Numerical Integration

Physics 258 - DS Hamilton 2004

This worksheet demonstrates how to do an integral numerically using Mathcad. The example is a "complete elliptic integral of the second kind". It is closely related to an integral for the large-angle pendulum lab, which is a "complete elliptic integral of the first kind"

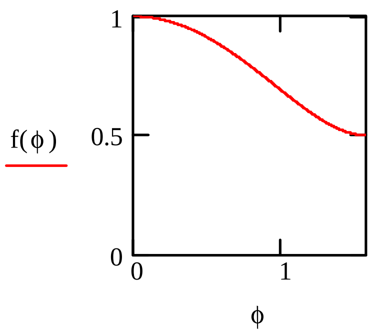
Begin with defining the integrand, the function to be integrated.

$\alpha := 120 \cdot \frac{\pi}{180}$ $\quad k := \sin\left(\frac{\alpha}{2}\right)$

A couple of parameters for the function. You can change the angle α if you want.

$f(\phi) := \sqrt{1 - k^2 \cdot \sin(\phi)^2}$

This is the function to be integrated.



It is a good idea to plot the function over the range of the integration, looking for singularities or other unusual behavior. This is a pretty benign function.

$a := 0$ $\quad b := \frac{\pi}{2}$

These are the lower and upper limits for the integral.

$n := 4$ $\quad i := 1 .. n$

Let's first do this by the simple "midpoint" rule. Divide the interval from a to b into n intervals. The index i just labels each interval.

$\Delta\phi := \frac{b - a}{n}$ $\quad \phi_i := a + i \cdot \Delta\phi$

The width of each interval is $\Delta\phi$, and each interval extends from ϕ_i to $\phi_i + \Delta\phi$.

$$mid := \left[\sum_{i=1}^{n} \left(f\left(\frac{\phi_{i-1} + \phi_i}{2}\right) \cdot \Delta\phi\right)\right]$$

The midpoint rule says to evaluate the function at the midpoint of the interval, multiply by the width of the interval, and then add up the results over all of the intervals.

$mid = 1.2111$

Because the function is almost linear, this gives a very accurate result, even with a relatively small number of intervals.

$TOL := 10^{-4}$

Mathcad has a built-in function to do this. Use the "Calculus" palette to get started. You can change the value of "TOL" in an attempt to improve the accuracy.

$$E := \int_{0}^{\frac{\pi}{2}} f(\phi)\, d\phi$$

$E = 1.2110560276$

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If you right click on the integral, you can change it from an "adaptive" to a "Romberg" algorithm.

$E = 1.2110560278$