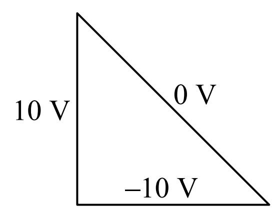
Physics 3201
Problem Set 7

Due: Thursday, October 17. Solutions will be posted at mid-day on Monday, October 21.

Notes: Because we are not yet ready for problems on Griffiths Section 3.3, this is another short problem set, although an unusual one, dedicated entirely to the numerical “relaxation” or Gauss-Seidel method for solving Laplace’s equation. For next week, please carefully read Section 3.3 on solutions via separation of variables.

1. *(15 points) Relaxation method for Laplace’s equation.* Use the numerical relaxation method in two dimensions to evaluate the potential for a 45-90-45 right triangle with electrodes along all three sides, as sketched below. (In actual 3D space, you can think of things as extending indefinitely along the z axis.) Assume the left side is held at 10 V, the bottom at -10 V, and the diagonal at 0 V. Note that it really doesn’t matter which values are assigned to the vertices, as these points are never actually used for calculations in the Gauss-Seidel method. Use an array with 20 points along the vertical edge and 20 points along the horizontal edge (a square 20×20 array will work fine, if you confine the actual calculation to a triangular region). You can accomplish this calculation using a slightly modified version of the Mathcad or C programs available in the “Resources” section of the course web page, or write a similar program of your own devising in Mathematica, MATLAB or another programming environment.

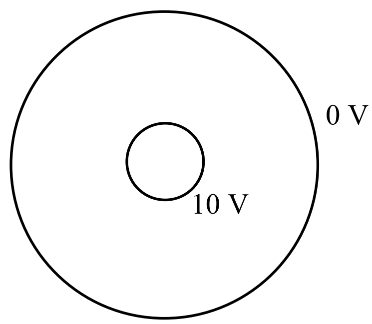


   Run your program until it has converged adequately. An easy way to judge convergence is to calculate the “maximum difference” for each iteration as in the sample programs; that is, the largest change between successive iterations for any point in the array. Verify explicitly that you have iterated until this maximum difference is less than 10^{-5}. For what value of the relaxation parameter w do you obtain stable results with a minimum number of iterations? What do you see if you make the value too large? (Note that the Mathcad sample program, with its contour plot, is very useful for visualizing the results — this could also be done easily in Matlab or Mathematica.)

2. *(15 points)* Now use your program to explicitly verify the additivity property of Laplace’s equation. Solve the configuration in Problem 1 for two simpler problems, in which only one of the electrodes at a time is non-zero. First set the left side to 10 V and the other two sides to zero, then solve again with the bottom set to -10 V and the other two sides at 0 V. Show that the sum of these two solutions is identical to the full solution from Problem 1 within your numerical accuracy. (Note that if you iterate

until the maximum difference between iterations is 10^{-5} V, the absolute numerical accuracy is considerably worse, about 10^{-2} V.) You can compare the solutions either by subtracting the sum from the full solution using a computer, or by manually checking the first 20-30 entries.

Honors: If you are taking the course for honors credit, try a variation with cylindrical symmetry: solve in cylindrical coordinates for the potential between two concentric cylindrical conductors with radii 5 cm and 20 cm, held at 10 V and 0 V as sketched below. Use an array in (r,ϕ) with a step size of 1 cm in r, and $2\pi/20$ in ϕ (that is, angular steps of 18°). How does the "nearest-neighbor" averaging have to be modified to take into account the r-dependent size of the differential elements in cylindrical coordinates? Because you can easily find the exact solution using Gauss' law, you can check the correctness of your method.



Once you have things working, change the potential of the inner cylinder to $V = 5\cos\phi$ V to make things a little less boring. Incidentally, the next meeting to discuss honors problems and related topics will have to be rescheduled because one of the students has a conflict.