Linear Regression III

Physics 258 - DS Hamilton 2004

I want to revisit the worksheet on linear regression (linfit.mcd) again. Let's consider a fit of the same data, but this time, fit to the function y=Bx (line through the origin). The derivation of the formula for the slope and its standard error are homework problems (8.5 & 8.18) in Taylor, 2nd ed. and is also mentioned in the write-up for the data analysis of the current balance experiment.

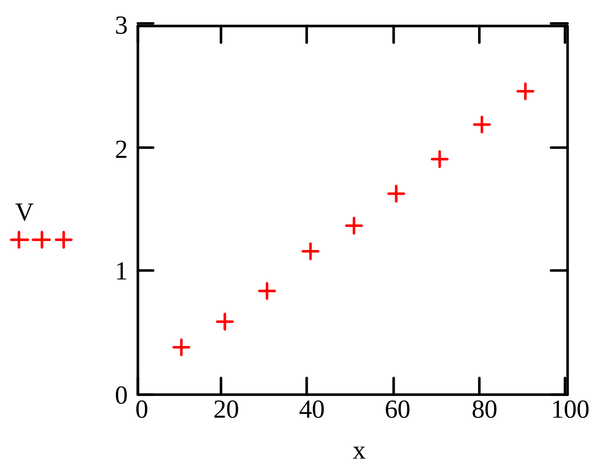
The data for this problem is from Bevington, page 98. The potential difference V has been measured as a function of position x along a current-carrying nickel-silver wire.

$$x := \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \\ 90 \end{bmatrix} \qquad V := \begin{bmatrix} 0.37 \\ 0.58 \\ 0.83 \\ 1.15 \\ 1.36 \\ 1.62 \\ 1.90 \\ 2.18 \\ 2.45 \end{bmatrix}$$

This data could also be entered into the worksheet as a table (Insert-Component-Input Table) or read in from a file. Here the Matrix pallet was used to generate two column vectors.

$n := \text{rows}(x)$ $\qquad n = 9$ $\qquad$ This is just the number of data points.

Always plot your data before you try and fit it to some analytical function. You need to "see" what your data looks like.



Type: V@x <enter> to generate a quick plot of the data. This can often suggest a specific relationship (linear, quadratic, exponential etc) that will make a reasonable model to describe the data.

$\text{slope}(x, V) = 2.6217 \cdot 10^{-2}$ This is the slope when fitting the data to y(x) = intercept + slope*x

Here is the new part

$y := V$ relabel the independent variable as y

$N := \mathrm{last}(y) \qquad N = 8 \qquad i := 0..N$ always be careful here with the range index

$$B := \frac{\sum_i \left(x_i \cdot y_i\right)}{\left[\sum_i \left(x_i\right)^2\right]} \qquad B = 0.0273$$

B is the (best estimate of the) slope.
Compare to 0.0262 from above.

$$s := \sqrt{\frac{\sum_i \left(y_i - B \cdot x_i\right)^2}{N - 1}} \qquad s = 4.556 \cdot 10^{-2}$$

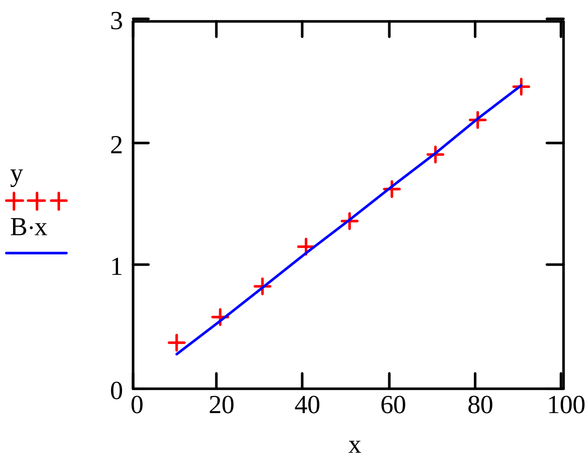
$$\Delta_B := \frac{s}{\sqrt{\sum_i \left(x_i\right)^2}} \qquad \Delta_B = 2.7 \cdot 10^{-4}$$

$$\frac{100 \cdot \Delta_B}{B} = 0.987$$

standard error in the slope B

$$BB := \frac{\sum \overrightarrow{(x \cdot y)}}{\sum \overrightarrow{x^2}} \qquad BB = 2.734 \cdot 10^{-2}$$

If you would like to do this with a "vectorization" of the data arrays, then the slope calculation would look like this.



Always do a visual check of the data and the fit.