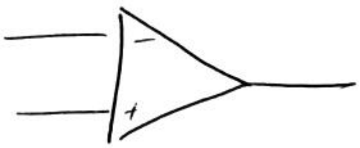
Operational amplifiers

Assume *ideal* for now

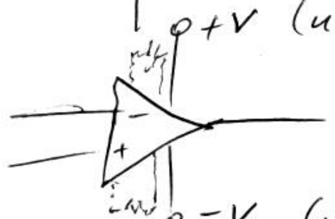
Usual version



actual connections:

comp (usually optional or absent)

+V (usually 15)



−V (usually −15)

bal. (optional, sometimes absent)

(no ground)

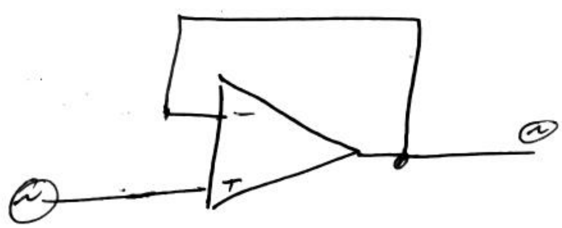
Properties (we will look at *real* op amps later)

1) No input current $(Z_{in} \sim \infty)$

2) output $= \big((+) - (-)\big) \times \infty$ $(Z_{out} \sim 0)$ — or ΔV_{in}

Intended for use with *− feedback*:

Follower illustrates



out = in

Output will *always* try to set itself so −, + inputs are at same V. To see, assume (+) at 1V, (−) at 1.1V

(−) correction

(In fact, this works even with finite gain, so long as $Z_{in} = \infty$.)

Compliance range:

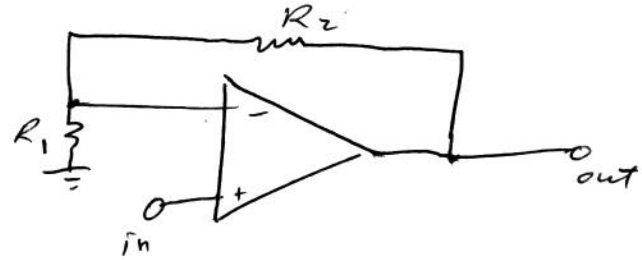
Typ. from $\simeq -V+2$ to $+V-2$ for older designs

"Special" types go all the way to one or both supplies, within ~ 10 mV or so.

Now quite common due to battery-powered devices.

Basic amplifiers (dc coupled):

Non-inverting:



① $V_- = V_{in}$ due to (-) feedback.

② $V_- = \frac{R_1}{R_1 + R_2} V_{out}$ (exactly, since $I_{op\,amp} \simeq 0$)

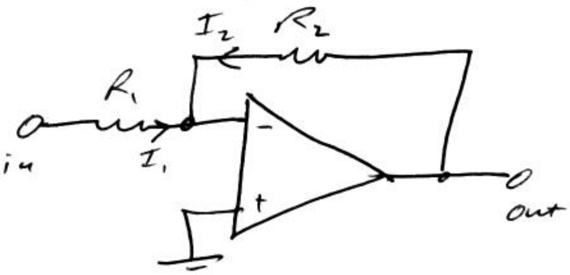
So $V_{out} = \frac{R_1 + R_2}{R_1} V_{in}$

$$\boxed{\frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}}$$

$Z_{in} \simeq \infty$ (for LF 411 $\simeq 10^{12}\,\Omega$)

Z_{out} very low, at least for low freq., small signals.

Inverting



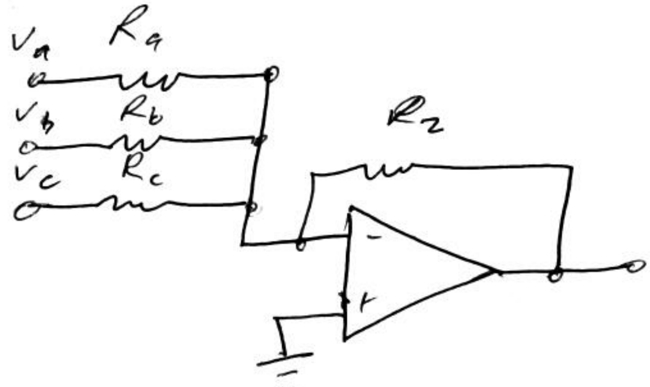
① $V_- = 0$ ② $I_1 = -I_2$ or $V_{in}/R_1 = -V_{out}/R_2$

So gain $\boxed{\dfrac{V_{out}}{V_{in}} = -\dfrac{R_2}{R_1}}$

$$\begin{cases} Z_{in} = R_1 \\ Z_{out} \simeq 0 \end{cases}$$

Most common configuration. This is partially because of the "virtual ground" at (−). It lets us build a

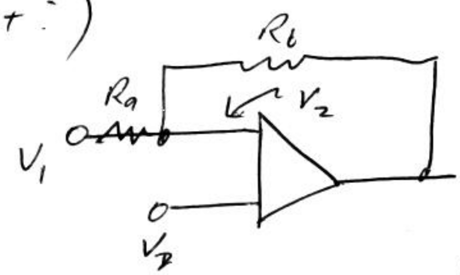
Summing amplifier



(−) still at ground, so $I_{tot} = 0 = I_a + I_b + I_c - I_2$

and $V_{out} = -R_2\left(\dfrac{V_a}{R_a} + \dfrac{V_b}{R_b} + \dfrac{V_c}{R_c}\right)$

(Variant:)



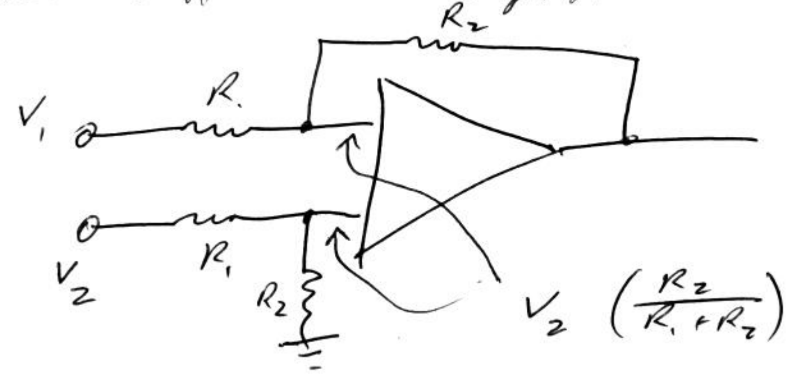
$$(V_1 - V_2)/R_a = -(V_{out} - V_2)/R_b$$

$$\frac{V_{out}}{R_b} = \frac{V_2}{R_b} + \frac{V_2 - V_1}{R_a}$$

$$V_{out} = (V_2 - V_1)\frac{R_b}{R_a} + V_2$$

The variant is a <u>differential amplifier</u>, but a bad one. However, inputs are asymmetric, and ~~input~~ output is not rel. to ground, but rather to V_2.

Better differential amplifier ---



we have
$$\frac{V_1 - V_2\frac{R_2}{R_1+R_2}}{R_1} = -\frac{V_{out} - V_2\frac{R_2}{R_1+R_2}}{R_2}$$

$$\frac{V_{out}}{R_2} = \frac{V_2}{R_1+R_2} + \frac{V_2\frac{R_2}{R_1+R_2} - V_1}{R_1}$$

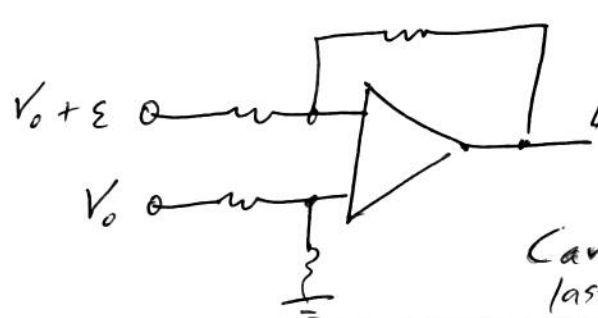
$$= \frac{V_2R_1 + V_2R_2 - V_1(R_1+R_2)}{R_1(R_1+R_2)}$$

$$= \frac{(V_2-V_1)\cancel{(R_1+R_2)}}{R_1\cancel{(R_1+R_2)}}$$

$$\boxed{V_{out} = \frac{R_2}{R_1}(V_2 - V_1)}$$

<u>But</u>, need precisely matched resistors!

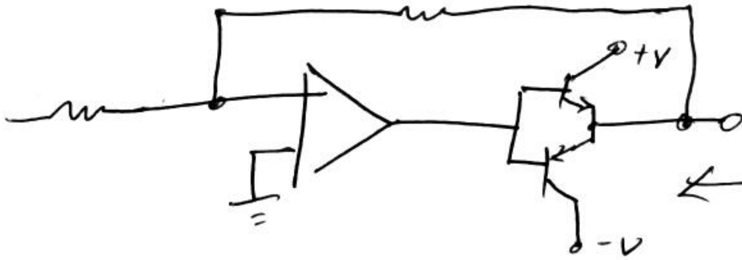
Problem is common-mode rejection ---



If not matched, V_{out} depends on V_0

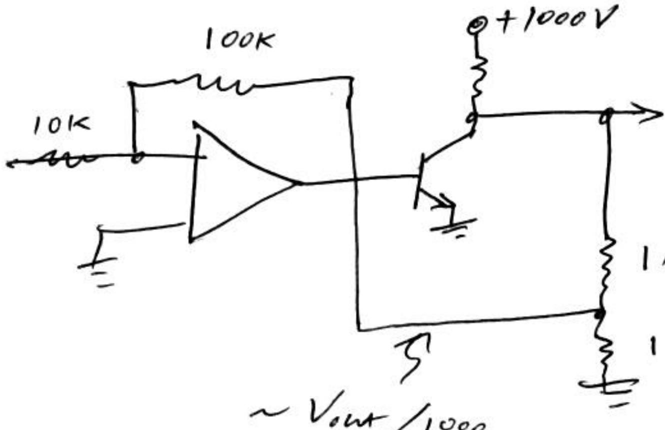
Can be addressed by hand matching, laser trimming, or <u>instrumentation amplifiers</u>.

Linearized <u>power</u> amplifier ---



(In practice, add
- protection
- bias
- thermal stabilization)

Can do same with HV output:



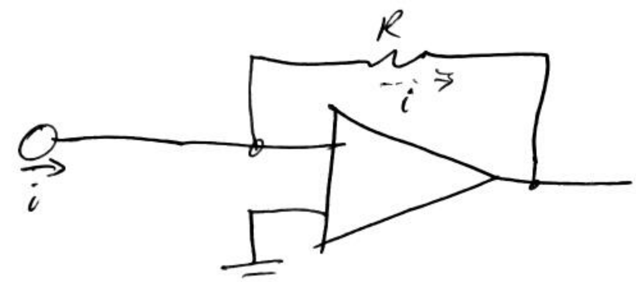
$$\frac{V_{out}}{1000} = 10\ V_{in}$$

Note F.B. ≳ 1K needed.

Even better: use single-chip solutions such as -

1) Texas Instruments OPA548T: up to 60V, up to 3A, ~\$15.
2) Apex/Cirrus PA340CC: up to 350V, up to 0.12 A, ~ \$25.

<u>Transimpedane amplifier</u>:

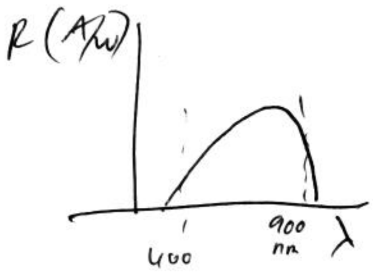


$V_{out} = -R\,i_{in}$

$Z_{in} = ?$ (0)

This is used with <u>photodiodes</u> and similar detectors

$I \propto$ photon flux
(eff'y about 50%)



At 633 nm, $E = h\nu = h\frac{c}{\lambda}$

$= (6.63\times10^{-34}\,J\cdot s)\,\frac{3\times10^{8}\,\frac{m}{s}}{633\times10^{-9}\,m}$

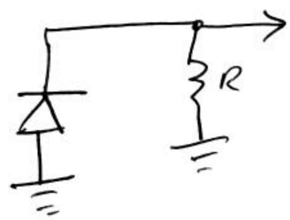
$= 3.14\times10^{-19}\,J$

$0.1\,mW \Rightarrow \cancel{3.14}\ 10^{-4}\ J/s$, or $3.2\times10^{14}\ \gamma/s$

current $I_p = .5\times\left(3.2\times10^{14}\,\frac{\gamma}{s}\right)\left(1.6\times10^{-19}\,\frac{C}{\gamma}\right)$

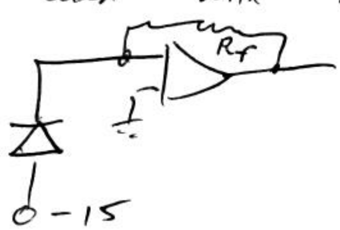
$= 26\,\mu A$ or $0.26\,\frac{A}{W}$

Sometimes used instead in <u>photovoltaic</u> mode,

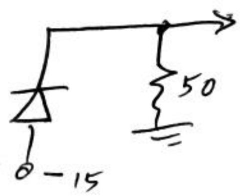


$\rightarrow V \sim -I_p R$ (up to ~ 1V max,) or less

Often used with reverse bias, to reduce capacitance:



or



V_{out} (to $-14\,V$!)

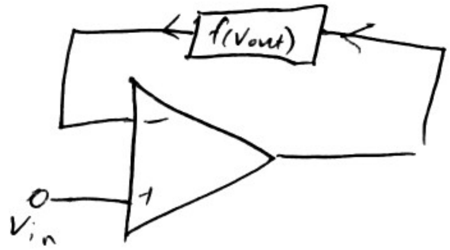
← For fast response, only usable for large signals

The op amp helps to minimize effects of stray C, too.
Since $Z_{in} \simeq 0$, the time constant RC does <u>not</u> apply.
Instead, a detailed analysis shows that

$$f_{3dB} \simeq \sqrt{\frac{f_T}{2\pi C_{diode} R_f}}\ ,\quad f_T = \text{unity-gain frequency or "gain-bandwidth product".}$$

For a perfect op amp, there would be no limit!

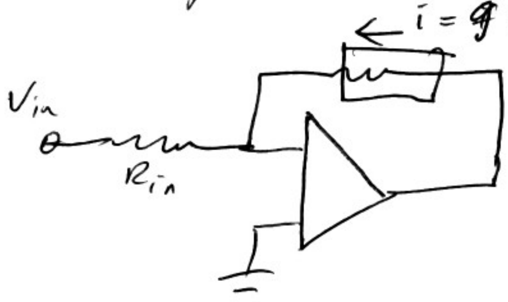
Arbitrary feedback:



$$f(V_{out}) = V_{in}$$

So $V_{out} = f^{-1}(V_{in})$

Can also do with (−) in, if device in loop produces a current rather than a voltage output:

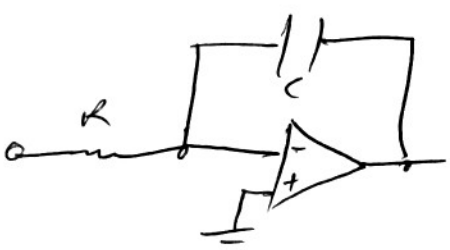


$$\frac{V_{in}}{R} = -g(V_{out})$$

$$V_{out} = -g^{-1}\left(\frac{V_{in}}{R_{in}}\right)$$

So if $g = e^V$, $V_{out} = -\ln \frac{V_{in}}{R}$ (see H&H for such a circuit)

Ex: integrator



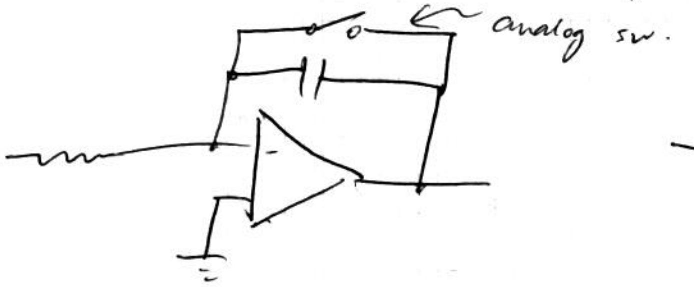
$$\frac{V_{in}}{R} = -\underbrace{C\frac{dV_{out}}{dt}}_{I_c}$$

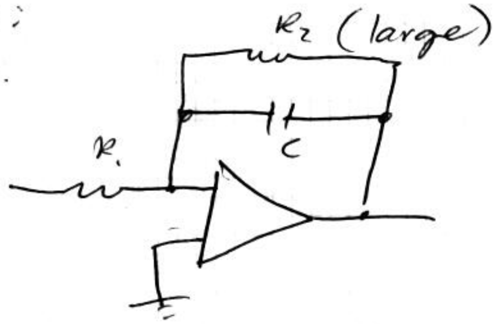
$$\boxed{V_{out} = -\frac{1}{RC}\int_0^{t_0} V_{in}\,dt + V_{initial}}$$

This is exact! The virtual ground keeps it that way.

(But in real op amps, also get $\int V_{offset}$)

Integrator needs a reset, or a resistor to set a finite dc gain, unless in a closed loop:

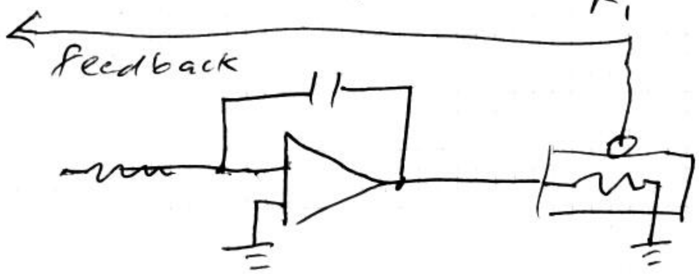




High freq: $-\frac{1}{RC}\int V_{in}\,dt$

Low freq: $-\frac{R_2}{R_1}$

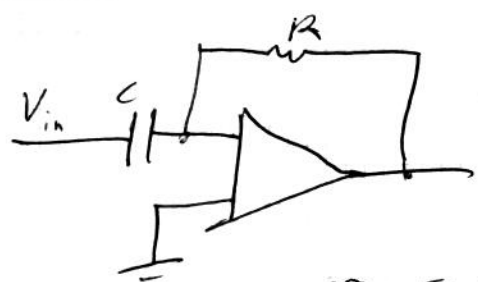
or



(gives ∞ gain, in effect) Often summed w/ linear gain.

<u>Differentiator</u>: same basic theme, but C <u>not</u> in feedback:

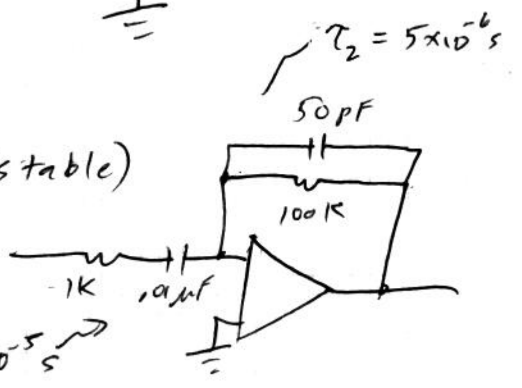
ideal



$$C\frac{dV_{in}}{dt} = -\frac{V_{out}}{R}$$

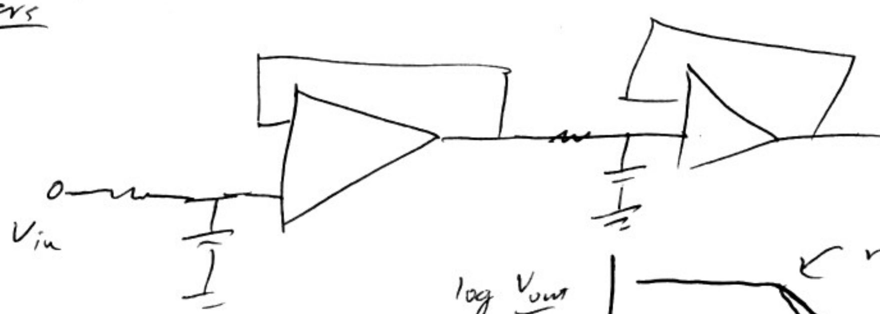
$$\boxed{V_{out} = -RC\frac{dV_{in}}{dt}}$$

actual (stable)

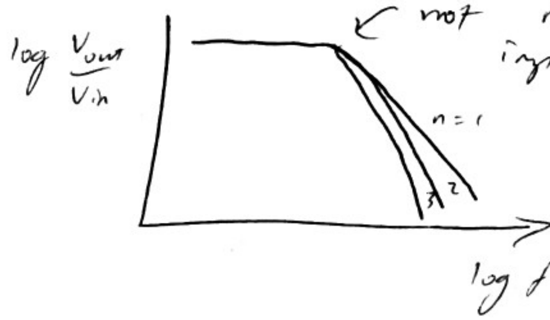


integrator for large ω

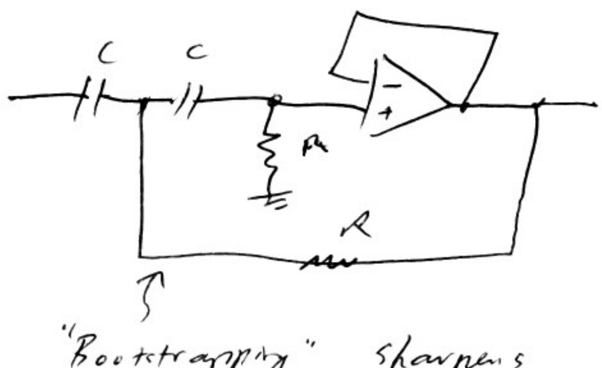
Filters



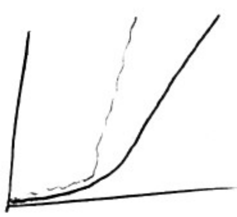
no good.



Instead, multi-pole active filters are used;
can synthesize any RLC filter without loading probs!

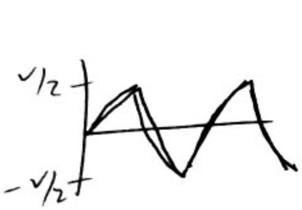


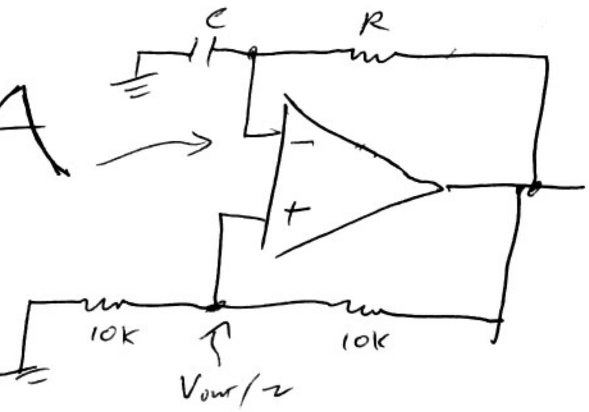
Basic Sallen & Key filter

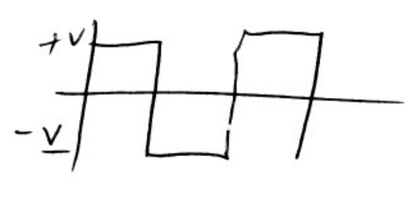


Oscillators -- classic cheap example is 555 timer chip --- uses positive feedback.

Simplest is relaxation osc ---







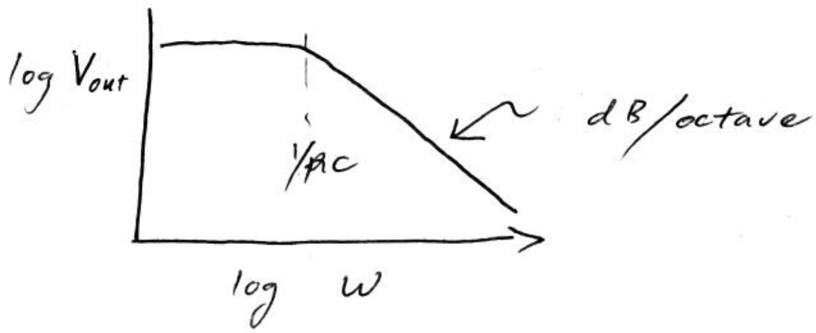
$T \sim 2.2RC$

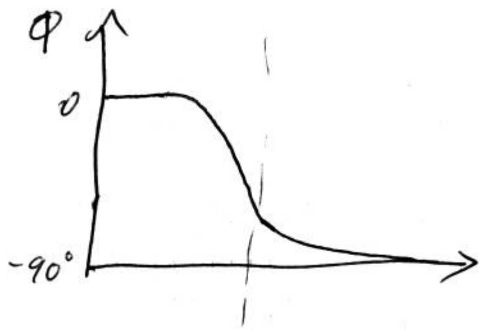
Real Op Amps I

ac Op amp behavior

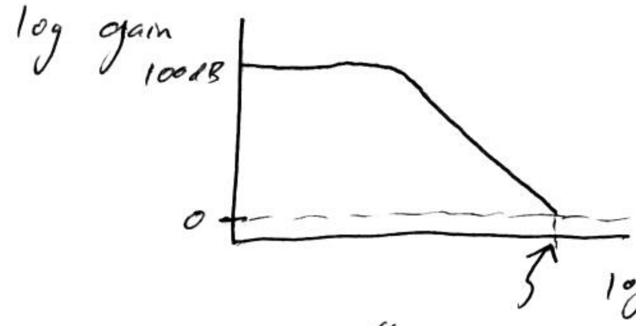
Finite gain; instability.

Recall the behavior of an RC low-pass:

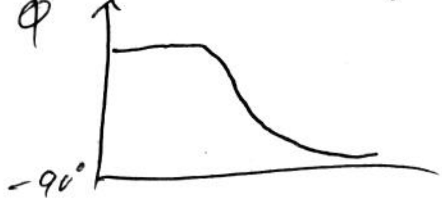




Most op-amps have a dominant "pole" so they look very similar ---



"unity gain" frequency



(× (−1) if inverting)

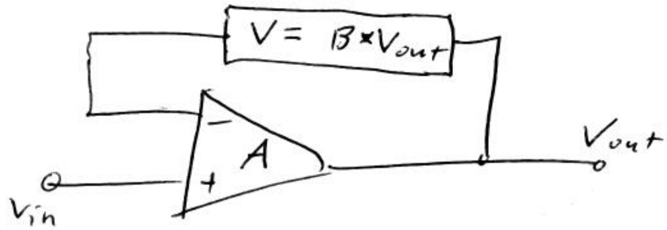
- Causes loss of accuracy

But what if we have additional phase lag?

If $|\Phi| > 180°$, neg. feedback → positive. So we get instability, and excursions at ω will be <u>amplified</u> ⇒ oscillation!

Look at these problems in turn:

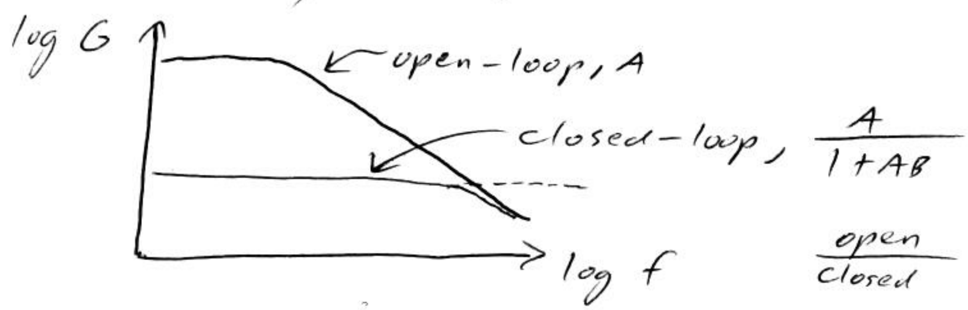
1) Finite gain. As an example, look at a non-inverting voltage amp (other configs. are similar):



$$\text{If } A = \infty, \quad G = \frac{1}{B} \qquad \left(V_{out} = \frac{1}{B} V_{in} \right)$$

But now,
$$V_{out} = A\,(V_+ - V_-)$$
$$= A\,(V_{in} - B V_{out})$$
$$V_{out} = \frac{A}{1+AB} V_{in} \qquad (1)$$
$\leftarrow G$

So, if we have a "single-pole" frequency roloff, 6 dB/octave,



$$\frac{\text{open}}{\text{closed}} \simeq AB$$

So, (a) Finite gain changes G
(b) Frequency response is limited by gain roloff.

Also, both Z_{in} and Z_{out} are ~~diminished~~ affected

If we have for open-loop,

using (1)

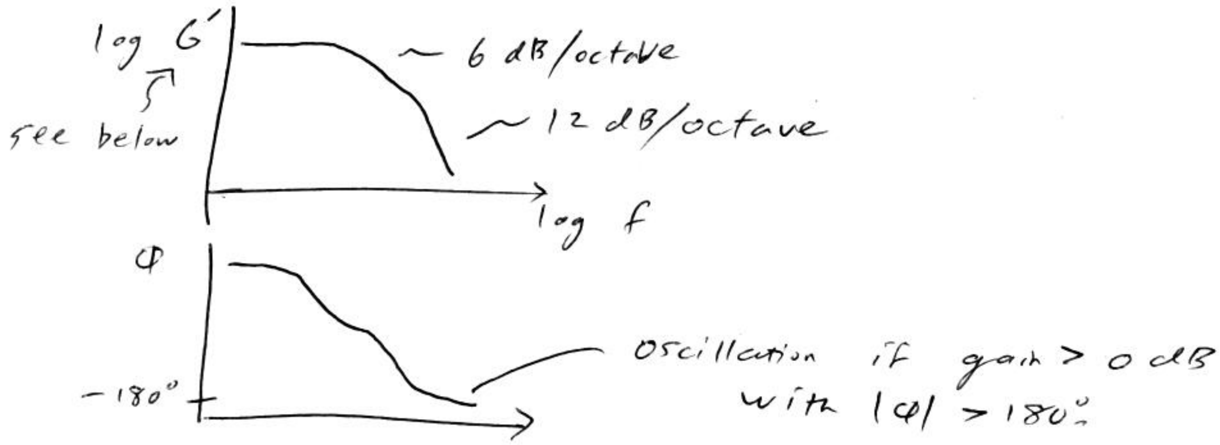
Then for closed loop,
$$I_{in} = \frac{V_{in} - B V_{out}}{R_{in}} = \frac{V_{in}}{R_{in}}\left(1 - \frac{AB}{1+AB}\right) = \frac{V_{in}}{R_{in}\,(1+AB)}$$

Thus $Z_{in} = \frac{V_{in}}{I_{in}} = (1+AB)\,R_i$ (not ∞)

Likewise, can show $Z_{out} = \frac{1}{1+AB} R_o$ (not zero)
$\leftarrow$ open-loop output imp.

2) Stability (a major issue in real designs)

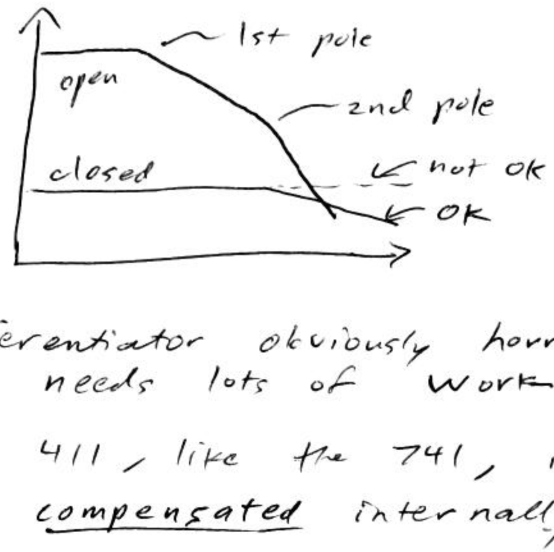
If there happen to be two poles, due to external components,

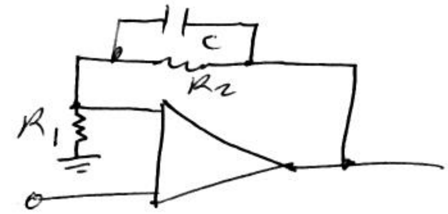


Assuming load / feedback are well-behaved, worst case is unity gain, because what counts here is the loop gain AB, which determines how much is fed back. (It's also $\simeq \frac{\text{open-loop gain}}{\text{closed-loop gain}}$.)

Compensation is used to assure stability.

Basic idea: make sure that closed-loop gain for $A = \infty$ intersects open-loop gain with slope << 12 dB/octave.





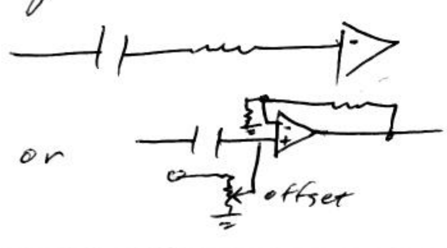
$$B = \frac{R_1}{R_2 /\!/ \frac{1}{i\omega C} \; (+R_1)}$$

Differentiator obviously horrible, needs lots of work to assure stability.

The 411, like the 741, is already unity-gain compensated internally.

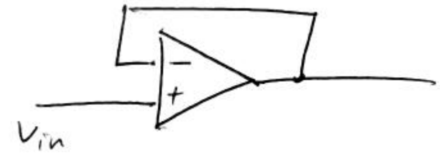
Note: poles, zeroes refer to response function, & Laplace transform language.

ac coupling: use when appropriate to eliminate offsets



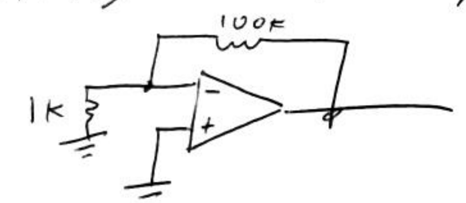
Real Op Amps II - Basic physical limitations

1) Input offset voltage: V_- held at $V_+ - V_{os}$, not V_+.
In a follower, replicated at output;



$V_{out} = V_{in} + V_{os}$

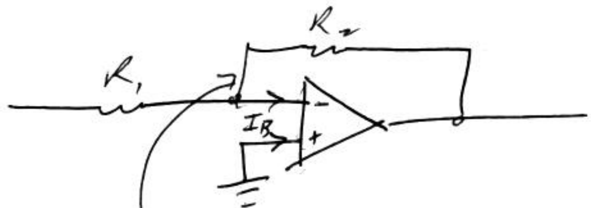
In amplifiers, can be amplified



$V_{out} = 100\, V_{os}$

LF411; 0.8 mV typ., 7 μV/°C
741: 2 mV, ~10 μV/°C ← has trimming provisions.
AD8551: 0.001 mV, 0.005 μV/°C

2) Offset & bias current



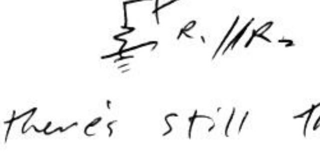
$|I_-| \simeq |I_+|$ = bias current

Ignoring V_{os}, $V_- = \Delta V_- = I_B\,(R_1 /\!/ R_2)$ rel. to perfect op amp.

$$\Delta V_{out} = -\frac{R_2}{R_1}\,\Delta V_-$$
$$\simeq R_2 I_B \text{ if } R_2 \gg R_1$$

741; 500 nA
LF411; 0.2 nA
AD549L: 60 fA (!) not pA!

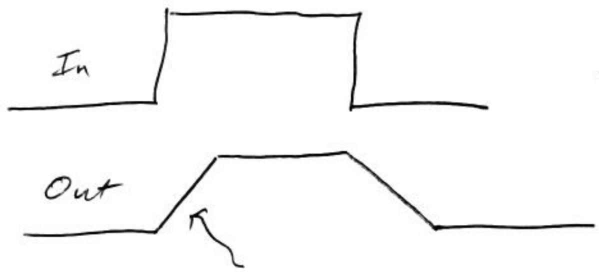
Can often be compensated;



741: 200 nA
LF411: 0.1 nA

However, there's still the input offset current $I_{os} \equiv I_+ - I_-$.
For many modern op amps, not that much smaller than I_{bias}.
For ac signals, ac coupling eliminates.

3) Slew rate = response to <u>large-signal</u> step function



Determined by current limitations of output stages.
≠ bandwidth

741: 0.5 V/μs
LF411: 15 V/μs
LF311: ~25 V/μs (with 500 Ω pullup to +5)
AD8009: 5500 V/μs

4) Output swing: "rail to rail" op amps can drive to < 20 mV wrt supply voltages --- some run off a single 1.5 V battery.

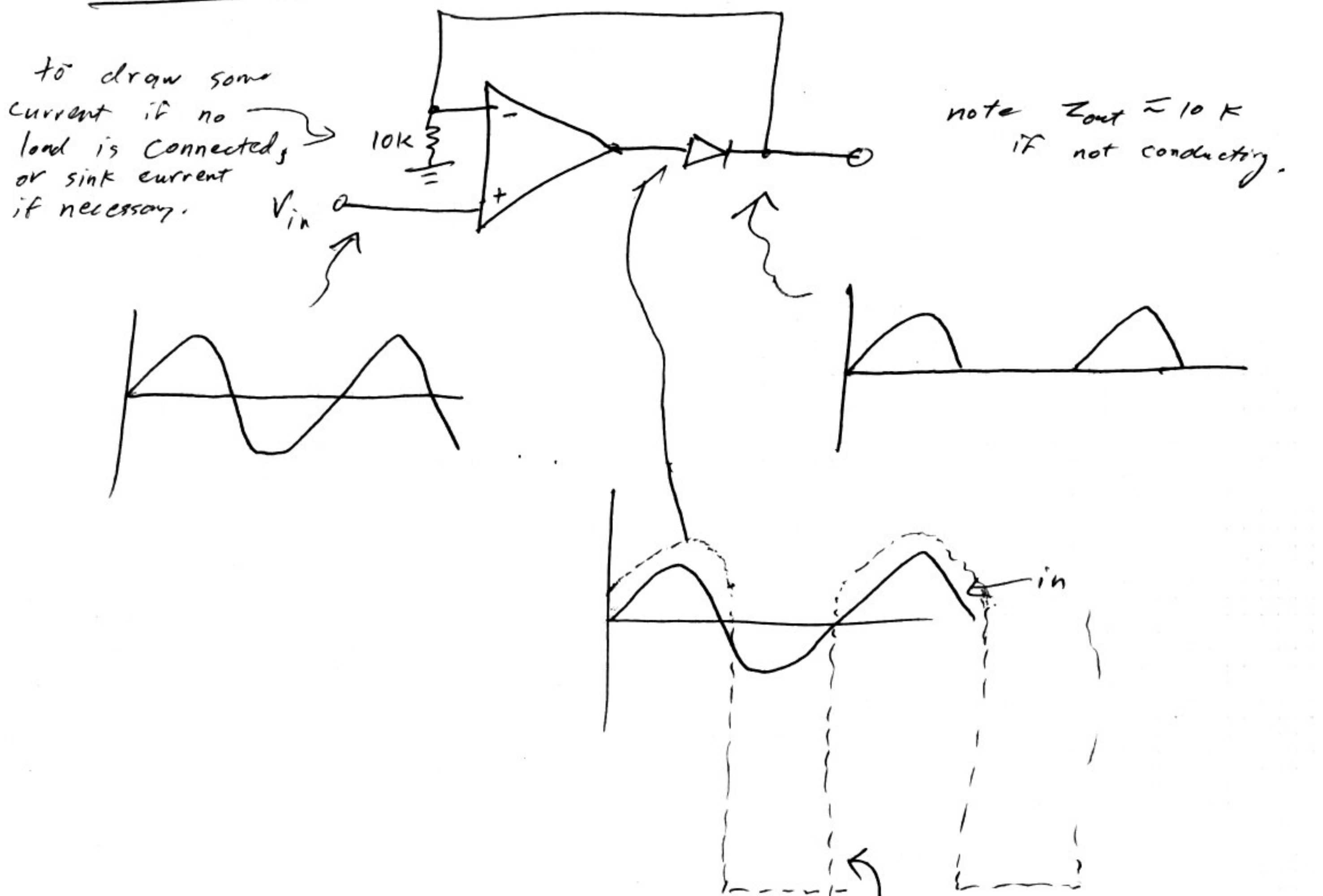
5) Common mode rejection -- typ. > 100 dB, usually limited by external circuitry.

6) Finite gain and phase shifts $\Phi(\omega)$ — to be discussed shortly

7) Noise: 1. low-frequency
2. white noise ($nV/\sqrt{Hz}$) } Will discuss later.

More op amp app's:

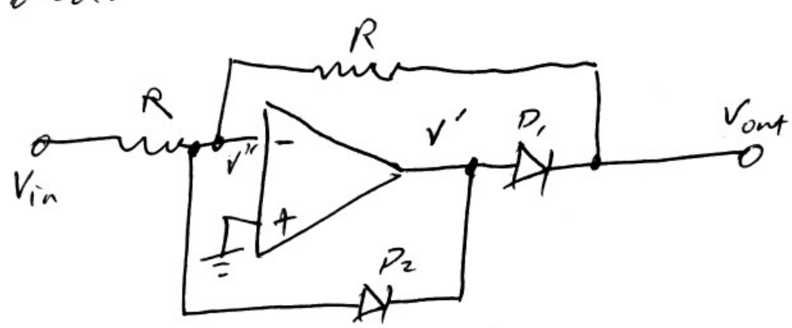
"Superdiode"



large slew rate needed

Full-wave variations (see H&H) can be used for measuring $\bar{V}$, etc., or for calculating $|V_{in}|$

Improved:



(−) in ⇒ D_1 conducts, gain −1

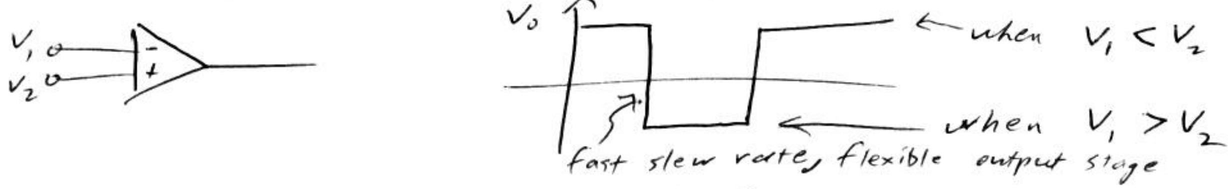
(+) in ⇒ D_1 reverse-biased

D_2 clamps V' at −.6V so $V'' = 0$.

Here $V_{out} = 0$ V, but impedance ~ R. ⇒ use follower if needed.

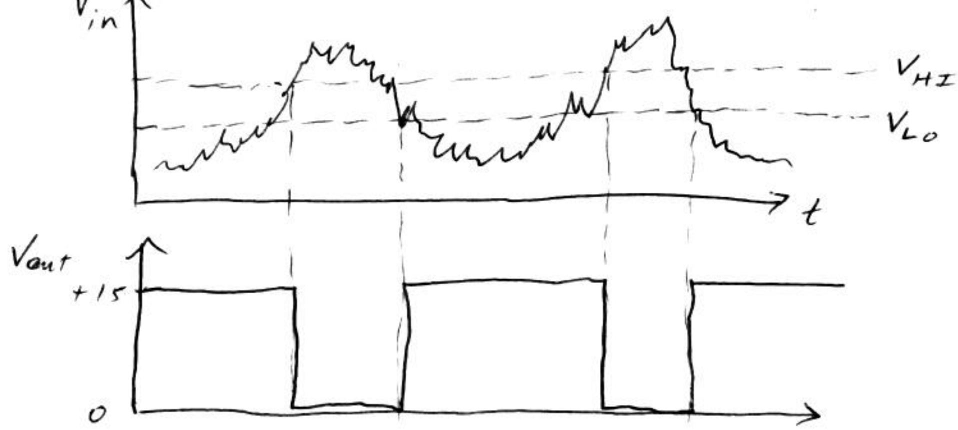
Comparators

They are really just op amp-like devices meant to operate without feedback, or even with (+) feedback. Typically they switch very quickly and cleanly between two output states.

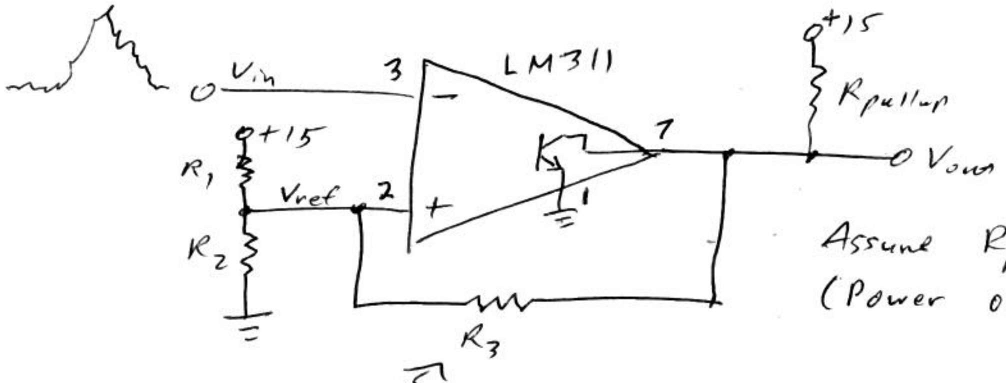


LM311 is a classic comparator (open collector output needs pullup)

A Schmitt trigger is often used to detect logic transitions on a noisy signal line:



Often built into line receivers. Can be manually set using a comparator and resistors — for example,



Assume $R_{pullup} << R_3$
(Power on V^+ = pin8, V^- = pin 4)

Parallels R_2 if output at 0 V, so

$$V_{ref} = V_{LO} = \frac{R_2'}{R_1 + R_2'}(15\text{ V}), \quad R_2' = R_2 // R_3$$

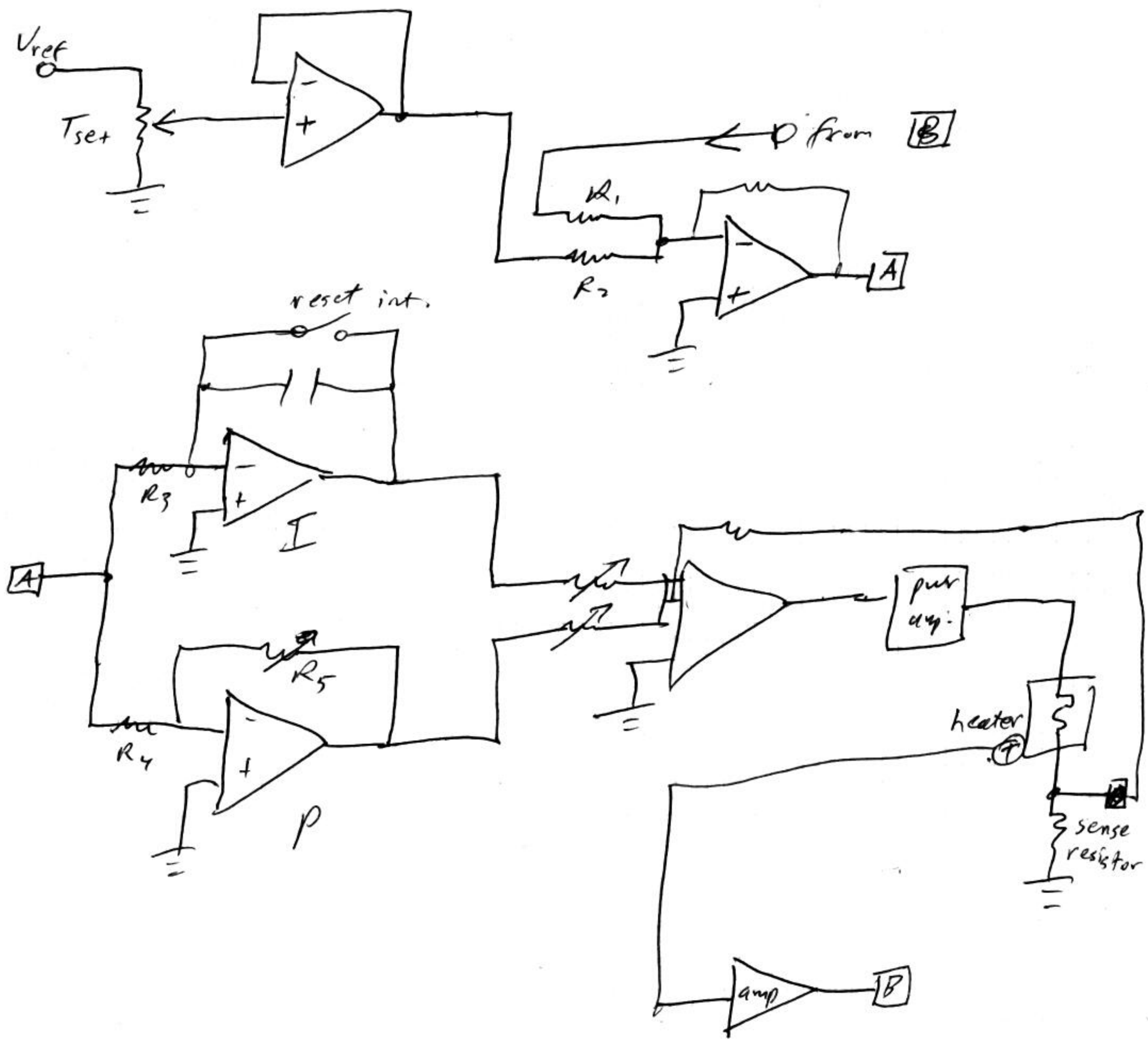
Parallels R_1 if output at 15 V, so

$$V_{HI} = \frac{R_2}{R_1' + R_2}(15\text{ V}), \quad R_1' = R_1 // R_3$$

and if R_3 is fairly large, $V_{HI} > V_{LO}$

If hysteresis small, $R_1 : R_2$ sets threshold, $R_3 : (R_1 // R_2)$ sets fractional hysteresis.

Typ. control system:



- Sometimes a differentiator is added — "PID" controller
- Why are buffers used?
- Why is integrator there?
- probably not stable as shown!
- In practice, usually use a thermistor + voltage source for T, and an instrumentation amp.