Physics 3201
Problem Set 8, problem 1 clarified on 10/23/13

Due: Thursday, October 24. Solutions will be posted at mid-day on Friday, October 25.

Notes: This problem set covers Section 3.3 of Griffiths. Please read it together with Section 3.4, which will be covered next.

1. In lecture we wrote the Fourier series in complex notation for a function $f(x)$ that is periodic on the interval $(-\pi \ldots \pi)$,

$$f(x) = \sum_{m=-\infty}^{\infty} c_m e^{-imx}, \text{ where}$$

$$c_m = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{imx} dx.$$

If the function $f(x)$ is real, the coefficients must occur in equal-magnitude pairs for negative and positive m, or more specifically with $c_{-m} = c_m{}^*$, where the asterisk denotes complex conjugation. Given this, and generalizing to a function with period $(-a \ldots a)$, show that the complex expansion above can be re-expressed for a real function $f(x)$ as a sum of sines and cosines, and show that the coefficients are given by the usual expressions:

$$f(x) = c_0 + \sum_{n=1}^{\infty} \left[c_n \cos\left(n\pi \frac{x}{a} \right) + d_n \sin\left(n\pi \frac{x}{a} \right) \right], \text{ where}$$

$$c_0 = \frac{1}{2a} \int_{-a}^{a} f(x) dx, \qquad c_n = \frac{1}{a} \int_{-a}^{a} f(x) \cos\left(n\pi \frac{x}{a} \right) dx,$$

$$\text{and } d_n = \frac{1}{a} \int_{-a}^{a} f(x) \sin\left(n\pi \frac{x}{a} \right) dx.$$

2. (15 points) Griffiths problem 3.13 (Problem 3.12 in 3rd Edition).

3. (15 points) Griffiths problem 3.16 (Problem 3.15 in 3rd Edition).

4. Griffiths problem 3.17 (Problem 3.16 in 3rd Ed.).

Honors: Use Neumann boundary conditions, but do so analytically rather than numerically. Repeat Example 3.5 of Griffiths, but instead of specifying the potential of the x=0 face as in the example, instead specify that it is a charged sheet with a constant surface charge density σ. Further, assume that the normal component of the electric field outside the surface (i.e., for x<0) is zero, as it would be if this were the inner surface of a conductor. You should be able to work out the coefficients completely, ending up with something resembling Eq. (3.52). If possible, plot $V(y,z)$ for x=0 using a program like Matlab or Mathematica. Could the surface at x=0 actually be a conductor, or would it have to be an insulator to be consistent with your results?

We will meet next on October 25 at 1:30 PM.