Preliminary Exam: Classical Mechanics, Monday August 21, 2017. 9:00-12:00

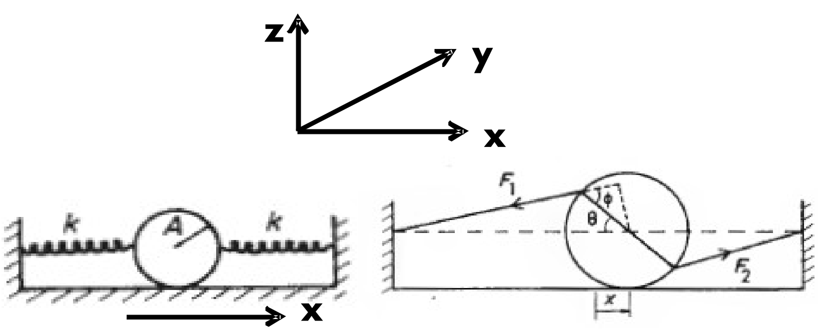
Answer a total of any **THREE** out of the four questions. Use the blue solution books and put the solution to each problem in a separate blue book and put the number of the problem on the front of each blue book. Be sure to put your name on each blue book that you submit. If you submit solutions to more than three problems, only the first three problems as listed on the exam will be graded.

1. A thin disk of mass M and radius A stands vertically on a horizontal table and is connected by two massless springs each of spring constant k to two fixed vertical walls as shown in the left-hand figure. The table is frictionless, and one can ignore gravity. The x and y axes are in the horizontal and the z axis is vertical. The center of the disk is free to move and the disk is free to rotate but the disk is constrained so that it always stays vertical. In equilibrium the springs are along the x direction. Each spring has an unconstrained length L_0 and initially both strings are stretched to a length L with $L > L_0$ in the equilibrium position as shown in the left-hand figure.

   (**a**) What is the frequency of small oscillations for displacements along the y direction?

   (**b**) What is the frequency of small oscillations for rotational displacements as shown in the right-hand figure?

   Hint: For (**a**) there is no rotation and the center of the disk oscillates in the y direction, and for (**b**) the center of mass of the disk does not undergo translational motion.



2. The orbit of a particle moving under the influence of a central force is $r\theta = C$. The quantity C is a constant.

   (**a**) Determine the potential energy as a function of r.

   (**b**) Calculate the angular momentum in terms of C and the total energy E.

3. A solid uniform circular cylinder of mass m and radius r rolls (under gravity) inside a fixed hollow cylinder of radius R $(> r)$, the axes of the cylinders being parallel to each other and horizontal. At any time t during the motion, the plane containing the axes of the cylinders makes an angle θ with the vertical. Assume that the smaller cylinder rolls without slipping inside the larger one and that the amplitude of motion is such that it does not fall off at any point (see Fig. 1). Take the earth's gravity to be a uniform acceleration g in the downward vertical.

(**a**) Express the angular speed of the small cylinder in terms of $\dot{\theta}$.

(**b**) Write down a Lagrangian for the motion described above.

(**c**) Find the period of small oscillations of the small cylinder.

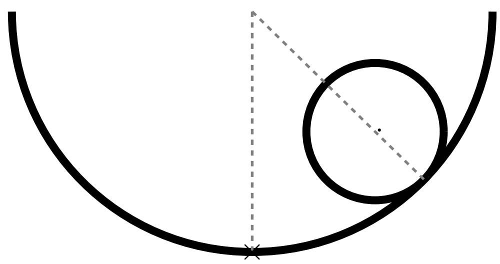


Fig. 1

4. A hemisphere of mass M is free to slide with its base on a smooth horizontal table. A particle of mass m is placed on the hemisphere at angle α (> 0) with respect to the vertical axis through its center (so the particle is free to move in the vertical plane containing the axis). Assume that there is no friction anywhere and no rolling of the particle in this problem, and that the motion begins when both the hemisphere and the particle are at rest (see Fig. 2). Take the earth's gravity to be a uniform acceleration g in the downward vertical.

(**a**) How many degrees of freedom are there in this problem?

(**b**) Write down a Lagrangian for the motion described above.

(**c**) Obtain the equation(s) of motion.

(**d**) Find an equation for the angle θ (with the above vertical axis) at which the particle leaves the hemispherical surface. Do not try to solve for θ.

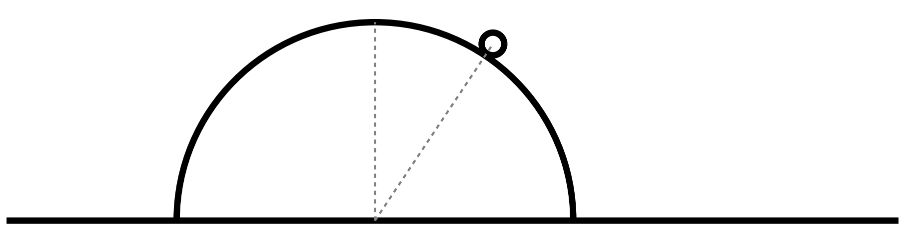


Fig. 2