CLASSICAL MECHANICS

Preliminary Examination

August 19, 2013

9:00am - 12:00pm in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book or on individual sheets of paper. Make sure you clearly indicate who you are and the problem you are solving on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

Problem 1

Three particles in a row with masses m_1, m_2, and m_3 are connected to each other and to the walls around by identical massless springs, as shown in Fig.1. The spring constant and the equilibrium length are *k* and *a* respectively, and the distance between walls is 4*a*. For the one-dimensional motion of all particles:

(a) Construct the Lagrangian of the system.
(b) Derive the Lagrange equations of motion.
(c) Find the eigenfrequencies of harmonic oscillations if the masses of all three particles are equal, m_1= m_2 =m_3=m. Describe the character of the motion for each eigenmode.
(d) Find the eigenfrequencies if m_1=m_3=m and m_2=M.

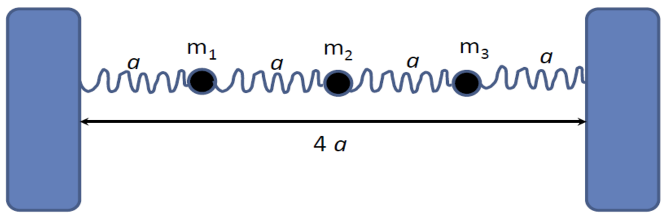

Fig.1

Problem 2

The one-dimensional harmonic oscillator can be studied using a complex variable α that encompasses both position and momentum at once:

$$H = \frac{p^2}{2m} + \frac{m\,\omega^2 x^2}{2}; \quad \alpha = \sqrt{\frac{m\omega}{2}}\,x + i\sqrt{\frac{1}{2m\omega}}\,p.$$

(a) Show that the Poisson brackets for the new variable α and its complex conjugate α^* are $[\alpha, \alpha]$ = $[\alpha^*, \alpha^*]$ = 0 and $[\alpha, \alpha^*]$ = $-i$.
(b) Express the Hamiltonian *H* as a function of α and α^*.
(c) Derive the equations of motion for the variables α and α^* using Hamilton's equations of motion, and show that $\dot{\alpha} = -i\omega\alpha$ and $\alpha(t) = \alpha(0)\exp(-i\omega t)$.
(d) Obtain the solutions *x(t)* and *p(t)* of the harmonic oscillator for the given initial values *x(0)* and *p(0)* using methods outlined in this problem.

Hint: The Poisson bracket [*f*, *g*] of two functions *f* and *g* is defined in classical mechanics as $[f, g] = \frac{\partial f}{\partial x}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial x}$, where *x* and *p* are the generalized coordinate and momentum. Poisson brackets play the same role in classical mechanics as operator commutators in quantum mechanics.

Problem 3

A particle of mass *m* is trapped in the field of the "spherical potential well" of radius R_0:

$$U(r) = \begin{cases} -U_0, & r \le R_0, \\ 0, & r > R_0, \end{cases}$$

where U_0 is a positive constant. For this central potential, the total energy *E* and angular momentum *L* are integrals of motion.

(a) Write the Hamiltonian describing the radial motion of the particle, and find the relationship between *E* and *L* values required to keep the trajectory inside the sphere of radius R_0.

(b) Describe particle trajectories at different values of *E* and *L*: calculate the radius $r_C = r_C(E,L)$ of the closest approach to the center of the sphere, and the angle $\Theta=\Theta(E,L)$ of reflection from the surface of the potential well at $R= R_0$.

(c) Establish conditions necessary for circular motion. Are these circular trajectories stable?

(d) Find a relationship between *E* and *L* for closed trajectories.

Problem 4

An excited diatomic molecule, moving with the velocity ***V*** in the Laboratory Frame (LF), decays into two identical atoms. The decay process is isotropic in the Center of Mass Frame (CMF), where the speeds of the atoms are equal to v_0. Calculate the atomic angular distribution function $\varrho(\Theta)$ in the LF, where Θ is the angle between the LF atomic velocity vectors.

Hint: The function $\varrho(\Theta)$ gives the probability density to detect the angle Θ between atomic velocity vectors in the LF, and it has to be normalized according to a standard rule: $\int_0^{\pi} \varrho(\theta)\frac{1}{2}sin\theta d\theta = 1$.