CLASSICAL MECHANICS

Preliminary Examination

Monday 08/18/2014

09:00 - 12:00 in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book or on sheets of paper stapled together. Make sure you clearly indicate who you are and what is the problem you are solving on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

You are allowed to use a result stated in one part of a problem in the subsequent parts even if you cannot derive it.

Problem 1. A particle of mass M moves under the action of a central force whose potential is $V(r) = Kr^4$ with $K > 0$.

a). Write down the expression for the effective potential which enters the equation of motion for the radial coordinate r.

b). For what energy and angular momentum will the orbit be a circle about the origin.

c). What is the period of the circular motion on this orbit?

Problem 2. A string of length l, with a mass M at one end and a mass m at the other is stretched over two frictionless pulleys. The mass m hangs vertically downwards (it moves up and down). The mass M is free to swing in the vertical plane. Let r be the distance of mass M from the pulley closest to M and θ the angular displacement of M from the vertical.

a). Write a Lagrangian for this system using coordinates (r, θ).

b). Derive expressions for generalized momenta p_r and p_θ in terms of time derivatives of the coordinates.

c). From (a) write the Hamiltonian of this system, and derive the Hamilton equations of motion.

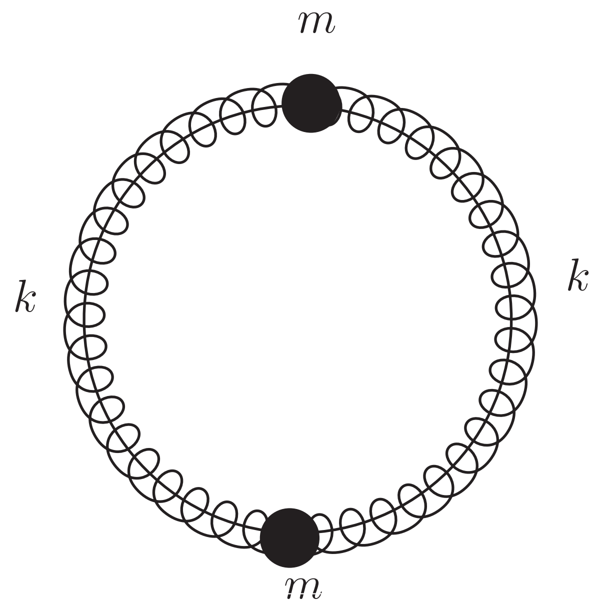
d). Explain why the Hamiltonian is a constant of motion. Is the Hamiltonian equal to the total energy?

Problem 3. Two identical masses m are constrained to move on a horizontal hoop. Two identical springs with spring constant k connect the masses and wrap around the hoop. One mass is subject to a driving force $F_d \cos \omega_d t$.

a). Write down the equations of motion for the system.

b). Find a particular solution for the motion of the masses.

c). Find general solution.



Problem 4. A ball of mass M collides with a stick with moment of inertia $I = \beta m l^2$ (relative to its center, which is its center of mass). The ball is initially traveling at speed V_0 perpendicular to the stick. The ball strikes the stick at a distance d from its center. The collision is elastic.

a). Find the resulting translational and rotational speeds of the stick.

b). Find the resulting speed of the ball.