

# Physics 3150, Laboratory 2

## Thévenin equivalent circuits and Characteristic curves

by Ed Eyler  
Last revised January 22, 2016

### ***Purpose:***

1. To experimentally verify Thévenin's theorem for a resistor network.
2. To build a simple homemade instrument, a *characteristic curve tracer*, and use it to produce real-time plots of  $I$  vs.  $V$  for a resistor, diode, and capacitor.

### ***An opportunity and a caution:***

In this and subsequent labs we will often be asking you to devise procedures and techniques on your own without specific instructions (but with help from the lab staff). Be sure to think carefully about your plans before arriving, so when we ask you to do something like “measure a phase shift” you'll have had time to devise a plan. You may even wind up doing something a bit different than the original design especially later in the semester—these are not always just cookbook labs!

**References:** Chapters 1–2 of Eggleston. Also, be sure to look at the sample lab writeup on the course web page — portions of it correspond closely to part I of this lab, though the actual numbers will probably differ.

### ***Equipment:***

1. Signal generator.
2. Digital oscilloscope.
3. Breadboard with power supplies and hookup wire.
4. A selection of 1/4W or 1/2W resistors and small capacitors.
5. Small-signal diodes, such as 1N914/1N4148.
6. Coaxial cables.

## ***I. Thévenin's Theorem***

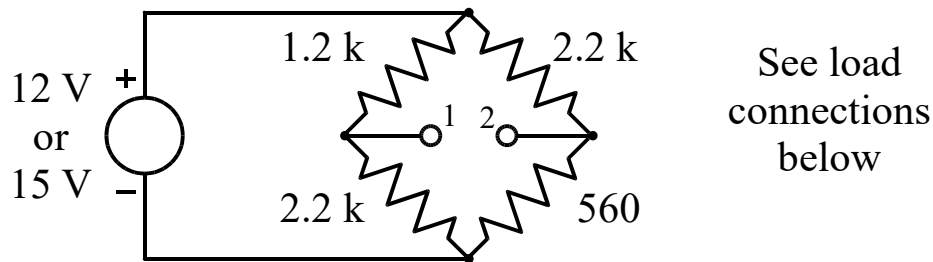
### ***Background:***

In its general form Thévenin's theorem tells us that any two-terminal network of passive linear components can be replaced by a single voltage source in series with a single impedance—it's impossible to tell the difference by any external measurement. As we discussed in lecture, the most direct way to find the Thévenin equivalent voltage  $V_{Th}$  is to measure the open-circuit voltage between terminals 1 and 2 when no load is connected. Next, you can measure the short-circuit current  $I_{sc}$  by connecting a low-resistance ammeter directly between the terminals. The Thévenin equivalent resistance  $R_{Th}$  is found from their ratio,

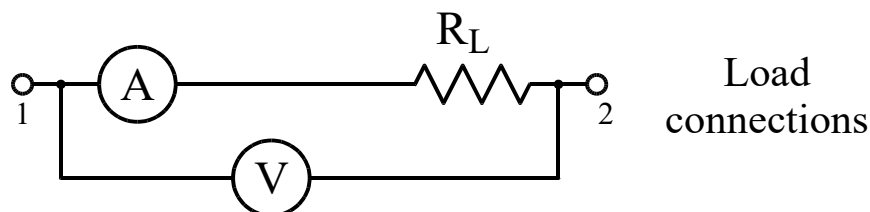
$$R_{Th} = \frac{V_{Th}}{I_{sc}}.$$

There are obvious pitfalls in doing this in the laboratory. Voltmeters have a finite input impedance and so may alter the open-circuit voltage. Ammeters not only exhibit a finite resistance, but may also cause trouble if the short-circuit current is so high that it blows a fuse (or something explodes). For this reason the short-circuit current is rarely measured directly, but we'll make an exception today by choosing a circuit that's relatively benign. Normally a gentler approach would be used if the network properties were unknown.

## A. Procedure



1. Construct the resistive bridge network shown above on your breadboard. This classic circuit is a non-trivial network, for which both of Kirchhoff's laws must be used in a direct analysis. Measure the behavior of this circuit by connecting at least three different resistors between terminals 1 and 2, measuring the load current and voltage in each case. Values for the load resistor  $R_L$  between about  $100\ \Omega$  and  $10\ \text{k}\Omega$  are recommended. After the first few measurements you will notice that the load current can be calculated more accurately than it can be measured, so you may as well stop measuring it. Why is this the case?



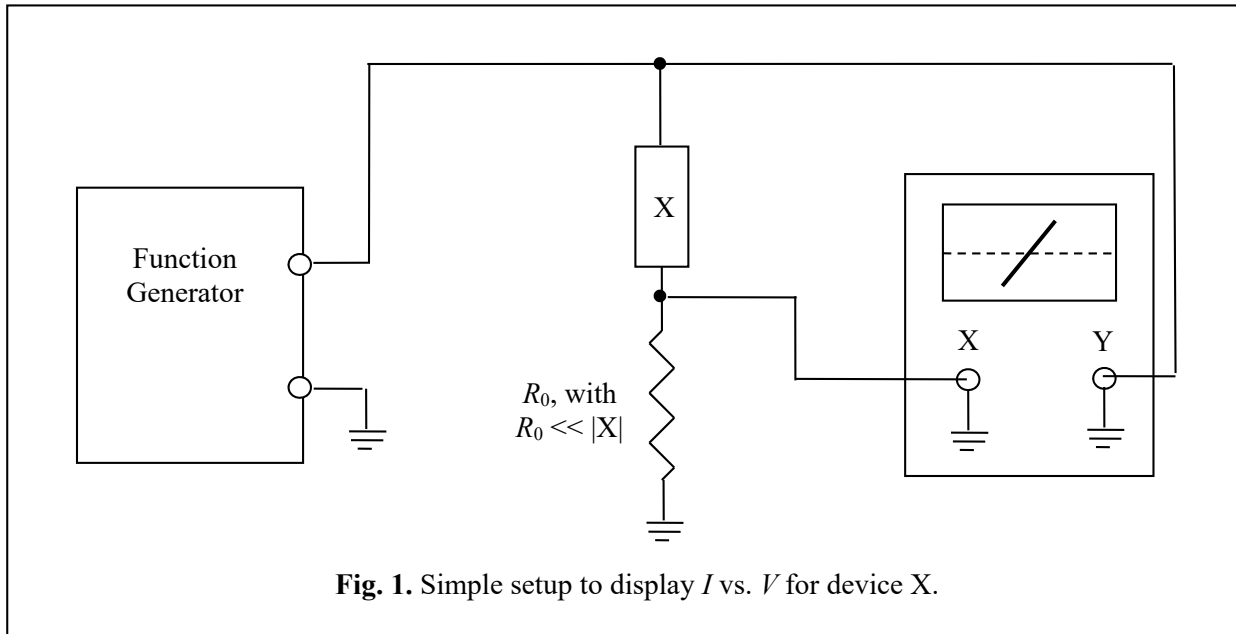
2. Measure the open-circuit voltage and short-circuit current of your bridge circuit, then construct the Thévenin equivalent circuit using a variable dc power supply. Use a series combination of resistors or a variable resistor (potentiometer) to approximate the required value  $R_{Th}$  within 5%. Now repeat a few of the measurements of part 1 on this equivalent circuit.

## B. Questions

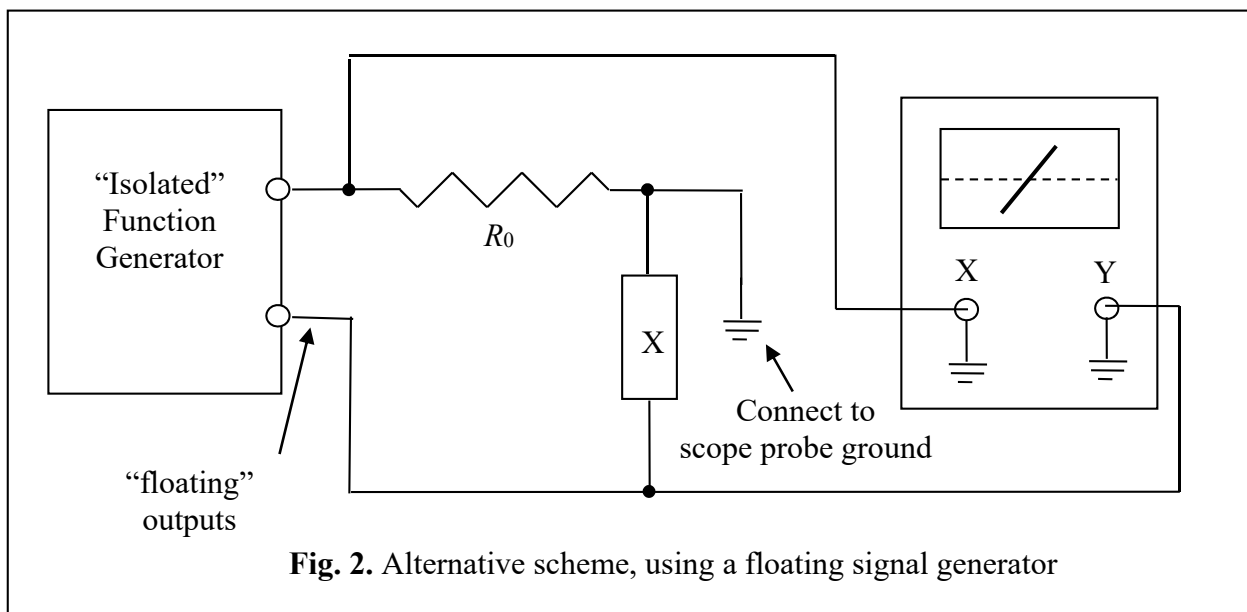
1. Use Kirchhoff's laws to write a set of equations for the voltage and current as a function of the load resistance  $R_L$ . Show that the solution in Eq. (4) of the online sample writeup satisfies your equations when the appropriate component values are used. Compare the predicted results with your measurements from part (1).
2. Comment on the accuracy of your results from part (2). What contribution to the error comes from the inexact value you used for  $R_{Th}$ ? From the finite resistance of the voltmeter? From the finite resistance of the current meter?

## II. Resistor and Diode Characteristic Curves

Any two-terminal electronic device whose current is a frequency-independent real function of the voltage can be fully characterized by a plot of  $I$  vs.  $V$ , often called the *characteristic curve*. You can obtain a good approximation to this curve by using a small *sense resistor* in series to sample the current through a device, then using an oscilloscope in x-y mode to display the current on one channel and the voltage on the other, as sketched below. If the resistor is small enough, the voltage drop across it will be small compared to the voltage drop across the test device  $X$ , and the difference can be ignored.



In principle, you can do even better by using the following clever scheme, worked out by Prof. Hines some years ago, that displays the  $I$  vs.  $V$  characteristics (or more precisely,  $I$  vs.  $-V$ ) with no approximations. The problem is that this involves *floating* the output of a sine or triangle



wave generator, so that neither terminal is at ground potential. Only certain specialized function generators or homemade units allow you to do this directly, since generally one terminal is connected directly to the instrument case. With other signal generators it would be necessary to use an isolation network, or to float the entire instrument using an ungrounded “cheater cord”. Both of these solutions tend to be problematic, often resulting in excess noise or safety issues, so we recommend sticking with the configuration of Fig. 1.

## A. Procedure

Construct the circuit in Fig. 1 on your breadboard. The sense resistor  $R_0$  should be very small compared to the impedance  $X$  of the device under test, so it will not significantly perturb the circuit, but it must also be large enough to produce a measurable voltage drop. A value of around  $10\ \Omega$  to  $50\ \Omega$  might work well for you. Make sure before you begin that you understand how the circuit works, and that you know how to interpret the vertical scale voltage readings in terms of current through the test device  $X$ . Incidentally, some of the digital ‘scopes have a somewhat grainy x-y display that can be annoying, but at least they work.

1. Connect a resistor as the device  $X$  ( $1000\ \Omega$  works well) and save the oscilloscope trace on a USB drive, being sure to record calibration data for both axes.
2. Repeat the procedure with a diode connected in the forward-biased direction as the test device  $X$ . Have your TA help you with setting the signal amplitude and offset to emphasize the most physically significant region of the resulting nonlinear response curve. What happens when you reverse the diode? In addition to sketching or photographing the scope traces, record a set of  $V$ - $I$  data points in numeric form, either manually or with a USB drive, so that you can replot the data on a semi-log scale (see Questions, below). If time permits, use a sensitive DMM to measure the reverse current  $I_0$  when a small reverse bias (a few volts up to perhaps 15 Volts) is applied to the diode. The best approach is probably to use a fairly large “sense” resistor in series with the diode, say  $1\ \text{M}\Omega$ , and to measure the small voltage drop across the resistor. It should depend only weakly on the voltage, except that if you exceed the *reverse breakdown voltage* the diode will suddenly go to a very low-resistance state, usually destroying it.
3. Finally, connect a small capacitor as the test device  $X$ . Set the signal generator to a sine wave for at least three widely varied frequencies. You will find that the results are no longer frequency independent due to the changing impedance of the capacitor, which will be discussed in lecture shortly. Just for fun, try triangle waves rather than sine waves. Can you explain what you see?

## B. Questions for Part II

1. From the slope of the oscilloscope trace, determine the resistance of your resistor, by converting your plot of  $V_x$  vs  $V_y$  to a plot of  $I$  vs.  $V_y$ .
2. According to the simplest theoretical treatment of a P-N junction, a diode that conducts leakage current  $I_0$  when reverse-biased should have a forward current given very approximately by an exponential function,

$$I = I_0 \left( e^{eV/k_B T} - 1 \right)$$

where  $k_B$  is Boltzmann's constant,  $1.38 \times 10^{-23}$  J/K, and  $e$  is the electron charge,  $e=1.6 \times 10^{-19}$  C. Make a semi-log plot of the  $I$  vs.  $V$  characteristics and compare a hand or computer fit to the results with the above equation. *Hint:* Notice that the factor of 1 can be neglected whenever the voltage is high enough that  $eV \gg k_B T$ , making your semi-log plot a straight line. Since you know the other constants, you should be able to roughly (*very* roughly) determine the room temperature  $T$ . What is your result, keeping in mind that it will be in degrees K, not C?

3. From the  $I$  vs.  $V$  curves, estimate the reactance  $X_C$  (i.e., the ratio of  $|V|$  to  $|I|$ ) of the capacitor at each of the driving frequencies for which you used sinusoidal excitation, and compare with the expected value of  $X_C = (\omega C)^{-1}$ . We will derive this result shortly in lecture. Use a numerical ac analysis with PartSim to verify your results for at least one of these frequencies.