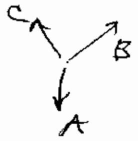
Week 2 ~~152~~ 123-5

3-2, 3, 9, 15, 26, 28, 46, 83, Bonus 80

3-2: Two vectors of equal magnitude can (yes) sum to zero, if their directions are opposite to each other
If the magnitudes are unequal, they cannot sum to zero

$\vec{A}, \vec{B}, \vec{C}$

3-3 Three vectors of equal magnitude can sum to zero
Yes, even if the magnitudes are unequal,
provided that $|\vec{A}-\vec{B}| \geq |\vec{C}|$
$|C|$ means: magnitude of vector $\vec{C}$



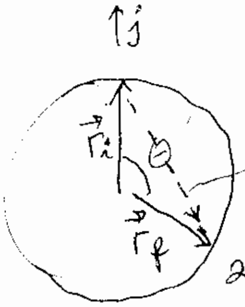
3-12 The vertical component of a projectile's velocity, at the peak of its trajectory is zero

3-13 At the peak of the trajectory of a projectile, the acceleration (vertical) is perpendicular to the velocity (horizontal)

3-15: Even though the magnitude of the velocity (speed) is constant, the direction changes, and hence there is an acceleration.

3:28 minute hand of clock has length $D = 5.5\,cm = \cancel{0.55\,m}$ $0.055\,m$

average velocity after 20 minutes



$\theta = \dfrac{360^\circ}{3} = 120^\circ$

dashed line $= \vec{r}_f - \vec{r}_i$

20 min = $\frac{1}{3}$ of an hour.
one hour = 360°

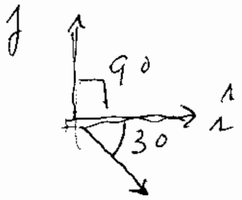
$\vec{r}_i = D * \hat{j}$

$\vec{r}_f = D\cos(120)\,\hat{j} + D\sin(120)\,\hat{i}$

$= -D\,\cancel{\cos}\,\sin 30\,\hat{j} + D\cos 30\,\hat{i}$

$= -D * 0.5\,\hat{j} + D * 0.866\,\hat{i}$

$-0.028\,\hat{j} + 0.048\,\hat{i}$



$\vec{r}_f - \vec{r}_i = 0.048\,\hat{i} - (0.055 + 0.028)\,\hat{j}$

$= 0.048\,\hat{i} - 0.083\,\hat{j}$

average vel $\vec{v} = \dfrac{\vec{r}_f - \vec{r}_i}{t} = \dfrac{0.048\,\hat{i} - 0.083\,\hat{j}\ m}{20\ min}$ m/min

$\boxed{\vec{v} = 0.238\,\hat{i} - 0.413\,\hat{j}\ \ cm/min}$

average velocity

3-46: Acceleration of the moon

$$|\vec{v}| = \frac{\text{Distance}}{\text{time}} = \frac{2\pi R = 2\pi(385 \times 10^3 \text{ km} \times 10^3 \text{ m/km}}{27 \text{ days} \times 24 \text{ h/d} \times 3600 \text{ s/h}}$$

$$= 2\pi \times \frac{3.85 \times 10^8 \text{ m}}{2.33 \times 10^6 \text{ s}} =$$

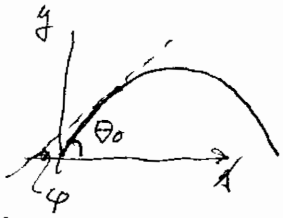
$$= 2\pi \times 0.165 \text{ m/s} \times 10^3$$

speed $= 1.037 \text{ m/s} \times 10^3$

$$\text{accel} = v^2/R = \frac{(1.037 \times 10^3)^2}{3.85 \times 10^8} = \boxed{2.80 \times 10^{-3} \text{ m/s}^2}$$

Bonus

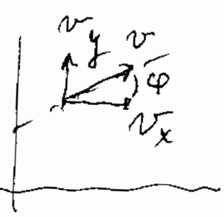
3-80: Eq 3.14: $y = x \tan\theta_0 - \frac{g}{2v_0^2 \cos^2\theta_0} \cdot x^2$



$$\frac{dy}{dx} = \tan\theta_0 - \frac{g}{v_0^2 \cos^2\theta_0} x = \text{slope} = \tan\varphi$$

Eq 3.10 $\left\{ \begin{array}{l} v_x = (v_0)_x = v_0 \cos\theta_0 \\ v_y = (v_0)_y - gt = v_0 \sin\theta - gt \end{array} \right\}$ slope $= \frac{v_y}{v_x}$

Show that the slope is compatible with the velocity Eq 3.10



$$\tan\varphi = \frac{v_y}{v_x} = \frac{v_0 \sin\theta_0 - gt}{v_0 \cos\theta_0} = \cancel{\tan\theta}$$

$$= \cancel{v_0} \frac{\sin\theta_0}{\cos\theta_0} - \frac{gt}{v_0 \cos\theta_0}$$

$t = \frac{x}{v_0 \cos\theta_0}$

$$\tan\theta_0 - \frac{g}{v_0^2 \cos^2\theta_0} \cdot (x = v_0 \cos\theta_0 * t)$$

agree

o.k.

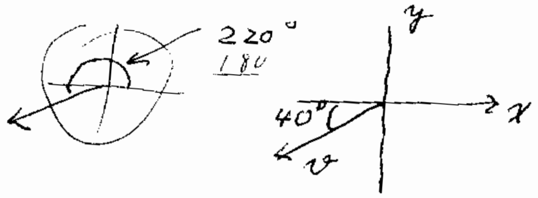
$$= \tan\theta_0 - \frac{g}{v_0 \cos\theta_0} \cdot t = \tan\theta_0 - \frac{gx}{(v_0 \cos\theta_0)^2}$$

agree

3-9 no, the subsequent motion is not strictly eastward. because, a brief acceleration eastward will deflect the northward motion. The briefer the acceleration westward the smaller the deflection (more later, Re: momentum)

3-15: When moving away from a straight line (like when going around a curve) you do accelerate towards the center of the curvature. The velocity direction is changed, even though the speed may not be changed, hence a non-zero acceleration occurs

3-26: speed = 18 m/s , direction



$v_x = -v \cancel{\sin}\cos 40° = -13.79$ m/s

$v_y = -v \sin 40° = -11.57$ m/s

also: $v_x = v\cos(220) = -13.79$ m/s
$v_y = v\sin(220) = -11.57$ m/s] same

3-28 see p. 2

3.83 : Asteroid being diverted by a rocket engine (5)

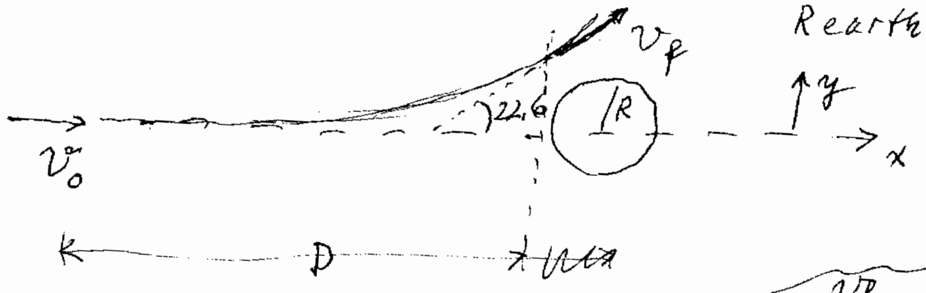
$v_{asteroid} = 21\ km/s = v_0$

Accel aster. $= 0.035\ km/s^2$ at right angle.

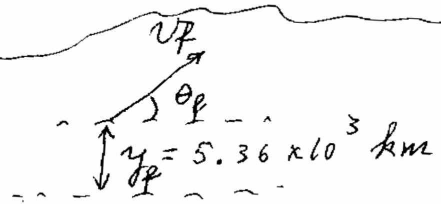
result: direction of motion will be changed by $22.6°$

Displacement at right angles $= 5.36 \times 10^3\ km$

$R_{earth} = 6.37 \times 10^3\ km$



method 1 : start from vertical displacement of 5.36×10^3 km



$D = (v_0)_x t$

$y_f = \cancel{(v_0)_y t_f} + \frac{1}{2}(a)_y t_f^2$ $\qquad$ t_f = time to come close to earth

$(v_0)_y = 0$

$y_f \to 5.36 \times 10^3\ km$; solve for t_f

$$\therefore\ t_f = \left(\frac{2 y_f}{a_y}\right)^{1/2} = \left(\frac{2 \times 5.36 \times 10^3\ km}{0.035\ km/s^2}\right)^{1/2} = 553\ s = \boxed{9.2\ min}$$

is larger than 4 min

final direction?

$$\frac{(v_f)_y}{(v_f)_x} = \frac{(a)_y t}{(v_0)_x} = \frac{0.035\ km/s \times 553 s}{21\ km/s} = 0.922 = \tan\theta_f$$

$\theta_f = \text{atan}(0.922) =$

$\boxed{\theta_f = 42.7°}$

incompatible with statement.

3.83 continued.

Asteroid being diverted

method 2: start from angular displacement, of 22.6°

$$\frac{(v_f)_y}{(v_f)_x} = \frac{a_y t}{(v_0)_x} = \frac{0.035\ km/s^2 \times t}{21\ km/s} \approx \tan(22.6°) = 0.416$$

$$\therefore t = \frac{21\ km/s}{0.035\ km/s^2} \times 0.416 = 249.7\ s = \boxed{4.16\ min}$$

yes, close to the 4 minutes assumed.

vertical displacement:

$$y = (v_0)_y t + \frac{1}{2} a t^2 =$$

$$= 0 + \frac{1}{2} \times 0.035\ km/s^2 \times (249.7\ s)^2 = \boxed{1{,}092\ km}$$

no, not compatible with claim of a vertical displacement of 5.36×10^3 km

so: The claims are incompatible with physics