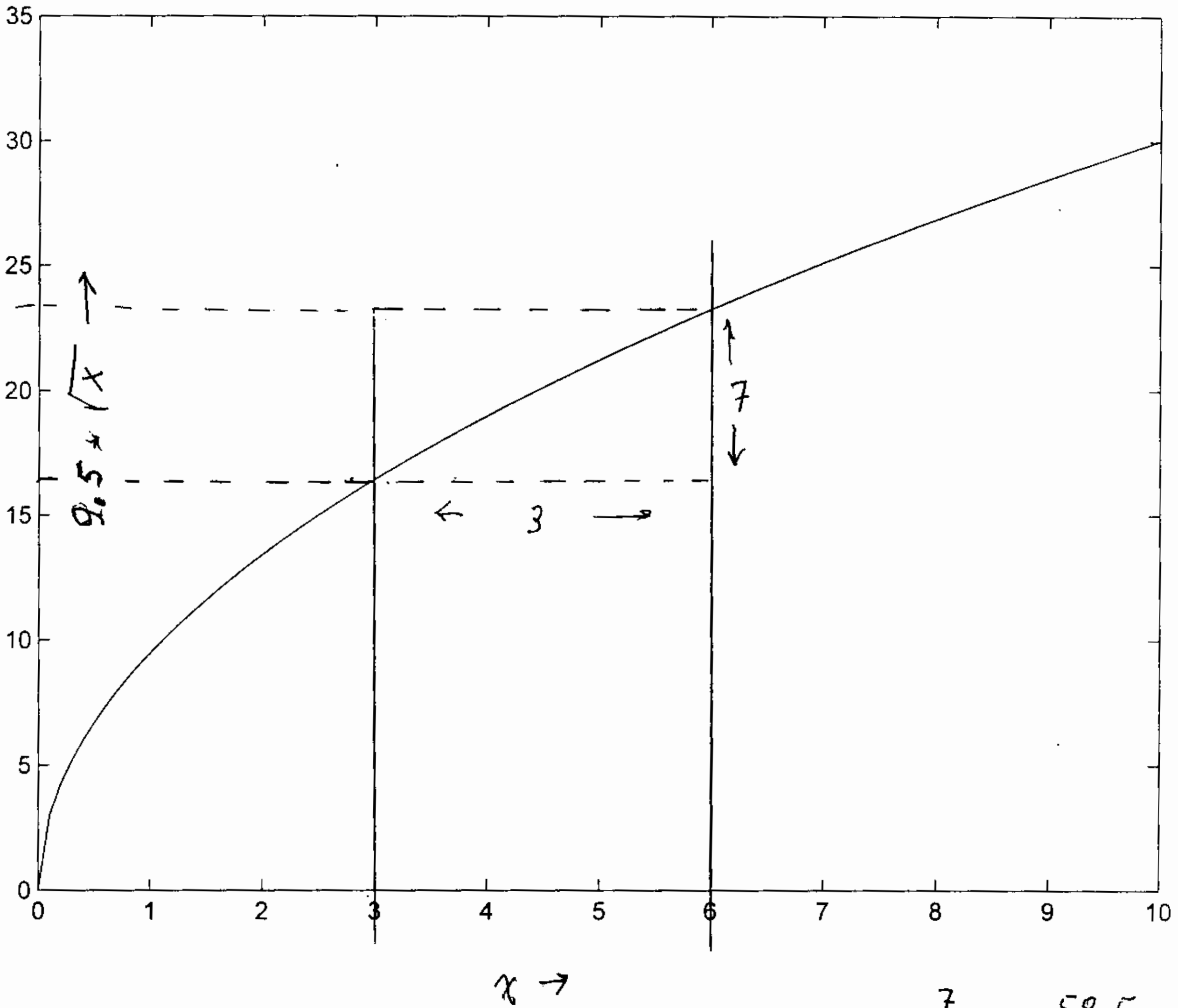
15×10^3 J/s × (time) = 144×10^6 J

$$\text{time} = \frac{144 \times 10^6 \text{ J}}{15 \times 10^3 \text{ J/s}} = 9.6 \times 10^3 \text{ s} = \boxed{2.7 \text{ hours}}$$

Problem 6.53 : Force $= a\sqrt{x}$ $a = 9.5\ N/m^{1/2}$

work done by force a) move from $x=0 \rightarrow x = 3\ m$

b) " " $x=3 \rightarrow x=6\ m$



$16 \times 3/2 = 24$

True area $0 \rightarrow 3 = 32.9$

$3 \rightarrow 6$ $16 \times 3 + \frac{\cancel{24}\ 7 \times 3}{2} = \cancel{84}\ 58.5$

True area $3 \rightarrow 6$ $= 60.2$

6-54. Force to stretch rubber band by distance x ③

$$F = F_0\left[\frac{L_0+x}{L_0} - \frac{L_0^2}{(L_0+x)^2}\right]$$

$$\int_0^x F\,dx = \text{Work}$$

change variables $y = \frac{L_0+x}{L_0}$ $\therefore F = F_0\left[y - \frac{1}{y^2}\right]$

$$\int_0^x F\,dx = \int_1^{1+\frac{x}{L_0}} F_0\left[y - \frac{1}{y^2}\right]dx$$

but $x = Ly - L_0$ $dx = L\,dy$

$$\text{work} = \int_1^{1+\frac{x}{L_0}} F_0\left[y - \frac{1}{y^2}\right]L\,dy = F_0 L\int_1^{1+\frac{x}{L_0}}\left[y - \frac{1}{y^2}\right]dy$$

$$\int y\,dy = \frac{1}{2}y^2 \quad ; \quad -\frac{1}{y^2}dy = \frac{1}{y}$$

$$\therefore \text{work} = F_0 L\left[\frac{1}{2}y^2 + \frac{1}{y}\right]_1^{1+x/L_0}$$

$$\boxed{\text{Work} = F_0 L\left\{\left(\frac{1}{2}\left(1+\frac{x}{L_0}\right)^2 + \frac{1}{1+x/L_0}\right) - \left(\frac{1}{2}1^2 + \frac{1}{1}\right)\right\}}$$

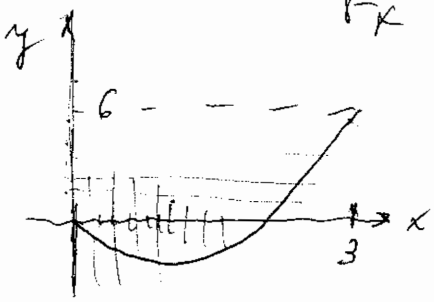
6-77

Partiil moves from origin $(0,0)$ to a point $P = (3,6)$
along the curve $y = ax^2 - bx$ $\quad a = 2m^{-1}$ $b = 4$

Force $= \vec{F} = (cxy)\hat{\imath} + (D)\hat{\jmath}$ $\quad c = 10\ N/m^2$ $D = 15N$

F_x

$2 \times 3^2 - 4 \times 3 = 18 - 12 = 6$ ok



? $U(x,y)$

$-U = \frac{1}{2} c x^2 y$

$\frac{\partial U}{\partial x} = -cxy$

$\frac{\partial U}{\partial y} = D - \frac{1}{2} c x^2$ $\quad \cancel{\frac{d}{dx}}\, dy$

$$\int \vec{F} \cdot (\hat{\imath}\, dx + \hat{\jmath}\, dy) = \int \underbrace{cxy}_{F_x}\, dx + \underbrace{D}_{F_y} * (dy)$$

$(a \cdot 2x - b)$

choose x as indep. variable :

$\frac{dy}{dx} \cdot dx = dy$

$$\int cxy\, dx + D \cdot \frac{dy}{dx}\, dx$$

$$= \int c\, x(ax^2 - bx) \cdot dx + D \int (2ax - b)\, dx$$

$$= ca \int x^3 dx - cb \int x^2 dx + 2Da \int x\, dx - bD \int dx = \int U(x)\, dx$$

$$\boxed{= ca \tfrac{1}{4} x^4 - cb \tfrac{1}{3} x^3 + 2Da \tfrac{1}{2} x^2 - bD x}$$

Bonus 6-71

power = $\cancel{P_0 t_0^2 \frac{1}{(1+t_0)^2}} = P_0 \frac{t_0^2}{(t+t_0)^2} = P_0 \frac{1}{(t/t_0+1)^2}$

work = $\int_0^\infty$ Power $\cdot dt = P_0 \int \frac{1}{(\frac{t}{t_0}+1)^2} dt$

new variable $\frac{t}{t_0} + 1 = u \qquad du = \frac{dt}{t_0}$

work = $P_0 \int_1^\infty \frac{t_0\, du}{u^2} = P_0 t_0 \left. \frac{1}{u} \right|_\infty^1 = \boxed{P_0 t_0}$