Physics 3201
Problem Set 5, with minor corrections to problem numbers on 9/28/13

Due: Thursday, October 3. Solutions will be posted on October 4.

Notes: This problem set covers Sections 1.1–1.5 and 2.1–2.5 of Griffiths. You should be reading Sections 3.1–3.2, which will be discussed next in lecture.

Exam 1: The first hour exam is scheduled for Thursday, October 10, from 9:30-10:45. It will cover the subjects corresponding to Problem Sets 1–5, most of which are discussed in Chapters 1 and 2 of Griffiths. No notes are permitted, although you will be provided with copies of all of the formulas on the inside covers of Griffiths.

1. ***Deferred from Problem Set 5***: Griffiths, problem 1.44 (problem 1.43 in 3rd Ed.).

2. Griffiths problem 1.47 (problem 1.46 in 3rd Ed.) Note that part (a) was done in lecture, which should make it pretty easy! In addition to the stated problem, generalize the concepts of part (b) to show that for a collection of point charges q_i with charge distributions represented by delta functions, the expression $U = \frac{1}{2}\int \rho V'\, d\tau$ for the electrostatic energy is equivalent to the customary expression for point charges,

$$U = \tfrac{1}{2}\sum_i q_i(\mathbf{r_i})\, V_i'(\mathbf{r}_i)\,.$$

3. Griffiths, Problem 1.64 (new in 4th Ed.). Feel free to perform the integration by table lookup or by use of Mathematica.

4. (Based on Ohanian problem 5.21). Consider a two-conductor capacitor consisting of two thin concentric spherical shells of radius a and b, where $b > a$.

   (a) Suppose that the inner shell has potential $V_a = 0$ and the outer shell has potential $V_b = V_0$, both measured relative to infinity. What are the charges q_a and q_b on the shells?

   (b) If the potentials are reversed so that $V_a' = V_0$ and $V_b' = 0$, what are the new values q_a' and q_b' of the charges on the shells?

   (c) Show that your results are consistent with *Green's reciprocation theorem*, which states that if the potentials V_i of a set of conductors labeled by i are altered, the new potentials and charges are related to the original ones by the expression

$$\sum_i q_i V_i' = \sum_i q_i' V_i\ .$$

5. Griffiths, Problem 2.39 (problem 2.36 in 3rd Ed.)

6. The idea of a single-conductor capacitor is briefly mentioned by Griffiths on p. 105, and was described as a limiting case in lecture. To see that this makes sense, consider a conducting sphere of radius R that has a potential V_0 relative to infinity.

   (a) What is the charge Q on the sphere, and what is its capacitance using the definition $C \equiv Q/V$?

   (b) By finding the electric field everywhere, directly calculate the total stored energy U in the electric field. Repeat and double-check your calculation by working with the potential and the charge instead of the field.

(continued)

(c) Show that the total stored energy U can also be expressed as $\frac{1}{2}CV^2$, consistent with the idea that this sphere behaves just like a "normal" capacitor.

Honors: Problem 2.56. In addition to the stated problem for the sun, find the equivalent gravitational energy for the earth. If the earth was formed from an accumulation of small rocks and dust that started with large separations but eventually consolidated due to gravitational attraction, how hot would the resulting proto-planet be just after its formation, assuming that energy loss from radiation during the formation process could be neglected? To make your estimate, use an averaged value of $C_p = 1000$ J/(kg·K) for the specific heat of the earth. We will meet next on Friday, October 11.