ELECTRICITY AND MAGNETISM

Preliminary Examination

Thursday January 12, 2012

09:00 - 12:00 in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

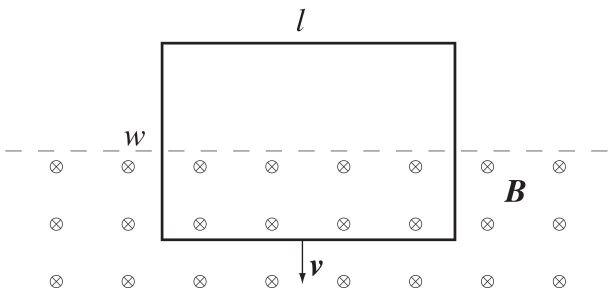
Each answer must be presented **separately** in an answer book or on individual sheets of paper stapled together. Make sure you clearly indicate who you are and the problem you are solving. Double-check that you include everything you want graded, and nothing else.

On the last two pages you find the forms of the standard vector calculus operations in the three most common coordinate systems.

Problem 1. Consider a rectangular loop of wire, with dimensions l and w, falling under gravity in a region of magnetic field B; consult the figure below. The loop has resistance R, self-inductance L, and mass m. At time $t = 0$ the loop is released from rest with its lower edge just entering the field region. Consider the motion of the loop only for times before its upper edge enters the field.

(a) Assume that the self-inductance can be ignored, but not the resistance. Find the current and velocity of the loop as functions of time.

(b) Now assume that the resistance can be ignored, but not the self-inductance. Find the current and velocity of the loop as functions of time for this case.

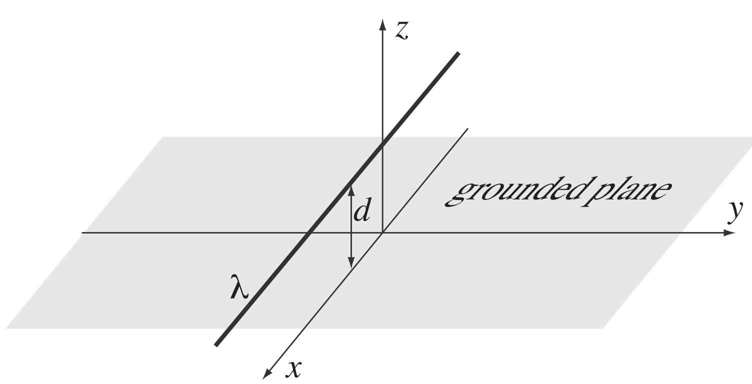


Problem 2. Consider an infinitely long uniform line of charge with linear charge density λ.

(a) Calculate the electric field at a distance s from the line of charge.

(b) Calculate the electric potential a distance s from the line of charge. Take $V(s = d) = 0$.

(c) Now consider this line of charge to be located above a grounded conducting plane, as shown. Take the grounded plane to be the xy plane and take the location of the line of charge to be at $y = 0$, $z = d$, i.e., parallel to the x axis a distance d above it. Calculate the potential for all points (x, y, z) above the grounded plane, for $z > 0$.



Problem 3. Given a sphere of radius R with uniform charge density ρ rotating at angular velocity ω about an axis through the center, find the magnitude of the magnetic field created at the center.

Problem 4.

(a) Write down the differential form of all four Maxwell's equations with charges and currents. Give a brief description of the physical meaning of each equation.

(b) In the absence of charges and currents, use Maxwell's equations in (a) to derive the wave equation for the electric field $\mathbf{E}$.

(c) Using the method of separation of variables, solve the wave equation in (b) in Cartesian coordinates for the x component of the electric field propagating along the z axis.

Standard vector operations in three common coordinate systems

Cartesian coordinates x, y, z

$$
\begin{aligned}
\nabla &= \hat{\mathbf{e}}_x \frac{\partial}{\partial x} + \hat{\mathbf{e}}_y \frac{\partial}{\partial y} + \hat{\mathbf{e}}_z \frac{\partial}{\partial z} \\
\nabla \cdot \mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
\nabla \times \mathbf{A} &= \hat{\mathbf{e}}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{\mathbf{e}}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{\mathbf{e}}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
\nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\end{aligned}
$$

cylindrical coordinates ρ, ϕ, z

$$
\begin{aligned}
\nabla &= \hat{\mathbf{e}}_\rho \frac{\partial}{\partial \rho} + \hat{\mathbf{e}}_\phi \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{e}}_z \frac{\partial}{\partial z} \\
\nabla \cdot \mathbf{A} &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\
\nabla \times \mathbf{A} &= \hat{\mathbf{e}}_\rho \left[\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] + \hat{\mathbf{e}}_\phi \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right] + \hat{\mathbf{e}}_z \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi} \right] \\
\nabla^2 &= \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}
\end{aligned}
$$

spherical polar coordinates r, θ, ϕ

$$
\begin{aligned}
\nabla &= \hat{\mathbf{e}}_r \frac{\partial}{\partial r} + \hat{\mathbf{e}}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\mathbf{e}}_\phi \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} \\
\nabla \cdot \mathbf{A} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta A_\theta) + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi} \\
\nabla \times \mathbf{A} &= \hat{\mathbf{e}}_r \frac{1}{r \sin\theta} \left[\frac{\partial}{\partial \theta} (\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\mathbf{e}}_\theta \left[\frac{1}{r \sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right] + \hat{\mathbf{e}}_\phi \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \\
\nabla^2 &= \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \sin\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \phi^2} \\
& \left[\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \equiv \frac{1}{r} \frac{\partial^2}{\partial r^2} r \right]
\end{aligned}
$$