Prelim Exam: Classical Mechanics, Monday May 10, 2020. 8:00am-11:00am

Answer **THREE** out of the four questions. If you submit solutions to all four then only the first three will be graded.

To take the prelims remotely students will need a good internet connection, a computer with a camera, a cell phone with a camera, and sufficient cell phone data capacity to switch to the data line on the cell phone if the wifi fails. Also students should keep their cell phone fully charged in case of power outages. Students should have the webex app on both their computer and cell phone. Each webex link will open at 7:45am. The computer camera will only be needed to check each student's ID prior to the start of the exam. The exam will be emailed to each participant at the starting time of the exam. Students should immediately download the exam to their computer and cell phone (and even print it out if they can) in case they lose the internet connection.

Students should write their solutions on blank 8.5 by 11 paper, putting their name on each page, the number of the problem and the number of the page in their solution (i.e. 2-1 means first page of problem 2). Also each problem solution should be on a separate set of pages (i.e. not putting parts of two different problems on the same page). At the end of the exam students should scan in their exams in sequence using the cell phone or a scanner (it might be easier to transfer the files to a laptop first) and email them in a file or files (ideally pdf) to philip.mannheim@uconn.edu and gayanath.fernando@uconn.edu no later than 15 minutes after the end time of the exam, and the files will be checked to see that they are readable or if a resend is required. Label both the email header and the file or files with your name and the name of the exam. In the email state which problems you have attempted and state how many pages there are for each of the problems.

During the exam students must keep the webex link live on both the computer and the cell phone, but only need to keep the cell phone camera on. Questions that arise should be asked through the chat on the computer webex, and students should arrange for at least the chat portion of the computer webex to be visible to them during the exam. Students can work on the same desk as they place their computer so that their hands are visible. The cell phone should be mounted (scotch tape on a hard vertical surface should suffice) so that the phone shows the computer screen and the entire work area. Proctors will monitor the students through the cell phone camera webex.

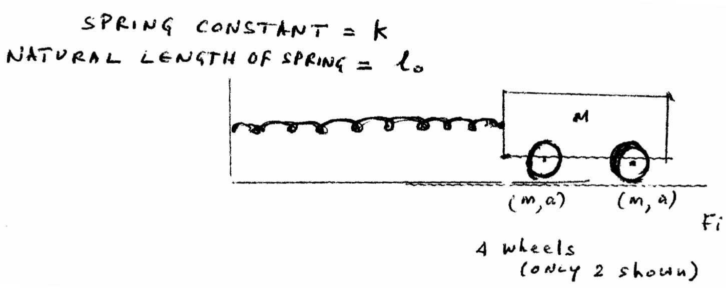

Fig. 1 (PROBLEM 1)

1. A small cart is attached to a light spring of natural length l_0 and spring constant k as shown in Fig. 1. The other end of the spring is attached to a (fixed) wall and the cart is free to move on a horizontal table. The mass of the body of the cart is M while the mass of each of the four identical cylindrical wheels is m. The moment of inertia about the axle of a wheel is $I = ma^2/2$ where a is the radius of a wheel. The spring is pulled horizontally by a distance l_0 (from its equilibrium position) and released from rest so that the cart rolls horizontally (with no slipping) on its wheels. Assume there is no friction between wheels and their bearings.

   (a) How many degrees of freedom are there in this problem?

   (b) Write down a Lagrangian $\mathcal{L}$ for the above system.

   (c) Does the cart hit the wall?

   (d) Find the speed of the cart (in terms of the given constant) when it passes through its equilibrium position.

   (e) Using the above Lagrangian $\mathcal{L}$, write down a Hamiltonian $\mathcal{H}$ for this problem. Is it conserved?

2. A planet of mass m is orbiting a massive star of mass M under the gravitational potential $-k/r$.
   (a) What is the limiting value of the reduced mass?
   (b) Explain why the orbit is confined to a plane containing the star assuming no other external forces are in play.
   (c) Write down the total energy of the system in terms of r, θ (plane polar coordinates), and their time derivatives.
   (d) If the planet has a speed u at a turning point of the orbit which is at a distance c from the star, find an expression for $\dot{r}^2$ at any point of the orbit in terms of r and constants c and u.

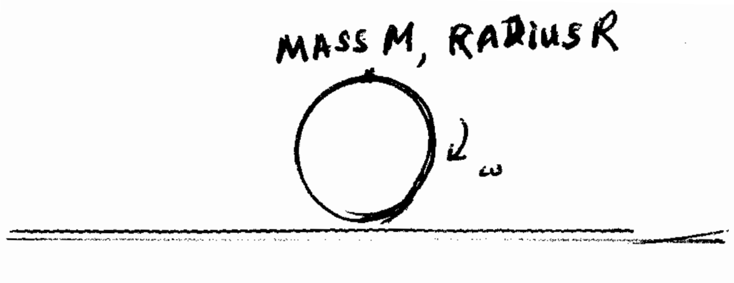

FIG 2b. (PROBLEM 3)

3. (a) Show that $Q = \ln(1 + \sqrt{q}\cos(p))$ and $P = 2(1 + \sqrt{q}\cos(p))\sqrt{q}\sin(p)$ yield a canonical transformation from (p, q) to (P, Q) as defined above.
   (b) A uniform disc of mass M and radius R is rolling on a horizontal plane with its flat face vertical and angular speed ω as shown Fig. 2. At time t, its highest point is held fixed. Stating your assumptions, find its new angular speed and kinetic energy. Be sure to identify conserved quantities here, if any, with a brief justification.

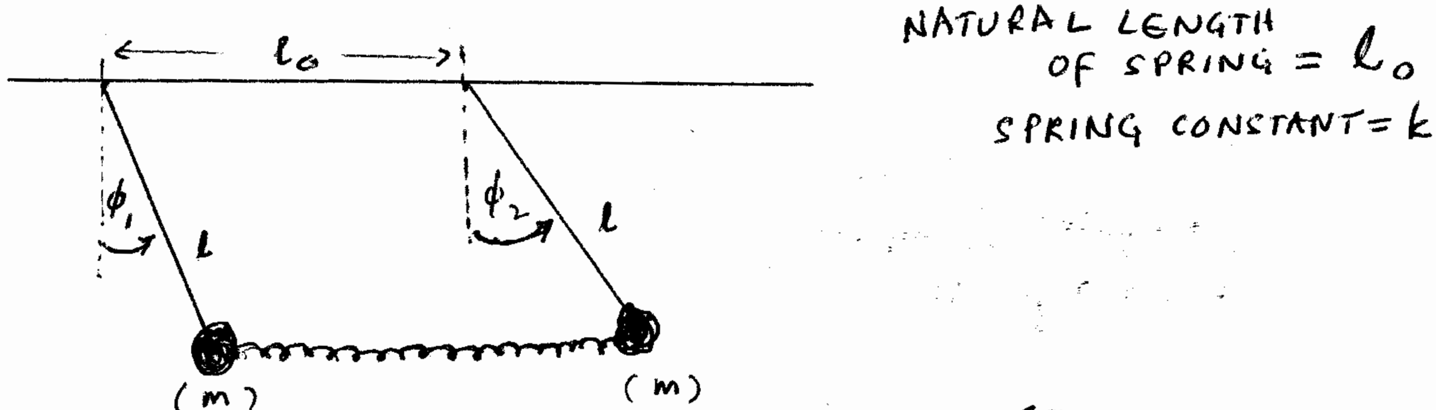

Fig. 3 (PROBLEM 4)

4. Consider two identical simple pendulums, hanging from a horizontal ceiling, each of mass m and length l joined by a massless spring of force constant k. As shown in Fig. 3 the positions of the pendulums are specified by the angles ϕ_1 and ϕ_2, with respect to the vertical. The natural length of the spring is equal to the distance between the two supports, so that the equilibrium position is at $\phi_1 = \phi_2 = 0$ with two pendulums vertical.
   (a) Set up a Lagrangian for the motion away from equilibrium assuming that the angles remain small at all times and obtain the equations of motion.
   (b) Find and describe the normal modes for these coupled pendulums.