CLASSICAL MECHANICS

Preliminary Examination

August 22, 2011

9:00am - 12:00pm in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book or individual sheets of paper. Make sure you clearly indicate who you are, and the problem you are answering on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

Problem 1. A particle of mass m moves in the central potential field with the potential energy $U(r) = \alpha r^2$, where r is the distance from the particle to the field center; α is a positive constant.

(a) Construct the Lagrangian of the radial motion of this particle for a given value of the angular momentum L_0.

(b) Using the Lagrange equation for the radial motion determine conditions, when the r-value does not change in time. Find the value r_e of this equilibrium radius as a function of L_0, m and α.

(c) Calculate the frequency of small radial oscillations around the equilibrium radius r_e, using leading terms of Taylor's expansion of the effective potential of the radial motion at r_e.

Problem 2. The ends of a light rod, with a mass attached to its center, can slide without friction on a circular wire which is made to rotate about a fixed, vertical diameter with angular velocity $\vec{\omega}$. Show that, if there is a steady state of motion in which the rod is *not* horizontal, its inclination θ to the horizontal is given by

$$\cos(\theta) = (\frac{g}{a\omega^2})$$

where a is the distance of the rod's center from the center of the circular wire.

Problem 3. In a space-time diagram (with axes x and ct using standard notation), the world lines of two free particles A and B are given by x=0 and x=vt respectively. Sketch these world lines as well as the world line of a photon leaving A (at the event E$\equiv (0, c\tau)$) and meeting B at the event F in the same space-time diagram, assuming $\tau > 0$. Find the space-time coordinates of F . Sketch the world lines in a separate space-time diagram with respect to B (i.e., with axes x' and ct' where t' is B's proper time). Now find the space-time coordinates of the event F here and show that the ratio of the photon frequency ν as observed by A, to the frequency ν' as observed by B is given by

$$\frac{\nu'}{\nu} = \sqrt{\frac{c-v}{c+v}}.$$

Hint: *You may assume that the number of wavelengths between the points of emission and observation of a photon is a relativistic invariant.*

Problem 4. The Lagrangian for a central force is given by

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - U(r),$$

where U(r) is the potential energy.

(a) Using Lagrange's equations for r and θ find the central force equations of motion.

(b) Using the integrals of motion of the equations of motion from part (a) derive the equation for the central force $F(r) = -\partial U(r)/\partial r$ for a given trajectory $r = r[\theta(t)]$:

$$F(r) = -\frac{J^2}{m}u^2(\frac{d^2u}{d\theta^2} + u),$$

where $J = mr^2\dot{\theta}$ =constant is the angular momentum and $u = 1/r$.

(c) Using the orbital equation from part (b) find the force F(r) which results in the orbit of a particle given by

$$r = a(1 + cos\theta),$$

were a is a positive constant.