Physics 3201
Problem Set 3

Due: Thursday, September 19. Solutions will be posted on September 20.

Notes: This problem set covers Sections 2.2 and 2.3 of Griffiths. You should be reading them, along with Sections 1.4, 2.4, and 1.5.

1. Griffiths, Problem 2.16. Verify directly that the field you find for $s < a$ satisfies the differential form of Gauss' law, by using the formula on the inside cover of Griffiths to evaluate the divergence in cylindrical coordinates.

2. A thick slab with uniform charge density ρ extends between the planes $z = -L/2$ and $z = L/2$. It extends indefinitely in the xy plane. Calculate (a) the electric field everywhere, and (b) the potential $V(z)$ for positions above the plane, $z > L/2$.

3. The "screened Coulomb" or Yukawa potential approaches that of a point charge at small r but quickly diminishes for large r:

$$V(r) = \frac{qe^{-\alpha r}}{4\pi\varepsilon_0 r}.$$

(a) What is the corresponding electric field $\mathbf{E}(r)$? Note that formulas for the vector differential operators in spherical coordinates are given in the inside front cover of Griffiths. We will discuss how these formulas can be obtained next week.

(b) What is the total charge Q contained within a sphere of radius r? Show that it goes to zero as $r \to \infty$, indicating that the total "screening" charge distributed through the volume away from the origin is $-q$.

(c) A modified version of this potential can be used to model the potential seen by an excited electron in a helium atom, where the inner electron provides r-dependent shielding. At small r, the potential is that of a point charge $+2e$ due to the two protons in the nucleus, but as $r \to \infty$ the potential should approach that of a point charge $+e$ (from the nucleus combined with the inner electron). Find a simple modification of the Yukawa potential that behaves this way.

4. Griffiths, problem 2.20.

5. Griffiths, problem 2.25(b).

6. Griffiths, problem 2.30.

Honors: We will meet next on Friday, September 27. Here's this week's problem:

According to the classical "plum pudding" model of J.J. Thomson, the helium atom consists of a cloud with a positive charge $2e$ uniformly distributed over a spherical volume, inside which two electrons are free to move. Suppose that the radius of the cloud is 0.5 Å and that the two electrons are symmetrically placed with respect to the center.

(a) What is the equilibrium separation of the electrons, at which their repulsion from one another is balanced by their attraction to the positive charge center?

(b) What is the frequency of *small* oscillations of the electrons along the radial direction? Assume that the electrons move in phase (with equal displacements), and make appropriate lowest-order approximations.

(c) What is the corresponding frequency for out-of-phase motion, where the displacements are equal and opposite? By the way, some of you may recognize that the motions in parts (b) and (c) are the two *normal modes* of vibration in the coupled system.