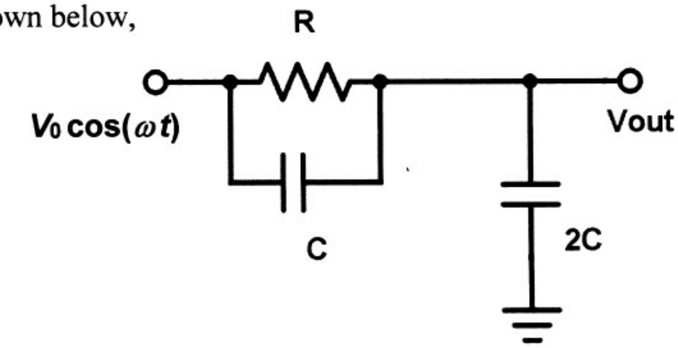
Physics 3150
Midterm exam, March 23, 2016

You are free to make reference to one textbooks and your class notes. Please write your answers on the exam pages. If you use extra pages, staple them to the exam before leaving.

1. For the filter-type circuit shown below,



(a) (12 points) Find an expression for V_{out} at frequency ω. Simplify the result to eliminate any compound fractions, although it's OK to leave a complex-valued denominator.

(b) (12 points) At very low or very high frequencies, the circuit can be simplified by eliminating one of the parallel components because it no longer contributes significantly. Show the resulting effective circuits in the limit of (i) very low frequencies, and (ii) very high frequencies, and find effective expressions for V_{out} in each case.

(c) (10 points) Show that the results from part (a) agree with those of part (b) if the appropriate limits.are taken in all expressions for V_{out}.

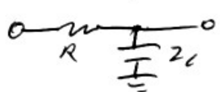
a) Voltage divider formed by Z_{2c} and $R // Z_c$:

$$V_{out} = V_0 e^{i\omega t} \frac{Z_{2c}}{Z_{2c} + Z_{R//Z_c}} = \frac{\frac{1}{2i\omega C}}{\frac{1}{2i\omega C} + \left(i\omega C + \frac{1}{R}\right)^{-1}} \left(V_0 e^{i\omega t}\right)$$

$$= \frac{1}{1 + \frac{2i\omega C}{i\omega C + \frac{1}{R}}} V_0 e^{i\omega t} = \frac{1 + i\omega RC}{1 + i\omega RC + 2i\omega RC} V_0$$

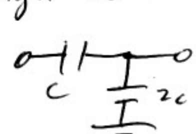
$$\boxed{V_{out} = \frac{1 + i\omega RC}{1 + 3i\omega RC} V_0 e^{i\omega t}}$$

b) Low ω: $Z_R >> Z_c$



$$V_{out} = \frac{\frac{1}{2i\omega C}}{\frac{1}{2i\omega C} + R} V_{in} = \frac{1}{1 + 2i\omega RC} V_{in}$$

High ω: $Z_c >> Z_R$

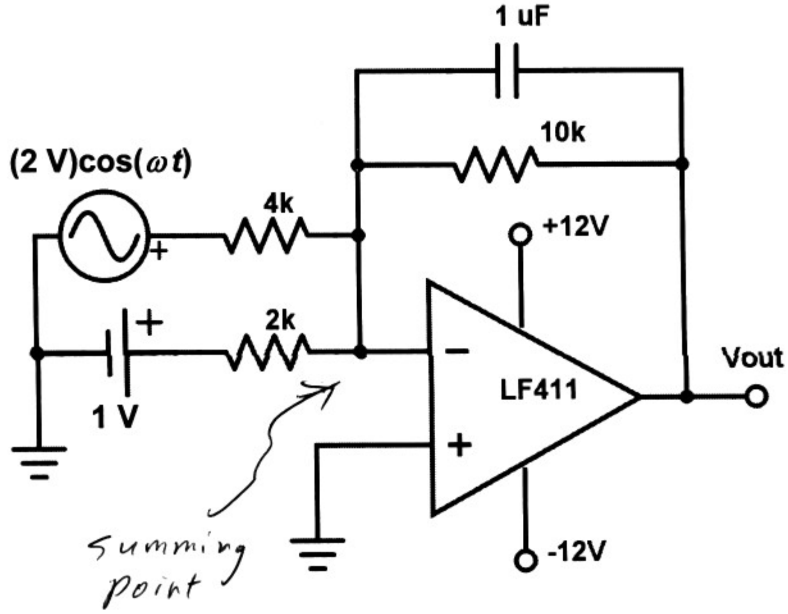


$$V_{out} = \frac{\frac{1}{2i\omega C}}{\frac{1}{2i\omega C} + \frac{1}{i\omega C}} V_{in} = \frac{1}{3} V_{in}$$

c) Low ω: From (a), $V_{out} \simeq \frac{1}{1} V_{in} = V_{in}$, From (b), $V_{out} = \frac{1}{1} V_{in} = V_{in}$ ✓

High ω: From (a), $V_{out} \simeq \frac{i\omega RC}{3i\omega RC} V_{in} = \frac{1}{3} V_{in}$, which matches (b) ✓

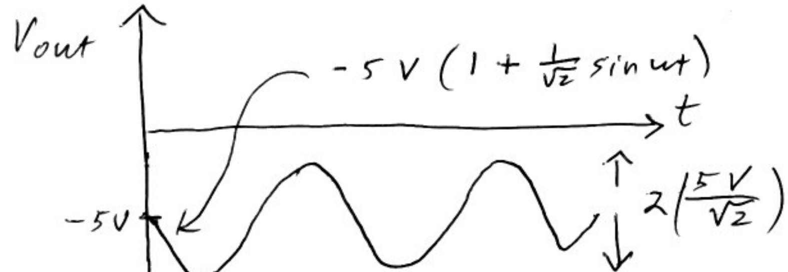
2. (34 points) An ideal op amp is connected as shown, with simultaneous inputs from a constant 1 VDC source and a 2 VAC source at frequency $\omega/2\pi = 1$ kHz, which happens to be the same as the 3dB frequency of the filter in the feedback loop. Find both the average dc level and the ac amplitude at V_{out}. By how much is the ac component phase shifted? (*Note*: if you cannot deal with the capacitor, solve the problem without it for 22 points.)



For the 1VDC input, the capacitor doesn't contribute,
so $V_{out}/V_{in} = -\frac{10K}{2K}$, and $V_{out,dc} = -5V$.

For the 2VAC input, we would have $G = \frac{-10K}{4K} = -2.5$ without the capacitor. At the 3dB point, the feedback voltage is reduced by $\sqrt{2}$ by the RC filter,
so $G(\omega) = \frac{-2.5}{\sqrt{2}}$, and $V_{out,ac} = -\frac{5V}{\sqrt{2}}\cos(\omega t + \phi)$ (see below)

The total output is just the sum,

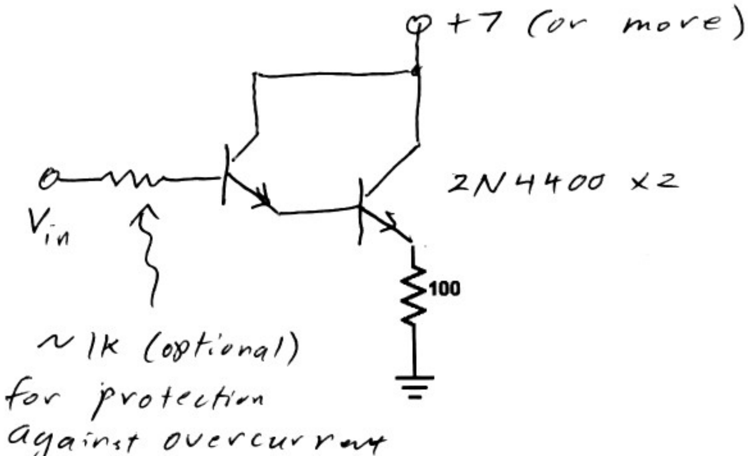


There is a 45° phase shift at the 3dB point, which is then inverted by the op amp, as shown.
This can be analyzed in detail by writing
$V_{out} = Z_{fb} I_{feedback} = (i\omega C + \frac{1}{R})^{-1} I_{feedback}$, etc.

3. (32 points) Design a voltage follower using 2N4400 npn transistors suitable for use after a filter circuit such as the one in Problem 1. It must be able to drive 0–5 Volts into a 100 ohm load resistance that's grounded on one side, and it must have an input impedance of at least 10k Ω. Use any power supplies and resistors that you wish. The current gain of the 2N4400 is specified to be $\beta > 50$. (*Hint*: If you can't meet the specifications with one transistor, consider using two, chained together in any way that works.)

If a single 2N4 were used, Z_{in} would be marginal:

$$Z_{in} \simeq R_L(1+\beta)$$
$$= 5100\,\Omega \text{ (or more)}$$



While it's entirely possible to connect two emitter followers together to circumvent this, an even more compact solution is the "Darlington" or "superbeta" configuration shown.

Note that there will be two diode drops between input and output, so for 0-5 V out, we need at least a 7 V supply.