CLASSICAL MECHANICS

Preliminary Examination

Monday 01/13/2014

09:00 - 12:00 in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

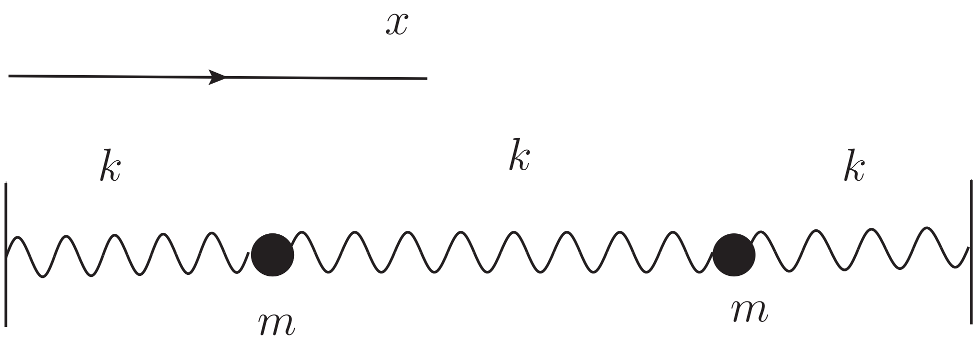
Each answer must be presented **separately** in an answer book or on sheets of paper stapled together. Make sure you clearly indicate who you are and what is the problem you are solving on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

You are allowed to use a result stated in one part of a problem in the subsequent parts even if you cannot derive it.

Problem 1. Consider two particles of mass m connected by a set of springs of spring constant k attached to two walls as shown in the figure below.

(a) Determine the frequency of oscillations of the spheres for small displacements along x direction (along the spring).

(b) Find normal modes of the oscillations. State how the particles move (direction and amplitude) for each normal mode.



Problem 2. A ball of radius R rolls without slipping between two rails such that the horizontal distance is d between the two contact points of the rails to the ball. If the two rails form a ramp and the ball, starting at rest descends a vertical distance h, show that the center of mass velocity is

$$v_{cm} = \left(\frac{10gh}{5 + \frac{2}{1 - \frac{d^2}{4R^2}}} \right)^{1/2}$$

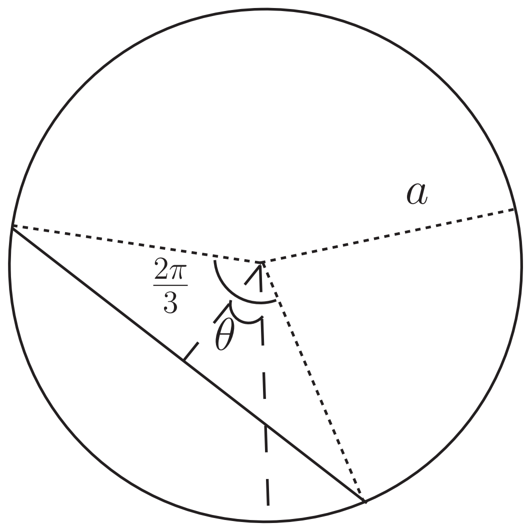
Problem 3. A uniform rod slides inside a smooth (frictionless) vertical circle of radius a. The rod of uniform density and mass M subtends an angle of $2\pi/3$ at the center of the circle.

(a) Show that the moment of inertia of the rod relative to its center of mass in terms of m and a (not the length of the rod) is equal to $\frac{1}{4}ma^2$.

(b) What is the moment of inertia of the rod about the center of the circle?

(c) Write the Lagrangian of the system using as the generalized coordinate the angular displacement θ of the rod measured from the center of the circle to center of mass of the rod.

(d) Compare the equation of motion of the rod to that of a simple pendulum and find its frequency of oscillation for small displacements.



Problem 4. Equations of motion of a particle are derived from the Hamiltonian

$$H = \frac{1}{2m}\Sigma_{i=1}^{3}\left[p_i - \frac{\partial f(x)}{\partial x_i}\right]^2 + V(x)$$

where p_i are momenta conjugate to coordinates x_i and $f(x)$ is some function of the coordinates x_i.

(a) Derive Hamilton's equations using Poisson brackets between canonically conjugate variables.

(b) Show that the transformation $p_i \to p_i - \frac{\partial f}{\partial x_i}$; $x_i \to x_i$ is canonical. Applying this transformation, show that the problem is equivalent to the motion of a particle with mass m in potential $V(x)$.

Now consider motion of a particle with charge e in a homogeneous magnetic field B_i. The Hamiltonian for this problem is

$$H = \frac{1}{2m}\Sigma_{i=1}^{3}\left[p_i - \frac{e}{2}\epsilon_{ijk}B_j x_k\right]^2$$

(c) By calculating the Poisson brackets, show that the transformation $p_i \to p_i - \frac{e}{2}\epsilon_{ijk}B_j x_k$ is not canonical and therefore the magnetic field cannot be eliminated from the Hamiltonian.

(d) Show that the following three quantities are conserved $\pi_i = p_i + \frac{e}{2}\epsilon_{ijk}B_j x_k$. What is their physical meaning? (Hint: magnetic field is constant, i.e. translationally invariant.)