Electronics - Physics 3150, Eggleston is primary text.

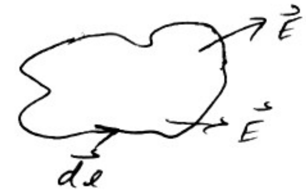
Read Ch. 1, start Ch. 2.

I. Resistors & dc Networks

A. Voltage or potential, V

We almost never use $\vec{E}$ directly, Instead electronics is expressed in terms of

$$V = -\int_a^b \vec{E}\cdot d\vec{\ell} \quad (\text{in Volts}),$$

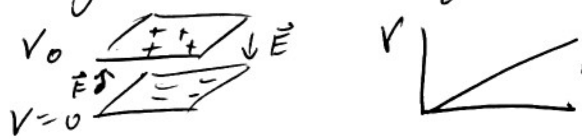


so $\vec{E} = -\vec{\nabla} V$

Since $\vec{\nabla}\times\vec{E} = 0$ (if $\frac{\partial \vec{B}}{\partial t} = 0$), the Voltage between two points a and b is path-independent, and $\oint \vec{E}\cdot d\vec{\ell} = 0$ around a closed loop.

The zero is arbitrary, and is often taken to be at earth ground, designated by $\overline{\underline{\perp}}$. Most instruments with ac line power plugs have one terminal grounded internally.

The voltage difference between two points describes the work needed to move a positive charge, according to $dW = q\,dV$ (+ if moving to higher V)



need to exert a force $\vec{F} = -q\vec{E}$ to move charge upwards.

B. Charge q or Q is in Coulombs,

$$q_{electron} = -e \cong -1.6\times10^{-19}\,C$$

C. Current I = charge/unit time, $\frac{dQ}{dt}$

Measured in Amperes (A)

So voltage is measured between two points; current flows through an area.

Physical principles for basic dc circuits:

1) Conservation of charge
2) Electrostatic force is conservative — potential change is path-independent.

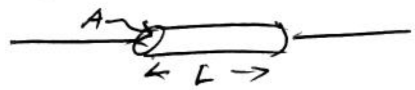
} will rephrase

3) Electromagnetism is described by linear equations
$\Rightarrow$ Can use superposition principle: sum of solutions is a solution.

4) Conductor (or wire) $\equiv$ equipotential: Every point on or in a conductor has the same voltage.

5) (Approximate) Ohm's "law" - For a resistor,

$$V = IR$$

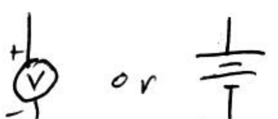


$\vec{J} = \sigma \vec{E}$, σ = conductivity
$\vec{J}$ = current density / unit area.

$I = JA$, $|V| = EL$, so $R = \frac{1}{\sigma}\frac{L}{A}$

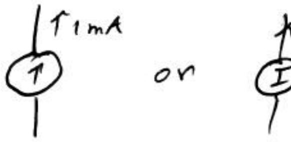
or $R = \rho \frac{L}{A}$, $\rho = \frac{1}{\sigma}$ = resistivity ($\Omega \cdot m$)

Schematic diagrams

a)  : Constant voltage source. May be referred to ground as zero of potential,

often used for a battery

 is very common.

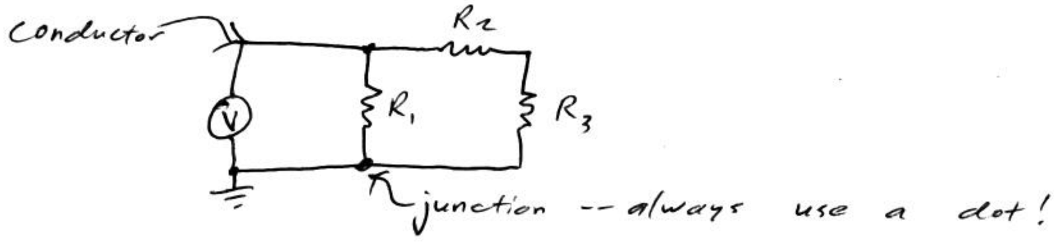
b)  : Constant current source; arrow points in direction that (+) charges would flow.

c)  R : Resistor

d)  : Capacitor (soon) $\left(V = \frac{Q}{C}\right)$

e)  : Inductor (soon)

Next -- codify rules so that we can solve problems like,



From (1), we can write a simple rule for any junction or "node" that isn't building up a net charge:

In time Δt, $\Delta q_{in} = \Delta q_{out}$

or $\left(\frac{dq}{dt}\right)_{in} = \left(\frac{dq}{dt}\right)_{out}$

or $I_{in} = I_{out}$

From this we get a useful rule if we define some consistent sign choices:

(a) Current is positive from (+) to (−).

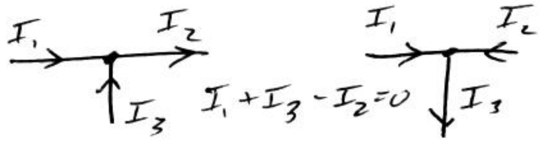
(b) For a resistor, $\Delta V = -IR$ in direction of current, $+IR$ the other way (i.e., must <u>push</u> charges through it.)

(c) For a voltage source V_s, $\Delta V = +V_s$ if traversed from the (−) to the (+) terminal, $-V_s$ the other way.

(d) At a junction, pick direction of current arrow in a new branch in any convenient direction, but then use it <u>consistently</u>.

We can now write Kirchoff's First Law (or node theorem),

> At any junction, the sum of all currents is zero or
>
> $$\sum_n I_n = 0$$
>
> $I_n = (+)$ for arrow into junction
> $= (-)$ " " out of "



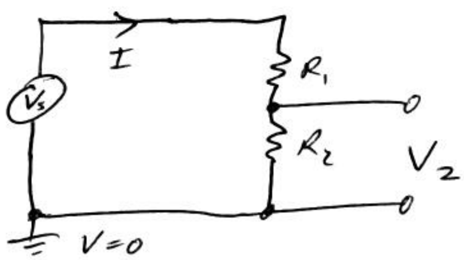
$I_1 + I_3 - I_2 = 0$

$I_1 + I_2 - I_3 = 0$

From (2) we have Kirchoff's Second Law (or loop theorem),

> Sum of all potential changes around any closed loop in the circuit is zero

Let's use this stuff:



Note I must be same everywhere, so only 2nd law needed:

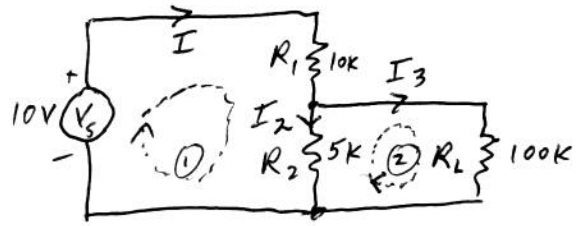
$$V_s - IR_1 - IR_2 = 0$$

also $V_2 = IR_2$ (ohm's law)

So $I = \dfrac{V_s}{R_1 + R_2}$

and $$\boxed{V_2 = V_s\left(\frac{R_2}{R_1 + R_2}\right)}$$ "Voltage divider" (worth remembering result!)

Note that if a measuring device is connected, the circuit is modified. With a load restance we have, for example,



Find V_2 = Voltage at load.
Since $V_2 = I_3 R_L$ (or $I_2 R_2$), find I_3:

At the node, $I = I_2 + I_3$ (a)

Loop (1): $V_s - IR_1 - I_2 R_2 = 0$ (b)

Loop (2): $-I_3 R_L + I_2 R_2 = 0$ (c) (so $V_2 = V_3$)

Add: $V_s - IR_1 - I_3 R_L = 0$

Use (a): $V_s - I_2 R_1 - I_3 (R_1 + R_L) = 0$

Use (c): $I_2 = I_3 \frac{R_L}{R_2} \Rightarrow V_s - I_3\left(\frac{R_1 R_L}{R_2} + R_1 \frac{R_2}{R_2} + R_L \frac{R_2}{R_2}\right) = 0$

So $V_2 = I_3 R_L = V_s \frac{R_L R_2}{R_1 R_L + R_1 R_2 + R_2 R_L}$

$$V_2 = V_s \frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_L}}$$ ← error term rel. to simple divider

And here, $V_2 = \frac{10\ (5000)}{10^4 + 5000 + \frac{50 \times 10^6}{10^5}}$

$V_L = V_2 = 3.23\,V$, vs. 3.33 for no load.

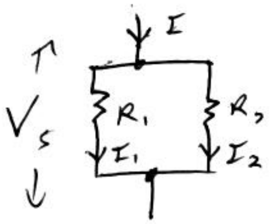
So to have a "stiff" divider that minimizes load sensitivity, need large R_L ("high impedance"). Specifically, need

$$\frac{R_1 R_2}{R_L} << R_1 + R_2 \quad \text{or} \quad R_L >> \frac{R_1 R_2}{R_1 + R_2} \leftarrow = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

For more complex networks Cramer's rule can be used for the simultaneous equations. Computers help greatly, either for symbolic solutions (Mathematica, Maple, etc.) or numerical results (Matlab for general work, SPICE specifically for simulation of circuits).

Quite often, circuit analysis can be simplified by looking for <u>series</u> or <u>parallel</u> combinations, and replacing them with their equivalent resistances:

same I $\Rightarrow$ $R_{eq} = R_1 + R_2$ <u>series resistances add</u>



same V $\Rightarrow$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \left(+ \cdots\right)$ <u>parallel resistances add reciprocally</u>

Proof: $V_s = V_1 = I_1 R_1 = V_2 = I_2 R_2$

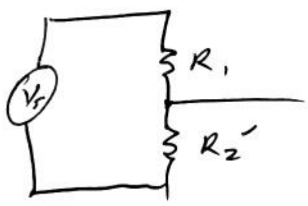
so $I_2 = I_1 \frac{R_1}{R_2}$ ("current divider")

and $I = I_1 + I_2 = I_1 \left(\frac{R_2 + R_1}{R_2} \right)$

Finally, $R_{eq} = \frac{V_s}{I} = \frac{I_1 R_1}{I} = \frac{R_1 R_2}{R_1 + R_2}$

Going back to p. 5, we recognize the condition on R_L as $R_L >> R_1 // R_2$, where $R_1 // R_2 \equiv \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$.

We could have solved the problem much more easily by replacing R_2 & R_L with $R_2' = R_2 // R_L$:

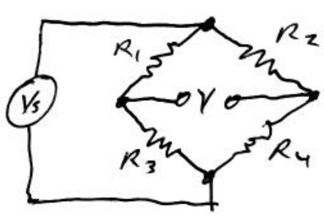


$V = \frac{R_2'}{R_1 + R_2'} V_s$, $R_2' = \left(\frac{1}{R_L} + \frac{1}{R_2}\right)^{-1}$

$= \frac{4.76}{10 + 4.76}(10V)$ $= \left(\frac{1}{100} + \frac{1}{5}\right)^{-1}$ kΩ

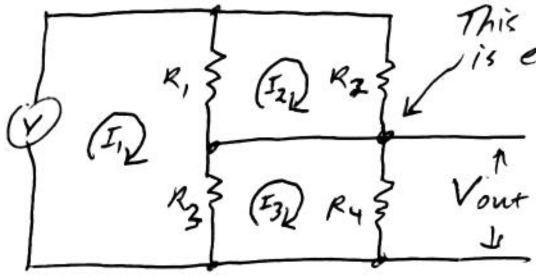
$= 3.22$ V ✓ $= \frac{100}{21}$ kΩ $= 4.76$ k

Sometimes this simplification is not possible, though —



$V = ?$ ("Wheatstone bridge" if a load is connected)

In such cases, or whenever a systematic algorithmic approach is desired, the "mesh loop" method works well:



This yields just enough undetermined parameters, one per loop.

$$\text{(1)}\quad V - (I_1 - I_2)R_1 - (I_1 - I_3)R_3 = 0$$

$$\text{(2)}\quad (I_1 - I_2)R_1 - I_2 R_2 = 0$$

$$\text{(3)}\quad (I_1 - I_3)R_3 - I_3 R_4 = 0$$

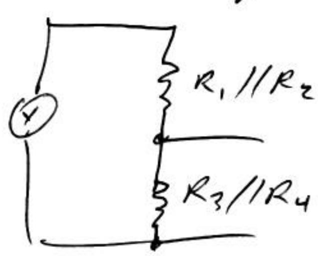
Solve for I_1, I_2, I_3 using (1)+(2), (1)+(3), or with Mathematica, Maple, etc.

Then use $V_{out} = I_3 R_4$.

For example, if $R_1 = R_2 = 2K$, $R_3 = R_4 = 6K$),

$$V_{out} = \frac{3}{4} V.$$

This example, though, is another trivial series-parallel combination,



$$V_{out} = \frac{R_3 // R_4}{R_1 // R_2 + R_3 // R_4}$$

If $R_1 = R_2 = 2K$, $R_1 // R_2 = \left(\frac{1}{2} + \frac{1}{2}\right)^{-1} K = 1K$

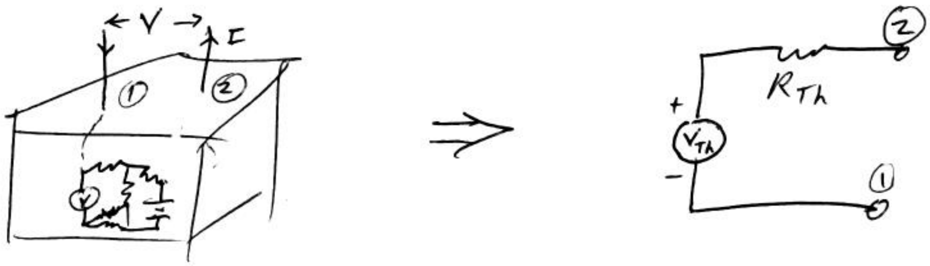
If $R_3 = R_4 = 6K$, $R_3 // R_4 = \left(\frac{1}{6} + \frac{1}{6}\right)^{-1} K = 3K$,

and $V_{out} = \frac{3}{1+3} V = \frac{3}{4} V$ ✓

By contrast, the Wheatstone bridge with a load has five resistors and three loops, and is not so simple.

Thévenin equivalent circuit

Any two-terminal network of resistors and voltage and current sources looks exactly like an equivalent circuit with a single voltage source and a series resistor.



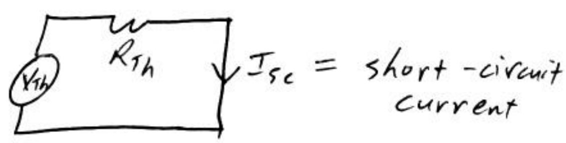
This follows from the facts that ① All of these devices are linear, and ② superposition principle applies. At the terminals, $V = V_{Th} + R_{eq} I$ because it's a linear superposition of the internal V's and IR's (and I's, for current sources):

$$V = \sum a_i I_i + \sum b_i V_i \quad , \quad I = \sum I_i .$$

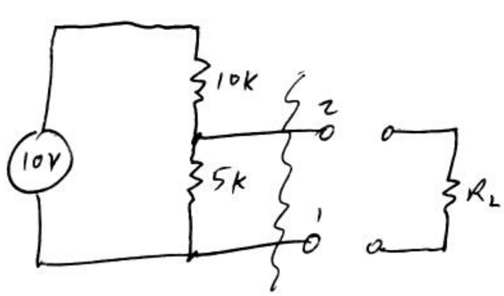
How do we find the equivalent circuit? In principle it's easy:

V_{Th} = open-circuit voltage $= V_{oc}$

and $R_{Th} = \dfrac{V_{Th}}{I_{sc}} = \dfrac{V_{oc}}{I_{sc}}$



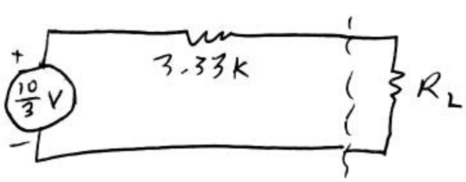
For the example circuit on p. 5,



$$V_{oc} = \frac{5}{15}(10\,V) = \frac{10}{3}V = V_{Th}$$

$$I_{sc} = \frac{10\,V}{10\,k\Omega} = 1\,mA$$

$$\text{So } R_{Th} = \frac{\frac{10}{3}\,V}{10^{-3}\,A} = 3333\,\Omega$$

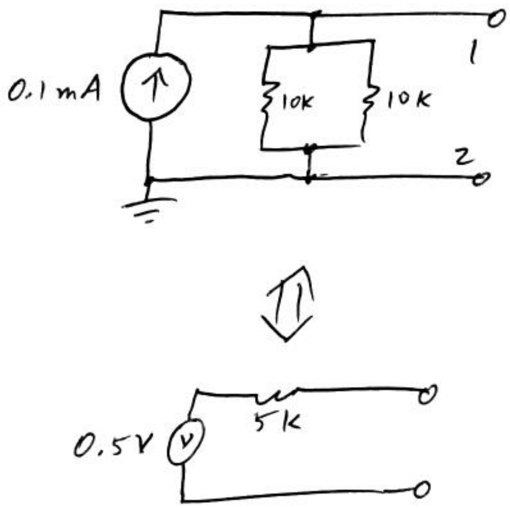


Now $V_L = \dfrac{R_L}{R_L + R_{Th}} V_{Th}$. With $R_L = 100k$,

$$V_L = \frac{100K}{103.33K}\left(\frac{10}{3}V\right)$$

$$= 3.23\,V \checkmark$$

Example with a current source:



First, note $10k \,//\, 10k = 5k$

so $V_{oc} = V_{Th} = (0.1\,mA)(5k\,\Omega) = 0.5\,V$

$I_{sc} = 0.1\,mA$, obviously.

and $R_{Th} = \frac{0.5\,V}{10^{-4}\,A} = 5000\,\Omega$

For example, with a 5k load resistor, $V_L = 0.25$, either way

Incidentally, in the lab it's often not safe to measure I_{sc}. If not, just measure V_L and I_L for at least two values of R_L. This is sufficient to find slope and intercept of V vs. $I \Rightarrow R_{Th}$ and V_{Th}.

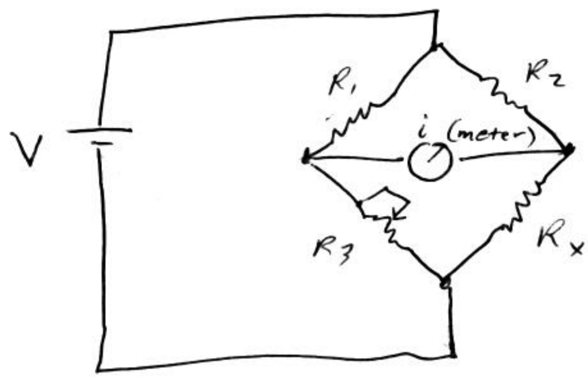
Every linear circuit also has a Norton equivalent circuit, though this is less useful in practice ---

$I_N = I_{sc}$, and

$V_{oc} = I_N R_N \Rightarrow R_N = \frac{V_{oc}}{I_{sc}} = V_{Th}$.

For ac circuits driven at a single frequency ω, all of this works even for capacitors and inductors if we replace resistance by the complex impedance --- will do this very shortly.

A non-trivial resistor circuit mentioned in the lab writeups is the <u>Wheatstone Bridge</u>,



$$\frac{R_1}{R_1+R_3} = \frac{R_2}{R_2+R_x} \quad \text{if } i=0.$$

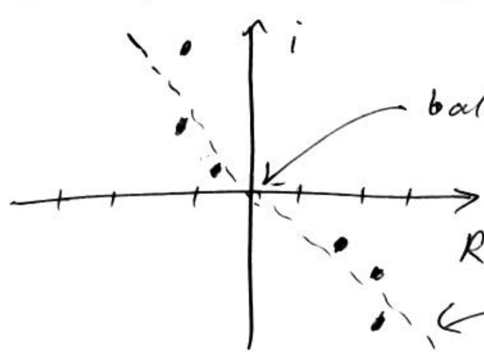
$$\frac{R_1}{R_2} = \frac{R_1+R_3}{R_2+R_x}$$

$$\cancel{R_1R_2} + R_1R_x = \cancel{R_1R_2} + R_2R_3$$

The most common use is to measure resistances. Adjust R_3 until $i=0$. Then we have a pair of voltage dividers, and

$$\frac{R_1}{R_2} = \frac{R_3}{R_x} \quad ; \text{ solve for } R_x$$

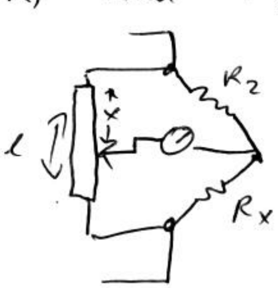
Alternatively, set close to balance and measure i. We can then use it to find the "fine" correction,



balance point

Find slope by solving for i in circuit above, and experimentally by fitting line to data.

Actual implementation often uses a slide wire for R_1 and R_3 --- at balance,



$$\frac{R_1}{R_1+R_3} = \frac{x}{\ell} = \frac{R_2}{R_2+R_x}$$

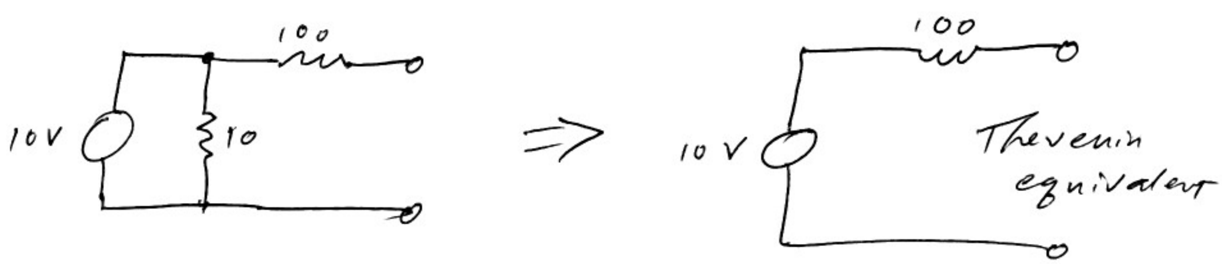
Power in dc circuits is dissipated only in the resistors, where

$$P = IV = I^2 R = \frac{V^2}{R}.$$

Be careful when using equivalent circuits:

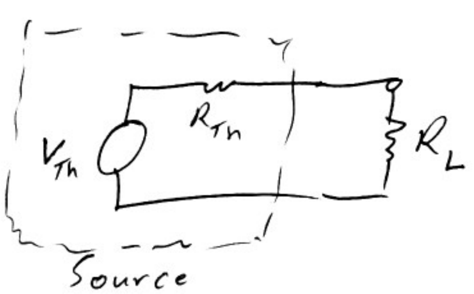
1) They *will* deliver the same power to an external load, but
2) They will *not* dissipate the same internal power, in some cases.

Example:



$P_{int} = 10\,W$ $\qquad$ $P_{int} = 0$

An interesting aside is a simple form of impedence matching: Given a fixed source configuration, to what R_L is the most power delivered?



Answer: $R_L = R_{Th}$

(see Meyer, p. 31, or Eggleston, p. 63)