Units and dimensions

How many basic units are needed to define a useful, complete system of physical units?
The conventional answer is three (or four, if charge is defined separately).

\[
\begin{array}{c|c|c}
\text{m} & \text{L} & \text{t} \\
\text{kg} & \text{m} & \text{s} \\
\text{g} & \text{cm} & \text{s} \\
\end{array}
\]

SI or MKS

Cgs

But, speed of light \( c \) has dimensions \( L t^{-1} \).

And actually, the meter is presently defined by setting

\[
c = 299,792,458 \text{ m s}^{-1}, \text{ exactly}.
\]

So, the standard of time also sets the scale of length. Knowing this, we might well choose a new unit system where we set

\[
c = 1 \quad (\text{dimensionless}).
\]

Then lengths have dimensions of \( s \) -- no problem! Can do the same with mass, if we wish.

Only one unit is required, and the rest are chosen for convenience in measurement and calculation. They allow dimensional checks and analysis.

In atomic units (described on a typeset page distributed separately) we make the choices

\[
h = e = m_e = 1.
\]

This is a natural system for atomic and molecular calculations.
More often we will stick to conventional units, primarily SI units.

In electromagnetism, several systems are in use. For our defining equations we will take the Coulomb force between two charges,

\[ F = \alpha_{\text{em}} \frac{q_1 q_2}{r_{12}^2} \]

and the expression for the \( \mathbf{B} \) field surrounding a long straight wire,

\[ |\mathbf{B}| = \beta_{\text{em}} \frac{I}{d} \]

Here are some of the choices for \( \alpha_{\text{em}} \) and \( \beta_{\text{em}} \):

<table>
<thead>
<tr>
<th>System</th>
<th>( \alpha_{\text{em}} )</th>
<th>( \beta_{\text{em}} )</th>
<th>Used with mech. units</th>
</tr>
</thead>
<tbody>
<tr>
<td>electrostatic (esu)</td>
<td>1</td>
<td>( \frac{2}{c^2} )</td>
<td>Cgs</td>
</tr>
<tr>
<td>electromagnetic (emu)</td>
<td>( c^2 )</td>
<td>2</td>
<td>Cgs</td>
</tr>
<tr>
<td>gaussian</td>
<td>1</td>
<td>( \frac{2}{c} )</td>
<td>Cgs (usual modern choice)</td>
</tr>
<tr>
<td>Heaviside - Lorentz</td>
<td>( \frac{1}{4\pi} )</td>
<td>( \frac{2}{4\pi c} )</td>
<td>Cgs</td>
</tr>
<tr>
<td>Rationalized</td>
<td>( \frac{1}{4\pi \varepsilon_0} )</td>
<td>( \frac{2\mu_0}{4\pi} )</td>
<td>SI or MKS</td>
</tr>
</tbody>
</table>

\( \varepsilon_0 \) is \( 10^{-7} \text{C}^2 \text{F}^{-1} \text{m}^{-1} \), \( \mu_0 = 4\pi \times 10^{-7} \text{N}^{-1} \text{A}^2 \text{m}^{-1} \)

Thus, dimensions vary with the unit system! In gaussian units, \( E \) and \( \mathbf{B} \) have the same dimensions, making it easy to construct a 4-vector. Also, \( V \) always appears in as part of the ratio \( V/c \).

But the SI system has more practical units for measurable quantities. In gaussian units, resistance is in \( \text{s} \text{cm}^{-1} \), and current in statamperes---ugh!
To assist with Cgs $\leftrightarrow$ SI conversions, there's a table of replacements (not equalities) in the back of Jackson's "Classical Electrodynamics."

To summarize our discussion of EM units --

1) In Cgs units, $\alpha_m$ is dimensionless and $\beta_m = \frac{2}{c}$, so there are no truly new dimensions for electromagnetic quantities.

2) In SI units, we define the ampere for convenience when we pick $\beta_m = 2 \times 10^{-7} N A^{-2}$. This is equivalent to stating that the ampere is that current which produces a force of $2 \times 10^{-7} N$ per meter of length between two parallel conducting wires, held 1m apart.

In turn, using $\mu_0 \varepsilon_0 = c^2 = (299792458 \frac{m}{s})^2$, the Coulomb is defined in terms of the Ampere and mechanical dimensions via Coulomb's law, $F = \frac{1}{4\pi \varepsilon_0} \frac{Q_1 Q_2}{r^2}$.

We will frequently write this as $K \frac{Q_1 Q_2}{r^2}$. (non-standard)

For physics 6110, we will predominantly use SI units, and secondarily, Atomic Units.

**CAUTION**: Friedrich uses CGS units. Fortunately, most equations are invariant, except where charges or electromagnetic invariance are involved.