STATISTICAL MECHANICS

Preliminary Examination

Tuesday 08/20/2013

9:00am - 12:00pm in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book or on sheets of paper stapled together. Make sure you clearly indicate who you are and what is the problem you are solving on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

On the last page you will find some potentially useful formulas.

Problem 1. Consider a system composed of a large number N non-interacting distinguishable particles. Each of the particles can be in one of the three states with energies 0, ϵ, and 2ϵ.

(a) Calculate the canonical partition function for the system with arbitrary N. (Hint: remember that the partition function for a system of a combination of two independent subsystems is the product $Z = Z_1 Z_2$.)

(b) What is the entropy of the system? Make a plot (approximately).

(c) What is the energy of the system at temperature T? Make an approximate plot. What is the maximal energy of the system that can be reached by heating it?

Problem 2. An ideal classical gas of N particles of mass m contained in a cylinder of height h and volume V is subject to a uniform gravitational field and temperature T.

(a) Find the pressure and internal energy density of the gas as function of h.

(b) What is the height of the center of mass of the gas?

(c) What is the heat capacity of the system?

(d) Use your results to find values in the limiting cases $kT \ll mgh$ and $kT \gg mgh$.

Problem 3. An ideal gas is contained in a box. Find the average number of particles hitting an area of size $1\,\mathrm{cm}^2$ on a side of the box, per second, with normal component of velocity larger than v_0. Estimate numerically the total number of particles hitting this area in one second at room temperature and atmospheric pressure for diatomic nitrogen (for N_2 at standard conditions $m = 28/N_A$ g, $V \simeq 22.4$ l).

Problem 4. Helium atoms can be adsorbed on a graphite surface and the resulting system can be modeled as an ideal gas at temperature T, free to move on the graphite surface, with energy $E = \mathbf{p}^2/2m - \epsilon$ ($\epsilon > 0$ is the magnitude of binding energy per He atom of mass m).

(a) Write down the canonical (classical) partition function $Z(N_a, T, A)$ for the He atoms on this surface. Here N_a is the number of adsorbed helium atoms and A is the area of the graphite surface. Obtain a closed expression for Z, defining any new variables you use.

(b) Calculate the chemical potential for the adsorbed atoms.

(c) Suppose we have a gas of He above this graphite surface. Repeat the above steps for this gas and find the relevant chemical potential in terms of pressure and other relevant thermodynamical variables.

(d) Briefly explain how you would obtain the equilibrium density of He atoms adsorbed on graphite (no calculations here).

Problem 5. Consider two interacting spin-1/2 particles, placed in a magnetic field B along a chosen axis, described by the Hamiltonian

$$\mathcal{H} = -\mu B(S_{1z} + S_{2z}) - JS_{1z}S_{2z}.$$

(The variables used above have their usual meanings.)

(a) Write down an appropriate partition function for this system.

(b) Calculate the average magnetization $M(B,T)$ at temperature T. Check whether there is any spontaneous magnetization in this system at any $T > 0$.

(c) Calculate the expectation value $\langle S_{1z}S_{2z}\rangle$ and find its limits as $T \to 0$ and $T \to \infty$ when $B \approx 0$. Briefly explain the physics of these limiting cases.

$$N_A \simeq 6 \times 10^{23}$$

$$\ln N! \approx N \ln N - N \quad \text{as} \quad N \to \infty$$

$$\int_{-\infty}^{+\infty} dx \exp(-\alpha x^2 + \beta x) = \sqrt{\frac{\pi}{\alpha}} \exp\left(\frac{\beta^2}{4\alpha}\right) \quad \text{with} \quad \mathrm{Re}(\alpha) > 0$$

$$\int_0^{\infty} dx\ x \exp(-\alpha x^2) = \frac{1}{2\alpha}$$

$$a = \sqrt{\frac{m\omega}{2\hbar}} \left(x + i\frac{p}{m\omega}\right)$$