Linear Regression II

Physics 258 - DS Hamilton 2004

I want to revisit the worksheet on linear regression (linfit.mcd). Instead of using the built-in functions of Mathcad, this worksheet will use the formulas derived in Taylor, and in Bevington and Robinson. What we gain is the ability to calculate the uncertainty (standard error) in the two fitting parameters, the slope and the intercept.

The data for this problem is from Bevington, page 98. The potential difference V has been measured as a function of position x along a current-carrying nickel-silver wire.

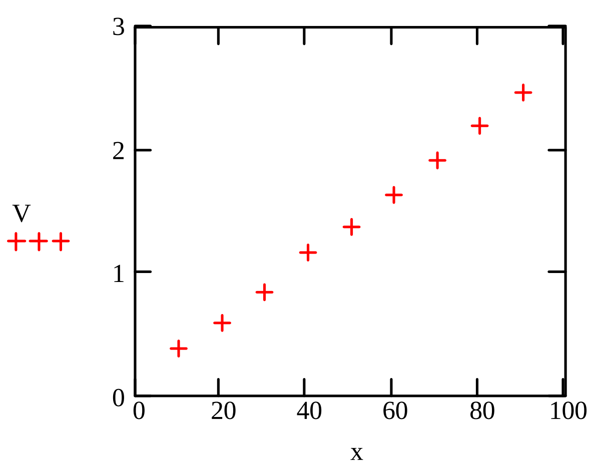
$$x := \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \\ 90 \end{bmatrix} \qquad V := \begin{bmatrix} 0.37 \\ 0.58 \\ 0.83 \\ 1.15 \\ 1.36 \\ 1.62 \\ 1.90 \\ 2.18 \\ 2.45 \end{bmatrix}$$

This data could also be entered into the worksheet as a table (Insert-Component-Input Table) or read in from a file. Here I used the Matrix pallet to generate two column vectors.

$n := \text{rows}(x)$ $\qquad n = 9$

This is just the number of data points.

Always plot your data before you try and fit it to some analytical function. You need to "see" what your data looks like.



Type: V@x <enter> to generate a quick plot of the data. This can often suggest a specific relationship (linear, quadratic, exponential etc) that will make a reasonable model to describe the data.

Here is the new part

$N := \mathrm{rows}(x)$ $\quad N = 9 \quad$ $i := 0 .. N-1$ $\quad$ be careful about the range variables

$y := V$ $\quad$ call the dependent variable "y"

$$\Delta := N\cdot\sum_i \left(x_i\right)^2 - \left(\sum_i x_i\right)^2$$

$$a := \frac{\left[\sum_i \left(x_i\right)^2\right]\cdot\sum_i y_i - \left(\sum_i x_i\right)\cdot\sum_i \left(x_i\cdot y_i\right)}{\Delta} \qquad a = 7.139\cdot 10^{-2}$$

compare to: $\mathrm{intercept}(x,y) = 7.139\cdot 10^{-2}$

$$b := \frac{N\cdot\sum_i \left(x_i\, y_i\right) - \left(\sum_i x_i\right)\cdot\sum_i y_i}{\Delta} \qquad b = 2.6217\cdot 10^{-2}$$

compare to: $\mathrm{slope}(x,y) = 2.6217\cdot 10^{-2}$

$$s_y := \sqrt{\frac{1}{N-2}\cdot\sum_i \left(y_i - a - b\cdot x_i\right)^2} \qquad s_y = 2.6387\cdot 10^{-2}$$

compare to: $\mathrm{stderr}(x,V) = 2.6387\cdot 10^{-2}$

And here is what we have been waiting for

$$\Delta_b := s_y\cdot\sqrt{\frac{N}{\Delta}} \qquad \Delta_b = 3.41\cdot 10^{-4}$$

uncertainty (standard error) in the slope:

$$\Delta_a := s_y\cdot\sqrt{\frac{\sum_i \left(x_i\right)^2}{\Delta}} \qquad \Delta_a = 1.92\cdot 10^{-2}$$

uncertainty (standard error) in the intercept