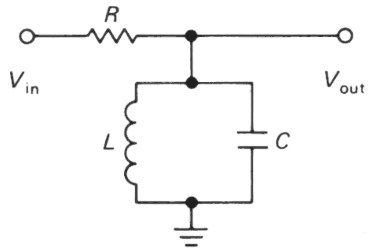
Parallel RCL Circuit

PHYSICS 258/259 D.S. Hamilton

Problem

Obtain the complex transfer function **H**(f) for the circuit shown below over the range 0 < f < ∞. Plot the magnitude of **H** and the phase angle as a function of frequency on a log scale.



The figure is a simple four-terminal RCL tank circuit. The inductor has a series resistance R_L and the capacitor has a parallel resistance R_C.

Parameters

$L := 22.0\cdot mH$ $C := 0.015\cdot \mu F$ $R := 39\cdot k\Omega$

$R_L := 15\cdot\Omega$ $R_C := 100\cdot M\Omega$

Solution

The complex transfer function can be written down by inspection using our previous Mathcad worksheet on the LC circuit.:

$$Z_{LC}(f) := \left[\left(j\cdot 2\cdot\pi\cdot f\cdot L + R_L\right)^{-1} + R_C^{-1} + j\cdot 2\cdot\pi\cdot f\cdot C\right]^{-1} \qquad Z(f) := R + Z_{LC}(f)$$

$$H(f) := \frac{Z_{LC}(f)}{Z(f)}$$

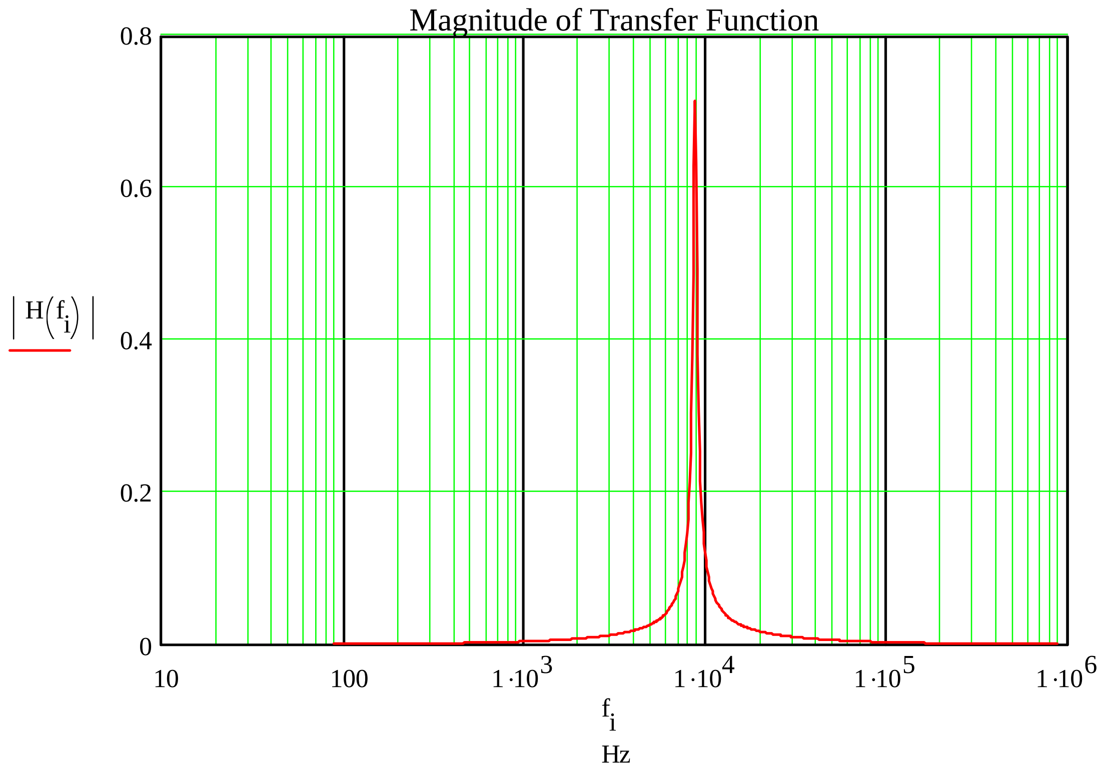
Parameters for plotting

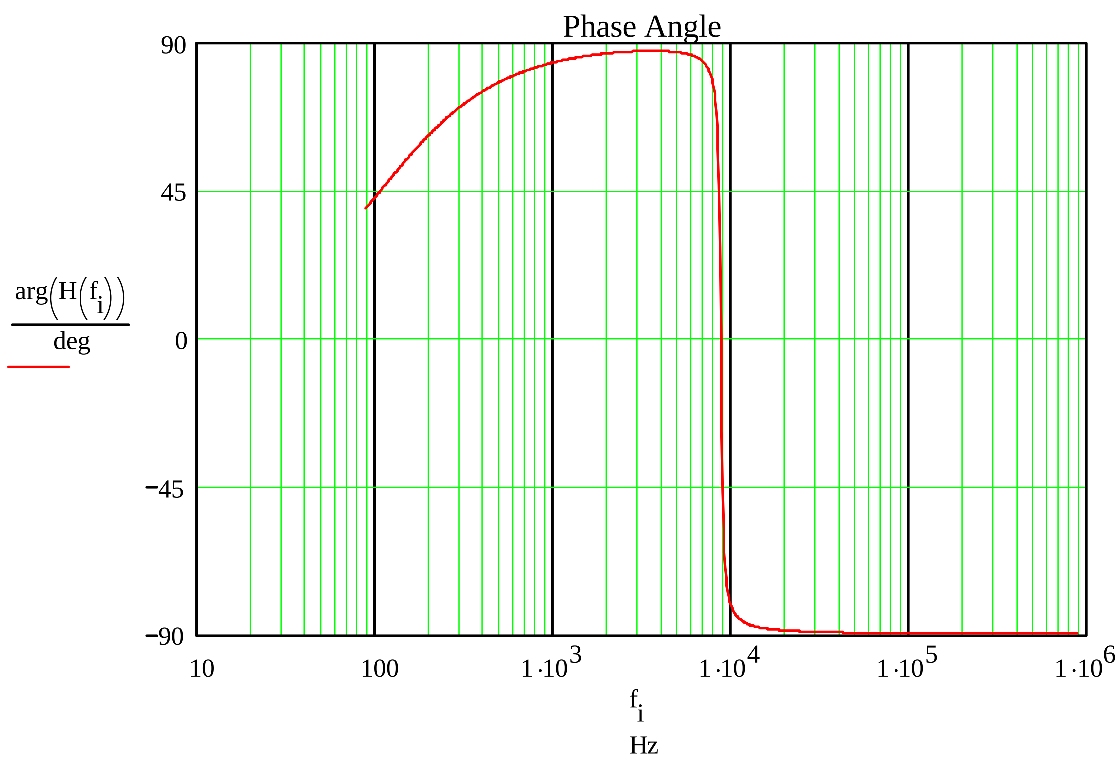
$$f_0 := \frac{1}{2\cdot\pi\sqrt{L\cdot C}} \qquad f_0 = 8.761\cdot 10^3\, s^{-1} \qquad \left|H\left(f_0\right)\right| = 0.715$$

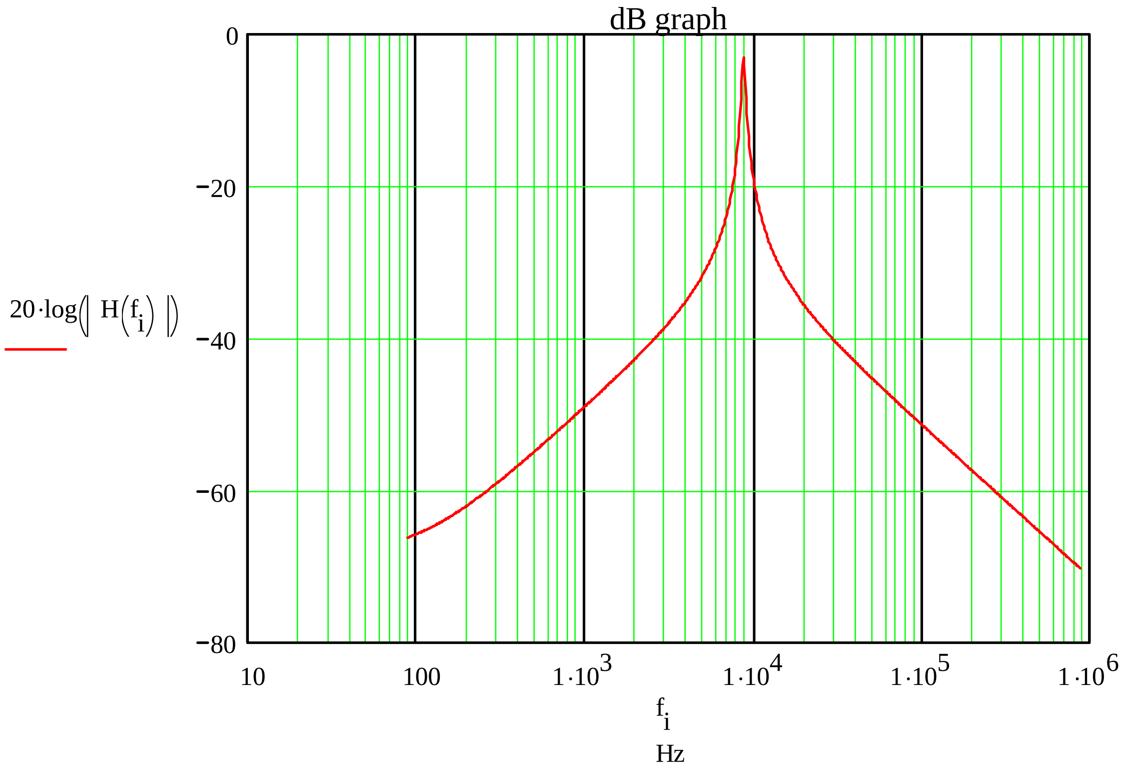
$$f_{low} := .01\cdot f_0 \qquad f_{high} := 100\cdot f_0$$

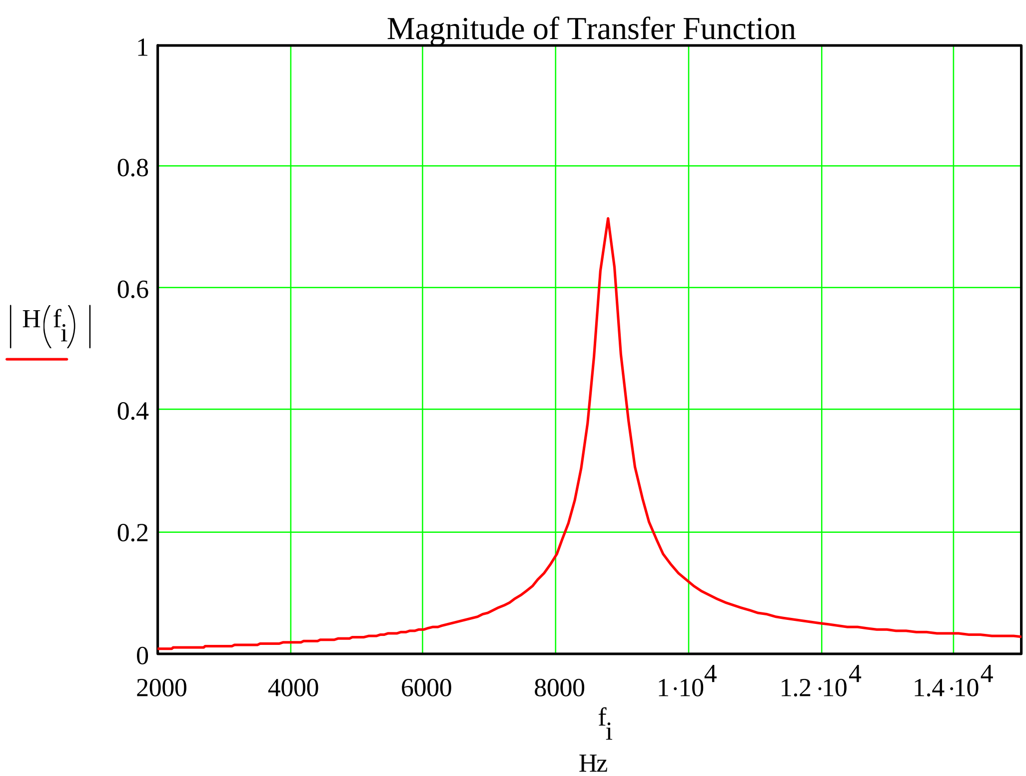
$$N := 800 \qquad i := 0 .. N-1$$

$$r := \log\left(\frac{f_{low}}{f_{high}}\right)\cdot\frac{1}{N} \qquad r = -5\cdot 10^{-3} \qquad f_i := f_{high}\cdot 10^{i\cdot r}$$


Magnitude of Transfer Function
0.8
0.6
0.4
0.2
0
|H(f_i)|
10
100
1·10^3
1·10^4
1·10^5
1·10^6
f_i
Hz


Phase Angle
90
45
0
−45
−90
arg(H(f_i))
deg
10
100
1·10^3
1·10^4
1·10^5
1·10^6
f_i
Hz


dB graph
0
−20
−40
−60
−80
$20 \cdot \log(|H(f_i)|)$
10
100
$1 \cdot 10^3$
$1 \cdot 10^4$
$1 \cdot 10^5$
$1 \cdot 10^6$
f_i
Hz


Magnitude of Transfer Function
1
0.8
0.6
0.4
0.2
0
$|H(f_i)|$
2000
4000
6000
8000
$1 \cdot 10^4$
$1.2 \cdot 10^4$
$1.4 \cdot 10^4$
f_i
Hz

$$Q := 2 \cdot \pi \cdot f_0 \cdot R \cdot C \qquad Q = 32.203 \qquad \frac{1}{Q} = 0.031$$

$$\Delta f_{3dB} := \frac{f_0}{Q} \qquad \Delta f_{3dB} = 272.06\ s^{-1}$$