Physics 3150, Laboratory X
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A. Procedure and Results

A.1. Voltage and current for a resistor bridge

We constructed a resistor bridge circuit as indicated in the lab writeup. Its schematic diagram is also shown below in part B. We measured the current through each of several load resistors R_{load} using a DMM in series, and the voltage across each using a DMM in parallel. The resistors have a tolerance of 5%. Table I summarizes the results.

Table I. Voltage and current data for the resistor bridge, for various resistors R_{load}. Calculated current I_{calc} is described in text. Estimated DMM uncertainties are 0.005 V for V_{load} and 0.05 mA for I_{load}.

R (Ω)	V_{load} (V)	I_{load} (mA)	I_{calc} (mA)
680	1.63	2.4	2.40
820	1.82	2.2	2.22
1000	2.05	2.1	2.05
1800	2.71	1.5	1.51
2000	2.87	1.4	1.44
22000	4.29	0.2	0.20

The DMM uncertainties are set by the finite resolution of the readout.[1] Expressed as percentage errors they range from 2% to 25% for the currents, but only from 0.1% to 0.3% for the voltage. Because the currents can be calculated from the voltages using the known resistance, $I_{load} = V_{load} / R_{load}$, the current readings are actually superfluous. The calculated values I_{calc} are shown in the last column of in Table I. Their uncertainty is set by the 5% resistor tolerance, though this could be reduced by measuring R directly.

These results are plotted in Fig. 1 together with a linear regression fit, which lies well within the error bars. For this fit the reduced value of χ^2 is 0.3, well below the expected value of ~1,[2] indicating that the statistical uncertainties are somewhat overestimated.

The coefficients of the fit give the parameters R_{th} and V_{th} of the Thévenin equivalent circuit (1212 Ω and 4.54 V), which can be compared with the results of Section A.2 below. This method is more time-consuming but has the advantages of being non-destructive[3] and yielding enough statistical data to reliably estimate the uncertainties.[4]

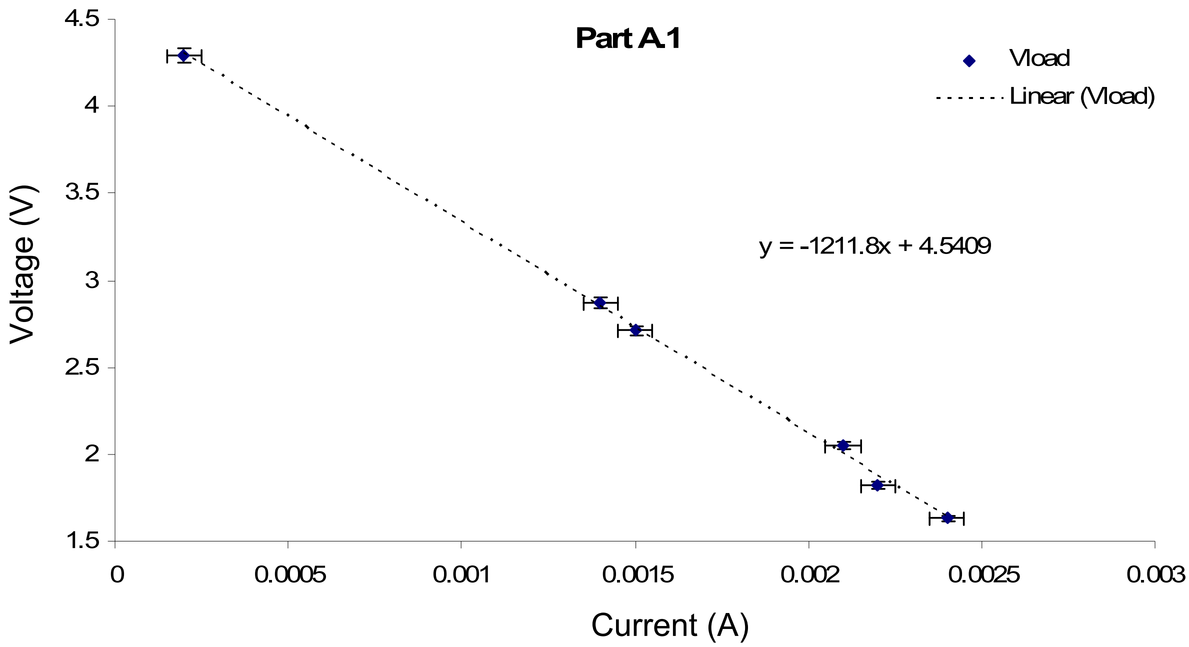

Fig.1 . Voltage vs. current for the resistor bridge circuit of part A.1, with a linear regression fit to determine R_{Th} and V_{Th}.

A.2. Thévenin equivalent circuit

We measured the short circuit current I_{sc}= (3.8 ± 0.05) mA and the open circuit voltage V_{oc} = (4.53 ± 0.005) V. Their ratio predicts a Thévenin equivalent resistance of R_{th} = (1200 ± 16) Ω.[5] We then constructed the equivalent circuit with a 1.2 kΩ (±5%) resistor and re-measured the data of Table I using the same set of resistors as before, obtaining the results shown in Table II.

Table II. Voltage and current data for the Thévenin equivalent circuit. The measured quantities and uncertainties are as in Table I.

R (Ω)	V_{load} (V)	I_{load} (mA)
680	1.63	2.4
820	1.83	2.2
1000	2.03	2.1
1800	2.69	1.5
2000	2.84	1.4
22000	4.29	0.2

A comparison with the results of Section A.1 is shown in Fig. 2. The result is gratifying; indeed it is much closer than the experimental errors and tolerances would lead one to expect. The fit results are hard to distinguish from those of the original resistor bridge circuit[6]—they fall within a few parts per thousand.

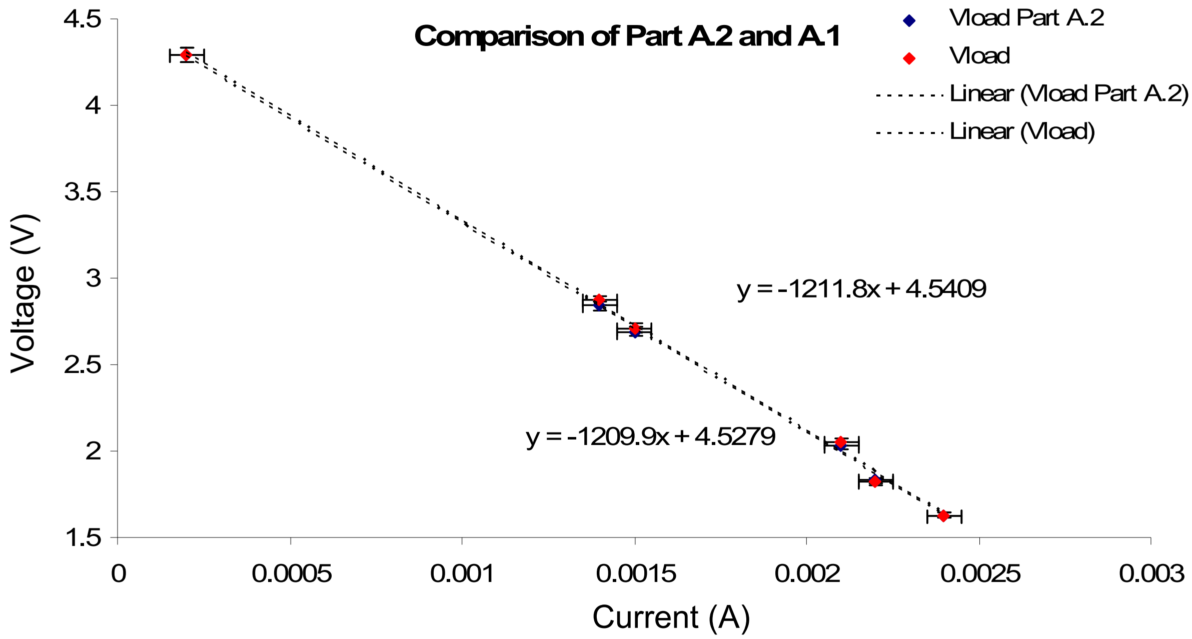

Fig. 2. Voltage vs. current measurements for Parts A.1 and A.2, with linear regression fits to each indicated by dotted lines.

A.3. *Familiarization with the oscilloscope*

The TA specified that no writeup is required for this section, as the main objective was simply to gain experience with the equipment.

B. Questions

B.1. Direct solution using Kirchoff's laws

For reference and nomenclature the circuit is sketched below. The load resistor R_{load} is not shown, but it connects between points 1 and 2. Using the "mesh loop" method, we write three loop equations for the currents I_1, I_2, and I_3:

Do this (1)
yourself! (2)
(But ask for help if needed.) (3)

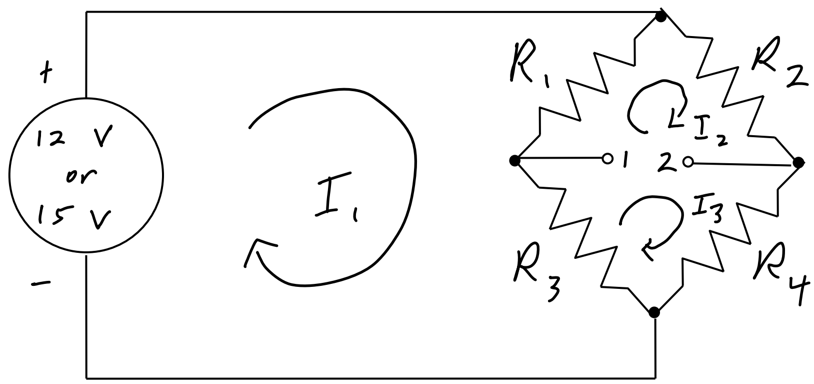

Fig. 3. Sketch of the bridge circuit, adapted from the lab writeup provided on the Physics 3150 web page.

These equations can be solved to find the current $I_{load} = I_3 - I_2$ that flows through the load resistor, and the corresponding voltage $V_{load} = R_{load} I_{load}$. This is most easily done by using a computer mathematics package such as Mathematica for symbolic solutions, or Matlab or SPICE for numerical solutions.[7] The solution for I_{load} is

$$I_{load} = \frac{V_{in}(R_2R_3 - R_1R_4)}{R_3R_4R_{load} + R_2R_3(R_4 + R_{load}) + R_1\left[R_4(R_3 + R_{load}) + R_2(R_3 + R_4 + R_{load})\right]}. \tag{4}$$

Simplifying by substituting numerical values (with SI units),[6]

$$I_{load} = \frac{5314.2}{1.4344 \times 10^6 + 1173 R_{load}}. \tag{5}$$

Note that Eq. (5) includes everything we need to know to find the Thévenin equivalent. Setting R_{load} to infinity gives the open-circuit voltage, V_{Th} = 5314.2/1173 = 4.530 V. Setting it to zero gives the short-circuit current, I_{sc} = 3.70 mA, and the ratio of these results yields the Thévenin equivalent resistance, R_{Th} = 1223 Ω. Comparing these to the results of Parts A.1 and A.2, we again find excellent agreement, much better than the resistor tolerances of 5% would suggest.

B.2. Remarks on accuracy

This subject was already discussed in some detail in the text—in general, the results agree more closely than the resistor tolerances of 5% would suggest. The input impedance of the voltmeter is a resistance of $\sim 10^7\ \Omega$, which will act in parallel with R_{load}. In the worst case when $R_{load} \sim 10^4\ \Omega$, this is only a 0.1% perturbation.

Likewise the ammeter input impedance is about $10^{-1}\,\Omega$ in series with the measurement terminals, so for the worst case when $R_{load} \sim 10^2\,\Omega$, it again causes a perturbation of only about 0.1%.

B.3. Peak-to-peak vs. rms voltage

In general it is not important which measure of ac voltage is used for comparisons, so long as the usage is consistent. However, as a practical matter digital scopes calculate the rms voltage by integrating root-mean-squared voltage over the entire waveform, which averages out noise fluctuations, whereas the peak-to-peak mesurements are very sensitive to noise spikes. In the present case this makes little difference, as the noise level was small.

For comparison with dc measurements by a DMM the rms voltage is required. If the peak-to-peak value was measured instead, it can be converted by dividing by $2\sqrt{2}$. It was also evident in the lab that DMMs are clearly not designed for measuring ac signals at rf frequencies, or indeed anything far in excess of 60Hz. They show a decreasing reading for increasing frequency, even for a constant-amplitude input.

Notes

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[1] It can be difficult to estimate errors, and a detailed statistical analysis of the data set is not always necessary, or even possible. In this case the DMM is not being read to its full 3.5 digit resolution, so it is a pretty safe assumption that the uncertainty does not exceed ±1/2 in the last displayed digit. Thus the uncertainties listed here should be an upper bound. Under other circumstances it might be necessary to look up or measure the absolute accuracy of the instrument. If so, this would be a *systematic* uncertainty, not a *random* or *statistical* uncertainty.

If a reading fluctuates, it's helpful to estimate the range of the fluctuations, taking an approximate average as the reading, with an uncertainty based on the fluctuations (in principle it should be based on the rms deviation and the number of samples used; in practice it's often easiest to repeat the first measurement three or four times to see how much it varies.

Of course, systematic errors are possible in electronics just as in any other experimental science: voltmeters can be miscalibrated, component values can be inaccurate, and the measuring instruments can perturb the circuit due to the finite input impedances. These issues are often important, and in most cases we will encounter, they cause uncertainties much larger than purely statistical fluctuations.

[2] A standard statistical measure of the quality of a least-squares fit is the "χ^2 statistic": the mean squared deviation of the data from the fit, with the deviation of each point expressed as a multiple of its uncertainty:

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(fit_i - data_i\right)^2}{\sigma_i^2}.$$

If the uncertainties are estimated accurately, each term in the summation should have a value of approximately one. With this perspective in mind, but refined by a more quantitative analysis, statisticians define the *reduced* χ^2 *statistic* to be $\chi^2 / (N\text{-}M)$, where N is the number of data points and M is the number of adjustable parameters used in the least-squares fit. The expected value of this reduced chi-squared statistic can be looked up in statistical tables: it is just slightly larger than one, with a slight dependence on the number of samples. So the rule of thumb is that *if the reduced chi-squared is close to one, the fit is good.*

This degree of statistical sophistication is normally not really required in your lab report, especially if your error bars behave as expected, encompassing the fit about 2/3 of the time. For further information ask your instructor or refer to a text, such as the ones listed in Note 4 below.

[3] It is often not safe to place an ammeter directly between two terminals; in the present case this works because the resistor network restricts the current to well below the limit of the DMM, typically 1 A.

[4] We don't show the uncertainties here, but formulas for estimating the uncertainties in the slope and the intercept are given in most books on data analysis, for example Bevington and Robinson, *Data Reduction and Error Analysis for the Physical Sciences, 3rd Ed.,* or Taylor, *An Introduction to Error Analysis*, 2nd Ed. Unfortunately many pre-programmed fitting routines do not provide these estimates, leaving the researcher to his own devices. One commonly-used exception is Origin graphics, which *does* provide estimates of the uncertainties of the fit results.

[5] Formulas for the propagation of uncertainties are also readily available in Bevington or Taylor (see Note 4).

[6] The Course staff doesn't always get things right, either! The fits in Fig. 2 would be much easier to distinguish if we made good use of colors or differing line types. In addition, the voltage used for the measurements and to obtain Eq. (5) from Eq. (4) is nowhere specified.

[7] See www.partsim.com for a free graphical interface providing SPICE simulation of basic circuits.