# Physics 3150, Laboratory 3 RC and RLC Circuits

February 8 and 10, 2016 Last revised February 10, 2016 by Ed Eyler

### Purpose:

- 1. To study the dynamics of RC circuits, and some of their common applications.
- 2. To experimentally investigate resonance in an RLC circuit.

# **References:**

Chapter 2 of Eggleston, or Chapters 2 and 3 of Meyer, or Chapter 1 of Horowitz and Hill for a more concise and practical viewpoint.

# Equipment:

- 1. Function generator with frequency sweeping capability (e.g., Instek AFG-2105/2112).
- 2. Digital oscilloscope, preferably one that can directly measure phase shifts.
- 3. Breadboard with power supplies and hookup wire.
- 4. A selection of 1/4W or 1/2W resistors.
- 5. 200 pF, 0.01  $\mu$ F, and 0.1  $\mu$ F capacitors.
- 6. High-*Q* inductors with 2 mH  $\leq L \leq$  50 mH.

# I. RC Circuits

A. Exponential time constant



- 1. Build the circuit above and drive it with a 50 Hz square wave. Monitor the output on the oscilloscope and make sure that you are using dc coupling.
- 2. Measure the time constant by determining the time for the output to drop to 37% of its maximum value after the falling edge of the square wave. Notice that this is not the same as the *fall time* defined on many built-in measurement functions, which is typically measured from 10% to 90%.
- 3. Similarly, measure the time for the output to rise to 63% of its final value after the rising edge of the square wave. Is it the same time as in part 2, within experimental error?
- 4. Compare the results to the calculated *RC* time constant.

### **B.** Integrator

- 1. The above circuit functions as an integrator for signals with sufficiently short time scales. Drive the circuit with a 10 kHz square wave. Qualitatively describe the output wave form and explain why it has the shape you observe. Quantitatively explain the p-p output voltage you measure. You may have to adjust the output level of the function generator to get good results.
- 2. Now drive it with a triangle wave. Again, qualitatively describe and explain the output wave form.
- 3. This is the same circuit as in Section I.A, where the output doesn't look much like an integrator. Under what conditions does it accurately integrate the input? Determine this experimentally, and in your writeup, explain your result using the relevant circuit theory.
- 4. What is the input impedance of this circuit at zero frequency? At very high frequencies?

### C. Differentiator



- 1. Build the circuit above and drive it with a 100 kHz square wave. Again, describe and explain the output wave form. Your writeup should include a quantitative explanation for the waveform and the maximum output voltage amplitude that you measure.
- 2. Do the same for a triangle wave and a sine wave.
- 3. What is the input impedance at zero and infinite frequency?

#### D. High-pass filter

- 1. The above circuit can also be viewed as a high-pass filter it blocks low frequencies. This is exactly what's used when you set an oscilloscope input to ac coupling, although in this case the low frequency cut-off is below 100 Hz. In this section, you will estimate the value of the "blocking capacitor" in the oscilloscope, assuming that the input resistance is 1 M $\Omega$ , which is typical for an oscilloscope without a probe.
- 2. Connect a signal generator directly to your oscilloscope, with a cable rather than a probe. Vary the frequency in the range from about 5-100 Hz until you find the value where the response is reduced by 3 dB when you switch the scope from dc to ac coupling.
- 3. Calculate the value  $C_{\rm B}$  of the internal blocking capacitor from your measurement of  $f_{3\rm dB}$ . Be careful not to confuse radians with cycles/sec, or decibels in power units with decibels in voltage units.

### **II. RLC Resonance**

#### A. Principles

The resonant behavior of RLC circuits is discussed in detail in Section 2.6.5.3 of Eggeleston and Section 3.6.2 of Meyer, as well as in lecture, so only a brief synopsis is needed here. The circuit configuration in this instance is a series-parallel combination, chosen to avoid problems with excessive loading of the signal generator:



The output voltage is found by treating the circuit as a complex voltage divider, in which  $Z_{LC}$  appears in series with  $Z_R$ :

$$V_{\text{out}} = V_{\text{in}} \frac{Z_{LC}}{R + Z_{LC}}$$
, where  $Z_{LC} = \frac{i\omega L}{1 - \omega^2 / \omega_0^2}$ .

Note that the frequencies  $\omega$  and  $\omega_0$  are expressed in radians/s, not in Hz. The resonance frequency is

$$\omega_0 \equiv \frac{1}{\sqrt{LC}}.$$

The squared magnitude of the gain G, (i.e., of the ratio  $V_{out}/V_{in}$ ) is thus

$$\left|G(\omega)\right|^{2} = \frac{\omega^{2}L^{2}}{\omega^{2}L^{2} + \left(R(1 - \omega^{2}/\omega_{0}^{2})\right)^{2}}$$

The quality factor Q, the ratio  $\omega_0 / \Delta \omega$  of the resonant frequency  $\omega_0$  to half-power width  $\Delta \omega$  of the resonance curve  $|G(\omega)|^2$ , is given approximately by

$$Q = \frac{R}{L\omega_0}.$$

#### **B.** Procedure

Construct an *LC*-parallel RLC filter as shown in the schematic diagram in Part A. Drive it with a sine wave, varying the frequency through a range that includes the circuit's resonance (which should be somewhere between about 3 KHz and 100 KHz depending on the value of L). Some of the inductors are difficult to insert in the breadboards, so be patient and try not to force them.

Find the resonant frequency  $\omega_0$ . Measure the amplitude of  $V_{LC}$  as a function of frequency, taking enough points to make a respectable plot. Measure the phase shift of  $V_{LC}$  relative to the input voltage at resonance, and also at frequencies chosen to be well below resonance and well above resonance. Note that the phase shift can be used to find the resonance frequency considerably more accurately than a direct attempt to maximize the amplitude. Finally, measure the series resistance  $R_L$  of the inductor with an ohmmeter; you will need this to answer the questions below.

For an elegant display of the resonance curve, learn to set up the frequency sweep option on one of the better signal generators to obtain the entire response curve in a single trace.

Just for fun, try using this filter to find the Fourier components of a square wave by gradually reducing the signal generator frequency so that the filter circuit passes through resonance with the successive Fourier components at  $\omega$ ,  $3\omega$ , etc. Since the filter's resonant frequency stays fixed, an output should be observed each time one of the Fourier components matches this resonant frequency.

# C. Questions for Part II

1. Plot the measured response of the parallel RLC resonant circuit. What is the observed Q factor, taking into account that it is defined in terms of the half-width in power, not in voltage? Compare your results with a calculated response curve for the same resonant frequency and Q.

2. Calculate the expected phase shifts at resonance, as well as at high and low frequencies, and compare with what you measured.

3. Can you account for the measured value of Q by using the values of the resistor R and the measured series resistance  $R_L$  of the inductor, or are additional loss mechanisms playing a role?