A scaling relation between pA and AA collisions

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Motivation

▶ Recent measurements of the two particle correlation function at the LHC and RHIC revealed a striking similarity between high multiplicity proton-nucleus (pA) and nucleus-nucleus (AA) collisions

▶ Same physics?? Collective flow in pA ?? Hydro in pA??
[Bozek et.al., Shuryak et. al., Kozlov et. al., ...]

▶ There are also some quantitative differences in the measurements

Idea: Come up with a framework that accounts for the similarities and the differences.

⇒ “Conformal dynamics”
Collective flow in nucleus-nucleus (AA) collisions
Flow in AA

- **Key measurement:** transverse momentum anisotropy

\[ v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \]

- **Interpretation:**
  - system behaves as a fluid with low viscosity
  - different pressure gradients in \( x \) and \( y \) \( \Rightarrow \) anisotropy in \( p_T \)
  - average eccentricity \( \epsilon_2 \equiv \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle} \Rightarrow v_2 \)
    (linear response: \( "v_2 = k \epsilon_2" \))
Flow in AA

• **Key measurement**: transverse momentum anisotropy

\[ \frac{dN}{d^2p_T} = \frac{dN}{p_Tdp_T} \sum_{n=1}^{\infty} (1 + 2v_n \cos(n\phi)) \]

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  ▶ different pressure gradients in $x$ and $y \Rightarrow$ anisotropy in $p_T$
  
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Flow in AA

• **Key measurement:** transverse momentum anisotropy

$$\frac{dN}{d^2p_T} = \frac{dN}{p_T dp_T} (1 + 2v_2 \cos(2\phi) + 2v_3 \cos(3\phi) + \ldots)$$

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Flow in AA

\[
\frac{dN}{d\phi} \propto 1 + 2v_2 \cos(2\phi - 2\Psi_2) + 2v_3 \cos(3\phi - 3\Psi_3) + \ldots
\]

- **The actual measurement**: two particle correlation fnc.

\[
C(\Delta\phi) \propto \left\langle \frac{dN}{d\phi} \frac{dN}{d(\phi + \Delta\phi)} \right\rangle_{\Psi_2, \Psi_3, \ldots}
\propto 1 + 2\langle v_2^2 \rangle \cos(2\Delta\phi) + 2\langle v_3^2 \rangle \cos(3\Delta\phi) + \ldots
\]

*notation*: \(v_2\{2\} \equiv \sqrt{\langle v_2^2 \rangle}, \quad v_3\{2\} \equiv \sqrt{\langle v_3^2 \rangle}, \quad \ldots\)
Flow in AA

A typical measurement:

\[ C(\Delta \phi, \Delta \eta) \]

[ATLAS, PRC 86 014907]

⇒ extract \( \langle v_2^2 \rangle \), \( \langle v_3^2 \rangle \) from a Fourier fit
Flow in nucleus-nucleus (AA) collisions

The triumph of linear response:

- To a good approximation:

\[ v_2 \{ 2 \} = k_2 \sqrt{\langle \epsilon_2^2 \rangle}, \quad v_3 \{ 2 \} = k_3 \sqrt{\langle \epsilon_3^2 \rangle} \]

[Niemi et. al. PRC87 054901]
The recent proton-nucleus (pA) results
The recent pA results

A typical event (low multiplicity)

[diagram showing CMS pPb √s = 5.02 TeV, N < 35, with conditions 1 < p_T^{trig} < 2 GeV/c, 1 < p_T^{assoc} < 2 GeV/c]

[data from CMS, slides from G. Roland, RBRC workshop Apr. 15-17, 2013, also PLB 724 213]
The recent pA results

A typical event (higher multiplicity)

CMS pPb $\sqrt{s} = 5.02$ TeV $35 \leq N < 90$

1 < $p_T^{\text{trig}}$ < 2 GeV/c
1 < $p_T^{\text{assoc}}$ < 2 GeV/c
The recent pA results

A somewhat rare event
The recent pA results

A very rare event (high multiplicity)
The recent pA results

Compare pA and AA at the same multiplicity

(a) CMS PbPb $\sqrt{s_{NN}} = 2.76$ TeV, $220 \leq N_{\text{trk}}^{\text{offline}} < 260$

1 $< p_T^{\text{trig}} < 3$ GeV/c
1 $< p_T^{\text{assoc}} < 3$ GeV/c

(b) CMS pPb $\sqrt{s_{NN}} = 5.02$ TeV, $220 \leq N_{\text{trk}}^{\text{offline}} < 260$

1 $< p_T^{\text{trig}} < 3$ GeV/c
1 $< p_T^{\text{assoc}} < 3$ GeV/c

PbPb

pPb
The recent pA results

$v_2$ and $v_3$

blue circles: $(v_2\{2\})_{\text{PbPb}}$
red triangles: $(v_2\{2\})_{\text{pPb}}$

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red triangles: $(v_3\{2\})_{\text{pPb}}$

$|\Delta \eta| > 2$, $0.3 < p_T < 3 \text{GeV}$, PbPb: 2.76 TeV, pPb: 5.02 TeV

[CMS, PLB 724 213]
The recent pA results

Transverse momentum dependence of $v_2$ and $v_3$

| $\Delta \eta$ | $0.3 < p_T < 3 GeV$, PbPb: 2.76 TeV, pPb: 5.02 TeV |

[CMS, PLB 724 213]
“Conformal dynamics” (as an elliptical cow approximation)

- **Initial state:** $N_{clust}$ independently distributed clusters such that the multiplicity $N \propto N_{clust}$

- **“Conformal dynamics”:** The density of clusters sets a momentum scale: only scale other than the system size $L$

\[
\tau_R \sim l_{mfp} \sim \frac{1}{T_i}
\]

⇒ Universal Knudsen numbers at fixed multiplicity

\[
\frac{l_{mfp}}{L} \propto \frac{1}{T_i L} = f \left( \frac{dN}{dy} \right)
\]

⇒ The pA system is smaller but hotter

- **Flow emerges as a collective response to the geometry:**

\[
v_{2,3} = k_{2,3} \left( \frac{l_{mfp}}{L} \right) \times \epsilon_{2,3}
\]

(response coefficient \times geometry)

(e.g. saturation inspired model: $N_{clust} = \pi Q_s^2 L^2 \Rightarrow \frac{l_{mfp}}{L} \propto \frac{1}{Q_s L} \propto \frac{1}{\sqrt{dN/dy}}$)
Linear response + conformal dynamics:

\[ v_2 = k_2 \left( \frac{dN}{dy} \right) \epsilon_2 \quad v_3 = k_3 \left( \frac{dN}{dy} \right) \epsilon_3 \]

How different are the geometries?
Independent cluster model [Bhalero, Ollitrault]

- Distribution of clusters:

\[ n(x) = \bar{n}(x) + \delta n(x) \]

\[ \langle \delta n(x)\delta n(y) \rangle = \bar{n}(x)\delta^{(2)}(x - y) \]

- Flow is sourced both by
  - average geometry \( \bar{n}(x) \)
  - fluctuations \( \delta n(x) \)

- \( \epsilon_2 \rightarrow v_2 \): driven by average geometry and fluctuations (AA)
  - fluctuations (pA)

- \( \epsilon_3 \rightarrow v_3 \): driven by fluctuations (AA and pA)
Eccentricity and elliptic flow

- **Linear response:** \( v_2 = k_2 \sqrt{\langle \epsilon_2^2 \rangle} \)

- **Conformal scaling:** \( k_{2,pA} = k_{2,AA} \equiv k_2(dN/dy) \)

- **Eccentricity in non-central AA**

  \[
  (\epsilon_2 \{2\})_{AA}^2 = \epsilon_s^2 + \langle \delta \epsilon_2^2 \rangle
  \]

- **Eccentricity in pA:**

  \[
  (\epsilon_2 \{2\})_{pA}^2 = \langle \delta \epsilon_2^2 \rangle
  \]

  \[
  \langle \delta \epsilon_2^2 \rangle = \frac{\langle r^4 \rangle}{N_{clust} \langle r^2 \rangle^2}
  \]
In order to compare the elliptic flow in pPb and PbPb justly one should “remove” the overall geometry from AA and isolate the fluctuation driven part:

\[ (v_2\{2\})_{PbPb,rscl} \equiv \sqrt{\frac{\langle \delta \epsilon_2^2 \rangle}{\langle \epsilon_2 \{2\} \rangle^2_{PbPb}}} (v_2\{2\})_{PbPb} \]

Conformal dynamics suggest that
\[ (v_2\{2\})_{PbPb,rscl} = (v_2\{2\})_{pPb} \] at the same multiplicity
Eccentricity and elliptic flow

The scaling factor \( \sqrt{\frac{\langle \delta \epsilon_2^2 \rangle}{\epsilon_2 \{ 2 \}^2_{PbPb}}} \) is a nontrivial function of multiplicity and is calculated by Glauber model (not a fit!).

No fine tuning!
Eccentricity and elliptic flow

- Don’t know the cluster distribution for pA. Does it matter??

\[ \langle \delta \epsilon^2 \rangle_{\text{hard sphere}} \approx 0.85 \]

Gaussian seems plausible. Compare with nuclear geometry.

\[
\begin{array}{c}
\text{eccentricity ratio} \\
\text{N_{offline}} \\
\text{trk}\end{array}
\]

\[
\langle \delta \epsilon^2 \rangle_{\text{gaus}} / \langle \delta \epsilon^2 \rangle_{\text{AA}}^{1/2}
\]
Eccentricity and elliptic flow

- Don’t know the cluster distribution for pA. Does it matter?? NO!
Don’t know the cluster distribution for pA. Does it matter?? NO!

Two very different distributions: \[ \sqrt{\langle \delta \epsilon_2^2 \rangle_{\text{hard-sphere}} / \langle \delta \epsilon_2^2 \rangle_{\text{Gaussian}}} \approx 0.85 \]

Gaussian seems plausible. Compare with nuclear geometry
Triangular flow

- Linear response: $v_3 = k_3 \sqrt{\langle \epsilon_3^2 \rangle}$
- Conformal scaling: $(v_3 \{2\})_{pA} = k_3 \sqrt{\langle \delta \epsilon_3^2 \rangle_{pA}} \approx (v_3 \{2\})_{AA} = k_3 \sqrt{\langle \delta \epsilon_3^2 \rangle_{AA}}$

\[
\langle \delta \epsilon_3^2 \rangle = \frac{\langle r^6 \rangle}{N_{\text{clust}} \langle r^2 \rangle^3}
\]

- Compare $\langle \delta \epsilon_3^2 \rangle_{pA}$ with that of nuclear geometry
Triangular flow

Expect $v_3$s to be the same.
Triangular flow

Expect $v_3$s to be the same.

blue circles: $(v_3(2))_{\text{PbPb}}$
red triangles: $(v_3(2))_{\text{pPb}}$
Transverse momentum dependence of the flow

- **Scaling argument** (dictated by “conformal dynamics”):

  \[ v_2(p_T) = \xi_2 \times \epsilon_2 \times f_2 \left( \frac{p_T}{\langle p_T \rangle} \right) \]

  - response coef.
  - geometry
  - universal function at fixed dN/dy

- **Input:**
  \[ \frac{\langle p_T \rangle_{pPb}}{\langle p_T \rangle_{PbPb}} \simeq 1.25 \]
  (ALICE, arXiv:1307.1094)

- **Expect:**
  - \[ \frac{L_{PbPb}}{L_{pPb}} = \frac{T_{t_PpPb}}{T_{t_PbPb}} \simeq 1.25 \] (pA is smaller and hotter)
  - \[ [v_2 \{2\} (p_T)]_{pPb} = [v_2 \{2\} \left( \frac{p_T}{\kappa} \right)]_{PbPb,rscl} \]
Scaling of $v_2(p_T)$

PbPb

pPb

original $v_2$

rescaled $v_2$

Notice the slopes at small $p_T$!
Scaling of $v_3(p_T)$

Notice the slopes at small $p_T$!
ATLAS recently adopted and extended our analysis recoil subtraction ⇒ remarkable agreement even at larger $p_T$!
The recent ALICE measurement reveals that $\frac{R_{PbPb}}{R_{pPb}} \sim 1.4$ at the highest multiplicity measured (ALICE, arXiv:1404.1194)

Compare with the conformal scaling result $\frac{L_{AA}}{L_{pA}} \sim 1.25$
Conclusions

- The similarities as well as the differences between the high multiplicity pA and AA can be explained in a quantitative fashion by a simple conformal scaling framework.

- Universal Knudsen number \( (l_{mfp}/L) \) at fixed multiplicity (pA is smaller but hotter).

- No need to fine tune parameters.

- It seems phenomenologically reasonable to conclude that the flow in pA and AA stem from the same physics.

- Not necessarily hydrodynamics, viscous corrections can be large.
Flow in AA

ATLAS

Pb-Pb \( \sqrt{s_{NN}} = 2.76 \) TeV

\( L_{\text{int}} = 8 \mu b^{-1} \)

\( 2 < p_T^a, p_T^b < 3 \) GeV

ATLAS, PRC 86 014907 (2012)
eg. saturation inspired model (early times):

Cluster density ↔ saturation momentum: \( Q_s^2 = \frac{N_{\text{clust}}}{\pi L^2} \)

Mean free path, relaxation time (at early times):
\[ \tau_R \sim \frac{l_{\text{mfp}}}{L} \propto \frac{1}{Q_s L} = \frac{1}{\sqrt{dN/dy}} \]

eg. Bjorken expansion (later times):

For flow a more relevant scale is \( \tau \sim L \)

Viscous corrections, etc:
\[ \frac{l_{\text{mfp}}}{L} \propto \frac{1}{T(\tau)L} \propto \frac{1}{(dN/dy)^{1/3}} \]

Consistent with more complicated hydro models
Jet energy loss heuristics (à la BDMPS)

- Different scales are involved:
  - Formation length, $l_{form} \propto \frac{\omega}{k^2_\perp}$
  - Mean free path, $l_{mfp}$
  - System size $L$

- Transverse momentum is accumulated by random walk, $\hat{q} \equiv d\langle k^2_\perp \rangle / dt$

- Depending on $\omega$ of radiated gluon, spectrum is different
  - $\omega \frac{dN_g}{d\omega \, dz} \sim \frac{\alpha_s}{\ell_{mfp}} \quad (\omega < \hat{q} \ell_{mfp}^2)$ (Bethe-Heitler)
  - $\omega \frac{dN_g}{d\omega \, dz} \sim \alpha_s \sqrt{\frac{\hat{q}}{\omega}} \quad (\hat{q} \ell_{mfp}^2 < \omega < \hat{q} L^2)$ (LPM)
  - $\omega \frac{d(\Delta N_g)}{d\omega} \sim \alpha_s \frac{(\hat{q} L^2)^2}{\omega^2} \quad (\omega > \hat{q} L^2)$ (“deep LPM”)
Jet energy loss heuristics for pA

- Depending on the energy, \( E \), of the hard parton the total energy loss is:

  \[
  \Delta E \sim \alpha_s \sqrt{E \hat{q}} L \quad (E < \hat{q} L^2), \quad \Delta E \sim \alpha_s \hat{q} L^2 \quad (E > \hat{q} L^2)
  \]

- **Conformal scaling:** \( \hat{q}_{pA} = \kappa^3 \hat{q}_{AA} \)

- **Semi-qualitative predictions:**
  - Larger transverse momentum broadening in pA
  - The transition from \( \Delta E \propto L \) regime to \( L^2 \) regime requires a larger parton energy!