123-5 Homework for week 1 Fall 2008

2-1, 6, 7, 9, 10 (proof) 15 51 57

2-1: average and instantaneous velocities are equal when there is ~~no~~ constant acceleration. Under special circumstances they can be equal even if accel is not ~~zero~~ constant, like in Fig 2.14 d

2-6 Of course, one can ~~be at~~ pass through position $x=0$, and still have a non-zero velocity.

2-7: Yes, one can have zero velocity (at an instant) and still have a non-zero acceleration. For example, a stone thrown vertically upwards at the instant the stone stops at maximum height.

2-9: No, eq. 2.10 is valid only when the acceleration is constant. The correct equation would be Here $a = bt$

$$v = v_0 + \int_{t_0}^{t} a \cdot dt = v_0 + \int_{t_0}^{t} (bt)\,dt = v_0 + b \cdot \frac{1}{2}\left(t^2 - t_0^2\right)$$

$$x = \int v\,dt = \int_{t_0}^{t} \left[\left(v_0 - \frac{1}{2} b t_0^2\right) + \frac{1}{2} b t^2\right] dt$$

Use $a = bt$

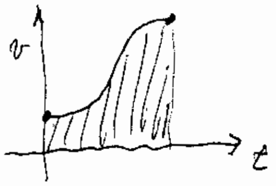
$$x = \left(v_0 - \frac{1}{2} b t_0^2\right)(t - t_0) + \frac{1}{2} \times \frac{1}{3} b \left(t^3 - t_0^3\right)$$

set $t_0 = 0$ then $x = v_0 t + \frac{1}{6} b t^3 = v_0 t + \frac{1}{6} a t^2$ correct.

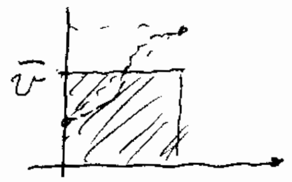
wrong: if a = constant expect $x = v_0 t + \frac{1}{2} a t^2$ wrong

2.10 In which graph in Fig 2.64 would $\bar{v} = \frac{1}{2}(v_0 + v_f)$?

answer: in graphs a and ~~d~~, because the area under the curve

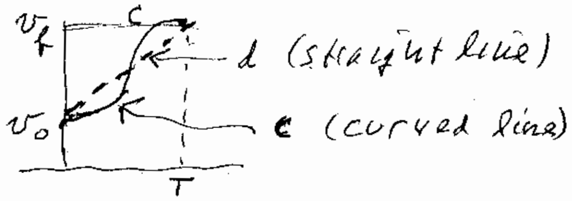


would be the same as



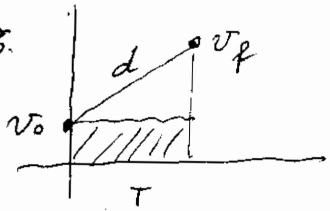
the area under the average velocity graph

mathem. proof! Area



i) area under curve (c) is the same as area under curve (d)

ii) area under curve (d) = ~~as~~



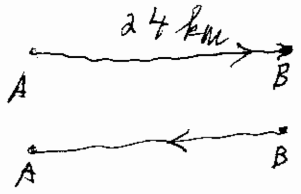
= area of rectangle $v_0 \times T$

plus area under triangle $= (v_f - v_0) \times T \times \frac{1}{2}$

the sum of both areas: $v_0 T + (v_f - v_0) T \cdot \frac{1}{2} =$

$$= \underbrace{\frac{1}{2}(v_f + v_0)}_{\bar{v}} T = \bar{v} T$$

2.15



in 2.5 h

in 1.5 h

a) Displacement at end of 2.5 h = 24 km (north)

b) average vel " " " " $= (x_B - x_A)/T_{AB} = \frac{24\text{ km}}{2.5\text{ h}} = \boxed{9.6\text{ km/h north}}$

c) average vel on homeward trip

$= (x_A - x_B)/T_{BA} = -24\text{ km}/1.5\text{ h} =$

$\boxed{\begin{array}{l} = -16\text{ km/h north} \\ = 16\text{ km/h south} \end{array}}$

d) displacement for entire trip: $\boxed{\text{is zero}}$

e) average velocity for entire trip $= (x_{final} - x_{initial})/\text{Total time}$

$= \boxed{0}$

since $x_{final} = x_{initial} = x_A$

2.51 : $x = bt^4$

i) instantaneous velocity = $v = \frac{dx}{dt} = \frac{d}{dt}(bt^4) = \boxed{4bt^3}$

ii) average velocity from $t=0$ to $t=t$ = $\int_0^t v\,dt \,/\, t$

$= \int_0^t \frac{dx}{dt}\cdot dt \,/\, t = [x(t) - x(0)]/t$

$= (bt^4 - 0)/t = bt^3 = \frac{1}{4} v$

2-57 : hockey puck: $v_i = 32$ m/s $= v_{initial}$

after slamming through a wall 35 cm thick $v_f = 18$ m/s

a) how much time spent going through snow?

assume deceleration is constant

$v_f^2 - v_i^2 = 2a(x_f - x_i)$

$(18\ m/s)^2 - (32\ m/s)^2 = 2 \times a \times 0.35\ m$

solve: $a = -700\,(m/s)^2 / 2 \times 0.35\ m = \boxed{-1{,}000.\ m/s^2}$

↑ is the de-acceleration

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\qquad t_i = 0$

$x_i = x_0 + v_0 t_i + \frac{1}{2}a t_i^2 = x_0$

$x_f = x_0 + v_0 t_f + \frac{1}{2}a t_f^2 = x_0 + 0.35\ m$

$0.35\ m = v_0 t_f - \frac{1}{2}(1{,}000)t_f^2 = 32 t_f - \frac{1}{2}(1{,}000)t_f^2$

solve quadratic equation for t_f

much Easier method: $v = v_0 + at$ $\qquad 18 = 32 + (-1000)t$

$t = \frac{14\ m/s}{1000\ m/s^2} = \boxed{0.014\,s}$

2.5 two definitions of average speed

a) $\bar{v}$ = average of instantaneous speed over a time interval

$$\bar{v} = \left(\int_0^t v_{inst}(t')\,dt'\right) \Big/ \int_0^t dt'$$

is the correct definition. See problem 2-10 for examples.

2.14 women's Olympic marathon

26 mi + 385 y in 2 h + 26 min.

$26 \text{ mi} \times 1609 \text{ m/mi} + 385 \text{ y} \times 0.9144 \text{ m/y} = 41834 + 352 = 42186 \text{ m}$

$2 \text{ h} \times 60 \text{ min/h} \times 60 \text{ s/min} + 26 \text{ min} \times 60 \text{ s/min} = 8760 \text{ s}$

$\text{speed} = 42186 \text{ m} / 8760 \text{ s} = 4.816 \text{ m/s}$