CLASSICAL MECHANICS/ ELECTRICITY AND MAGNETISM

Preliminary Examination

January 13, 2010

9:00 - 15:00 in P-121

Answer a total of **SIX** questions, choosing **THREE** from each section. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book or individual sheets of paper. Make sure you clearly indicate who you are, and the problem you are answering on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

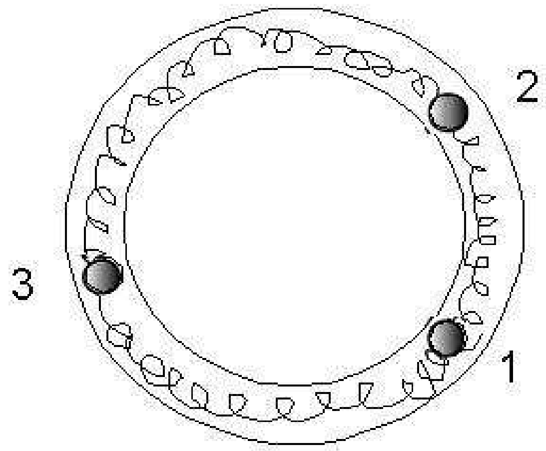

Figure 1: For problem **A1**.

SECTION A - Classical Mechanics

A1. Three particles, each of mass m, are constrained to lie on a (horizontal) circle and are connected by identical springs lying on the circle, each of spring constant k. Find the general solution for the motion of these particles and the frequencies of small oscillations associated with this system.

(Hints: derive equations of motion for angular positions ϕ_i of each ith particle from the Lagrangian; write the equation of motion as a linear system of equations containing a matrix **A** of constants and a vector $\vec{\phi} = (\phi_1, \phi_2, \phi_3)$. Assume perfect springs and neglect friction.)

A2. A ping-pong ball of mass m is placed on top of a basketball of mass M and the two are dropped from a height h ($h \gg$ diameter of basketball) on a hard floor (similar to a basketball court assume all collisions are elastic). Assuming $m \ll M$, how high would the ping-pong ball bounce back?

Hint: Air resistance is very small but it creates a very small finite separation between the basketball and ping-pong ball when they hit the floor. This results in the basketball hitting the ping-pong ball after collision with the floor.

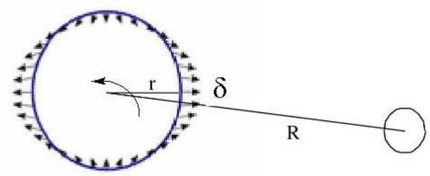

Figure 2: For problem **A3**.

A3. Assume that the Earth is initially a rotating radially symmetric sphere. The Earth's moon perturbs the shape of the gravitational potential surface of the Earth by an amount

$$V_T(r,\psi) = kGm\ (r^2/R^3)\ P_2(\cos\psi),$$

where m is the mass of the moon, and r and R are measured from the center of mass of the unperturbed sphere, with R the distance to the moon. P_2 is a Legendre function of order 2 and (the polar angle) ψ is measured counterclockwise from the Earth-moon line. The angle δ (which represents a particular value of ψ) is shown in the figure and identifies a point on the deformed surface of the Earth representing a phase lag between the peak tidal deformation and the Earth-moon line, and k is a constant measured to be 0.303. Assume a moment of inertia I for an Earth rotating at angular velocity ω.

(a) Find an expression for the deceleration of the Earth's rotation due to the torque exerted by the moon on the deformed Earth.

(b) By analogy to the Q of a resonant circuit, how can the tidal phase δ be related to energy dissipation in heat?

Hints: Note: $P_2(x) = (3x^2 - 1)/2$. First, find the force exerted by the tidal perturbation on the moon by evaluating the relevant gradient of the potential.

A4. A perfectly rough, solid, uniform sphere rests symmetrically upon a circular cylinder, which is fixed with its axis horizontal. If the sphere is slightly disturbed, it rolls down. Show that it begins to slide when the angle θ between

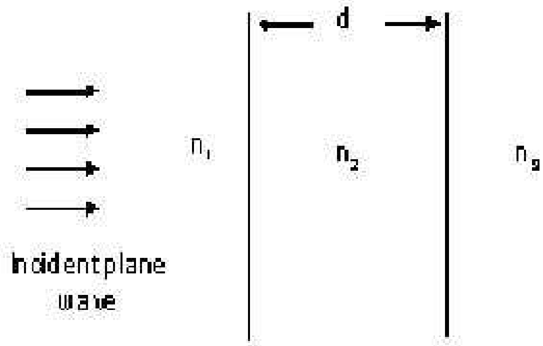

Figure 3: For problem **B1**.

the line of centers and the upward vertical is given by

$$2\sin\theta = \mu\ (17\cos\theta - 10),$$

where μ (> 0) is the coefficient of friction between the two surfaces. Also, show that such sliding occurs before the bodies separate (i.e., first it rolls, then it slides after which the bodies separate).

SECTION B - Electricity and Magnetism

B1. A plane wave is incident on a layered surface shown below. The quantities n_1, n_2, n_3 are the refractive indices of the three layers and d is the thickness of the middle layer. The layers 1 and 2 are semi-infinite. Derive an expression for the reflection coefficient of a plane wave incident on the layered surface from medium 1.

The medium 1 (index = n_1) is part of an optical system (e.g., a lens); medium 3 is air (n_3 =1). It is desired to put an optical coating (medium 2) on the surface so that there is no reflected wave at frequency ω. Determine the thickness d and index n_2 for this case?

Hint: The reflected waves from the two interfaces must have a phase difference of π at interface 1 to have no reflected wave to medium 1 i.e., when the reflected wave from n_2-n_3 interface arrives at n_1-n_2 interface, it must have undergone a phase change of π.

B2. Consider a center fed linear antenna (whose length d is much smaller than the wavelength λ and, $k = 2\pi/\lambda$) carrying a current I ($I = I_0 \sin(\omega t)$). Derive expressions for electric and magnetic fields at a distance $r (r \gg \lambda, d)$ from the antenna. Show that the total power radiated is given by

$$P = I_0^2 \, (kd)^2 \; / \; (12c)$$

(in CGS units). Answer in MKS or SI units is also OK.

B3. You are walking along a hallway of a building wearing polaroid sunglasses and looking at the reflection of a light fixture on the waxed floor. Suddenly at a distance d from the light fixture, the reflected image momentarily disappears. Show that the refractive index n of the reflecting floor can be determined from the ratio of distances

$$n = d/(h_1 + h_2),$$

where h_1 is your height and h_2 is the height of the light fixture. You may assume light from the fixture is unpolarized with a mixture of 50% TE and 50% TM, and that polaroid sunglasses filter out horizontally polarized light. Explain your reasoning.

B4. A conducting sphere of radius R is placed in the field of a point charge q at a distance a ($a > R$) from the center of the sphere. The system is immersed in a homogeneous dielectric of permittivity ϵ. Find the potential at any point and the charge distribution σ induced on the sphere in the following two cases:

(a) the potential of the sphere is maintained at a constant value V (and the potential at infinity is zero).

(b) the charge on the sphere is Q.