Inferring the Initial Condition for the Balitsky - Kovchegov Evolution Equation

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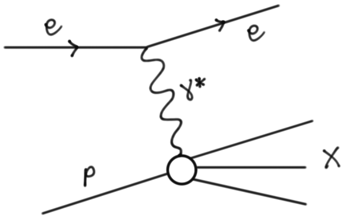
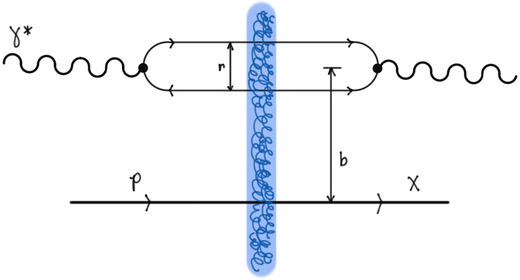
JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Outline

Deep Inelastic Scattering in the Dipole Picture



IMF picture



Dipole Picture

$$\sigma_{T,L}^{\gamma * p}(x, Q^2) \sim \frac{\sigma_0}{2} \times \mathcal{N}(r, y) \times \{\mathrm{LCWF}\}$$

rcBK: $\mathcal{N}(r, x = x_0; Q_{s0}^2, \gamma, e_c, C^2) \to \mathcal{N}(r, y)$

Motivation

- Obtain posterior distribution for model parameter via Bayesian inference constraining against HERA *ep* scattering cross section data.
- Provide predictions and uncertainties for other observables.

Previous fits to HERA data:

- H.Mäntysaari, T. Lappi (2013): 1309.6963v1
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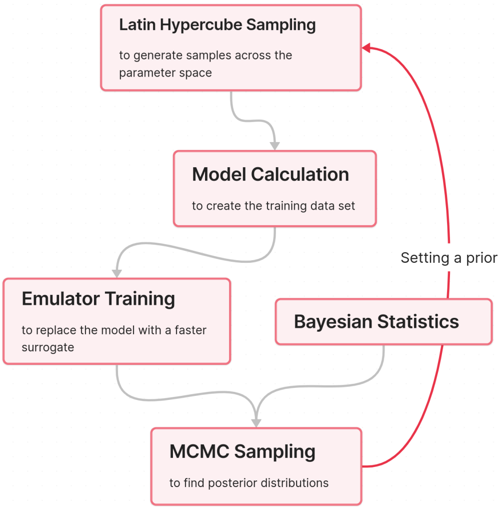
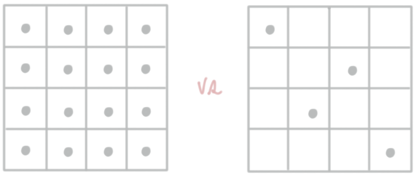
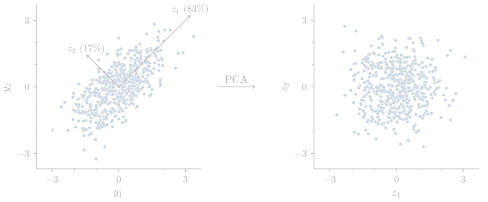


Latin Hypercube Sampling
to generate samples across the parameter space
Model Calculation
to create the training data set
Emulator Training
to replace the model with a faster surrogate
Bayesian Statistics
MCMC Sampling
to find posterior distributions
Setting a prior

Latin Hypercube Sampling

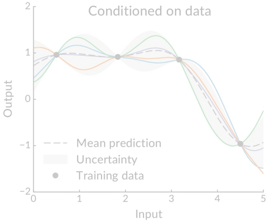


Principal Component Analysis

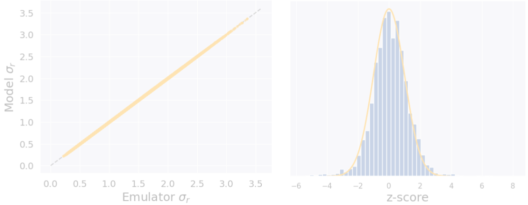


Gaussian Process Emulator

GPs learn the parameter dependence of the model!



Validation



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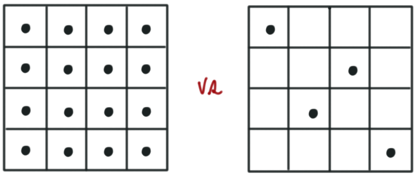

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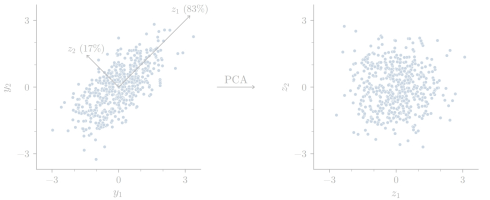

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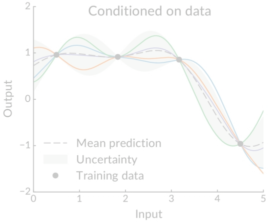

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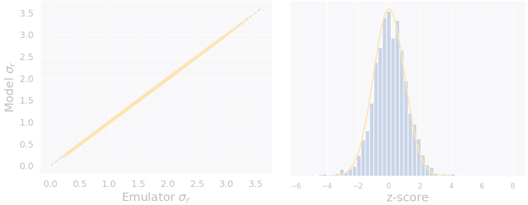

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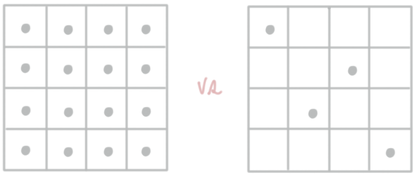

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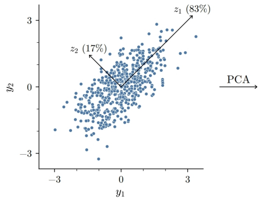

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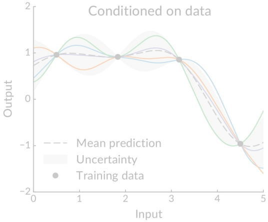

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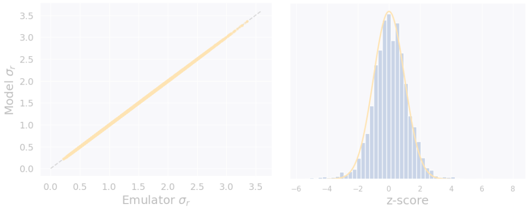

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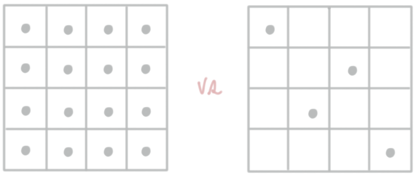

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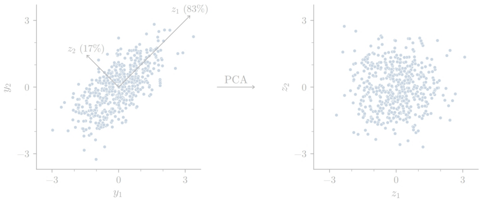

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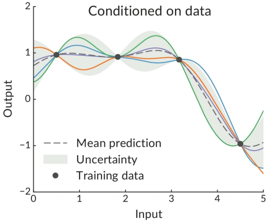

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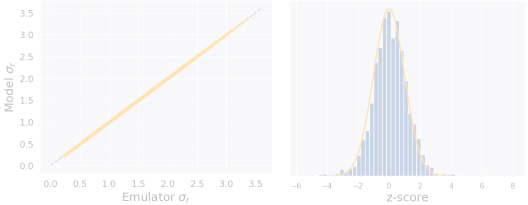

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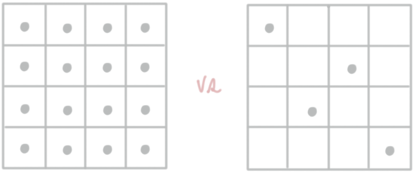

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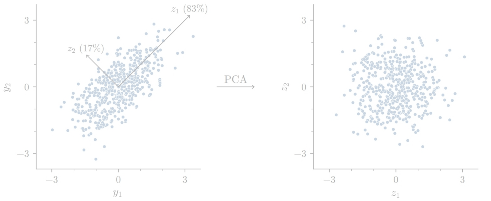

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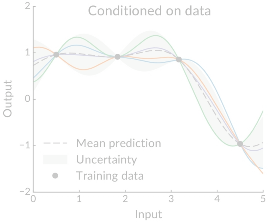

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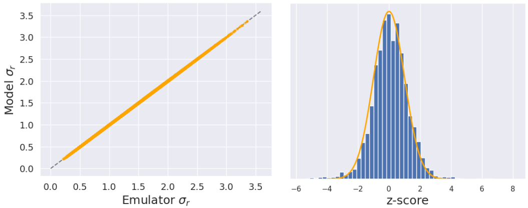

MCMC Sampling

Large number of walkers explore the parameter space and every step is accepted with a probability of $P(\theta_{i+1})/P(\theta_i)$

Bayesian Statistics

$$P(\theta) = \text{posterior} = \text{likelihood} \times \text{prior}$$

1. Likelihood: how well constraining data matches the model at a certain design point; encodes model and experimental error
2. Prior: knowledge of preferred area of the parameter space (e.g. flat distribution)

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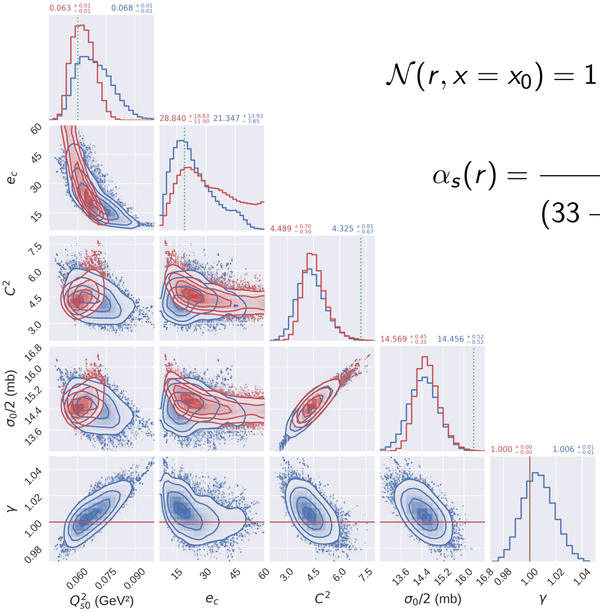
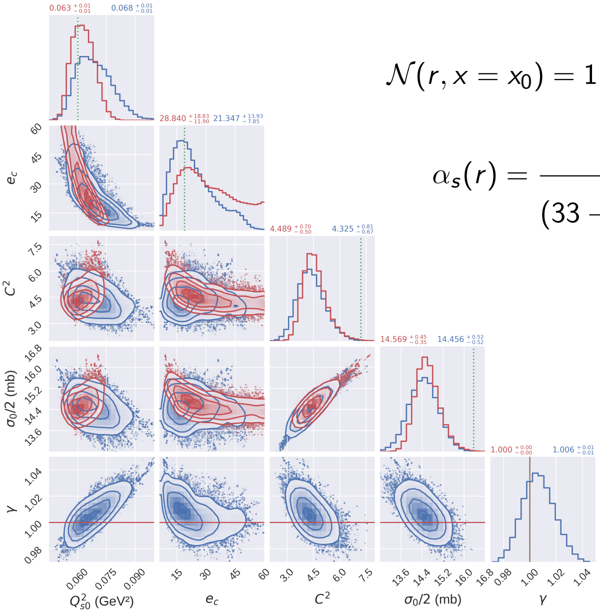


$$\mathcal{N}(r, x = x_0) = 1 - \exp\left[-\frac{(r^2 Q_{s0}^2)^{\gamma}}{4} \ln\left(\frac{1}{r\Lambda_{\rm QCD}} + e_c \cdot e\right)\right]$$

$$\alpha_s(r) = \frac{12\pi}{(33 - N_f)\log\left(\frac{4C^2}{r^2\Lambda^2_{QCD}}\right)}$$

1. HERA data prefers a $\gamma = 1$
2. With γ as a free parameter, Q_{s0} is allowed to have a wider posterior distribution
3. $C^2 \sim \alpha_s^{-1} \sim N(r, y)^{-1} \sim \sigma_0/2$



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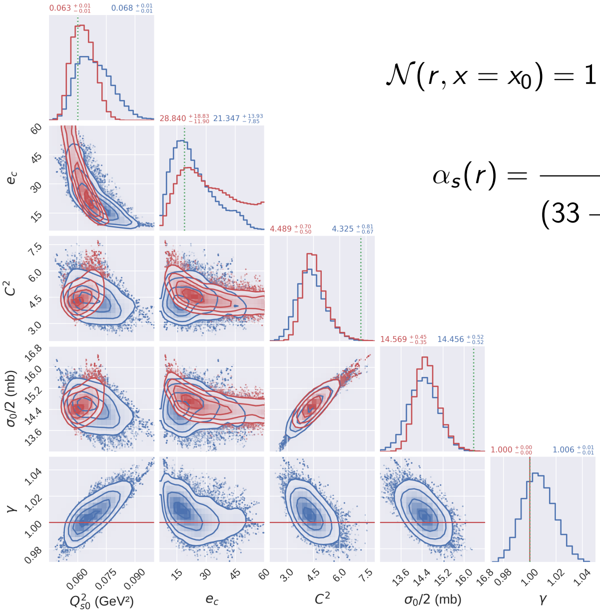

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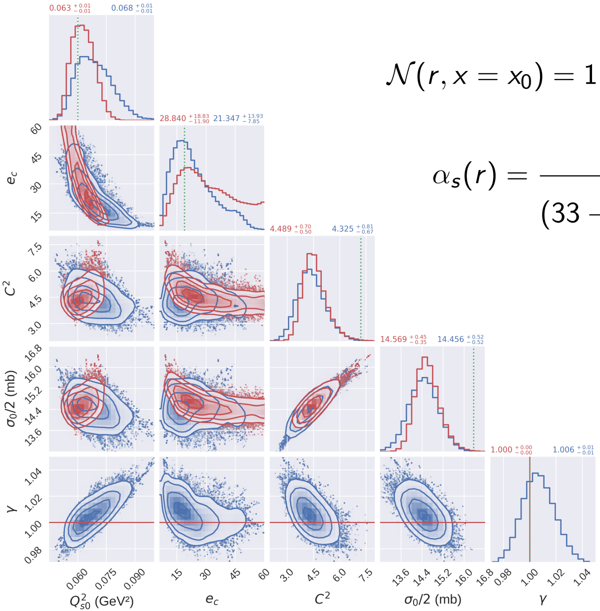

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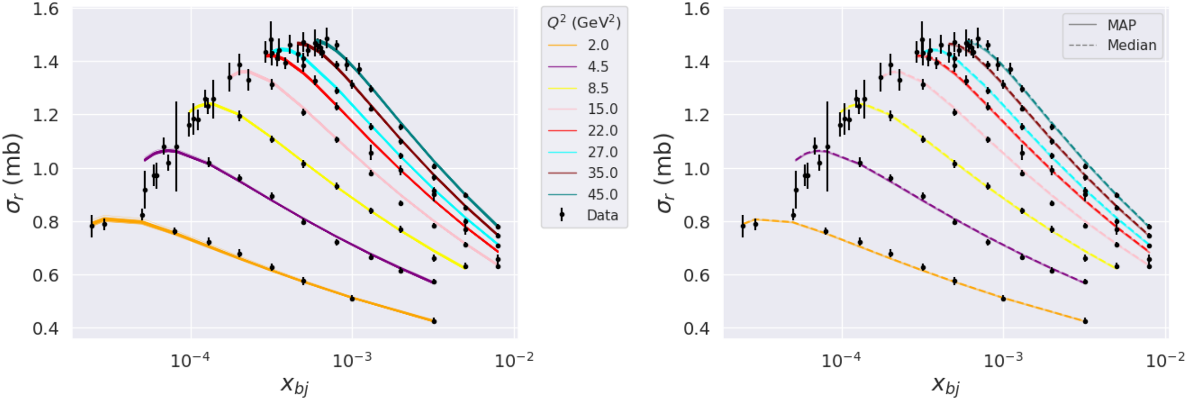
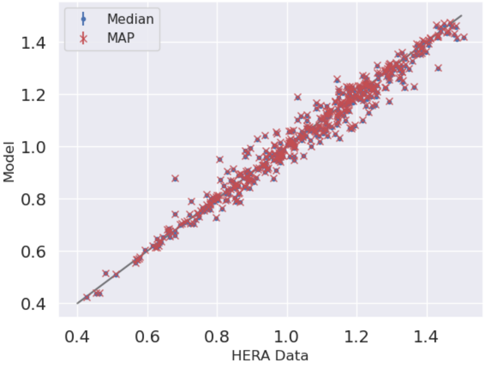
Posterior Samples, Median and MAP curves

- $\sqrt{s} = 318$ GeV



Fitting Values

5 - parameter	Q_{s0}^2	γ	e_c	C^2	$\sigma_0/2$	χ^2/dof
MAP	0.073	1.010	16.356	4.234	14.395	1.024
median	0.067	1.006	21.347	4.325	14.456	1.041



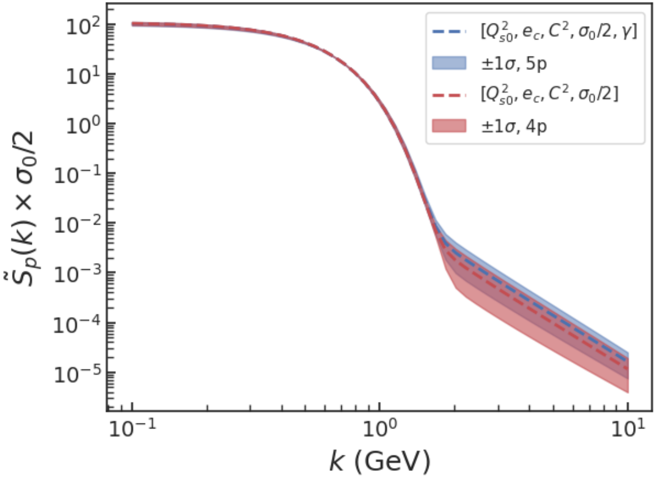
2D Fourier Transform, $\tilde{S}(k)$

$$\mathrm{d}\sigma^{q+A\to q+X} = xg(x, \mathbf{k}_T^2)\tilde{S}_p(\mathbf{k}_T)$$

where $\tilde{S}_p$ is the 2DFT of the proton-dipole scattering matrix,

$$\tilde{S}_p(\mathbf{k}_T) = \int \mathrm{d}^2\mathbf{r}_T \, e^{i\mathbf{k}_T\cdot\mathbf{r}_T} \times [1 - \mathcal{N}(\mathbf{r}_T, x = x_0)]$$

- Previous fits have $\gamma > 1$, which result to negative 2DFT values
- Uncertainty estimates are now provided!



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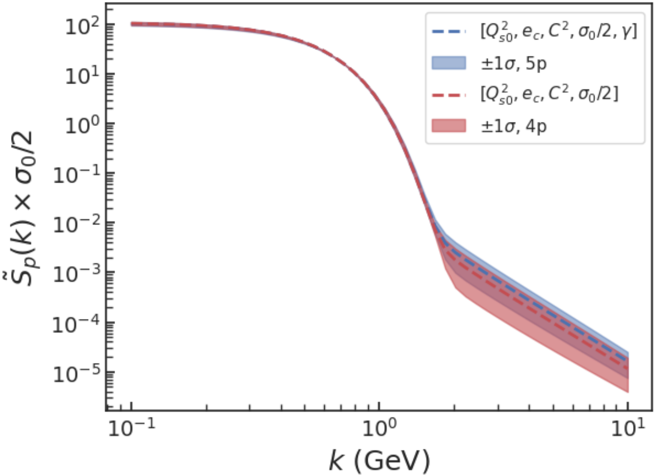

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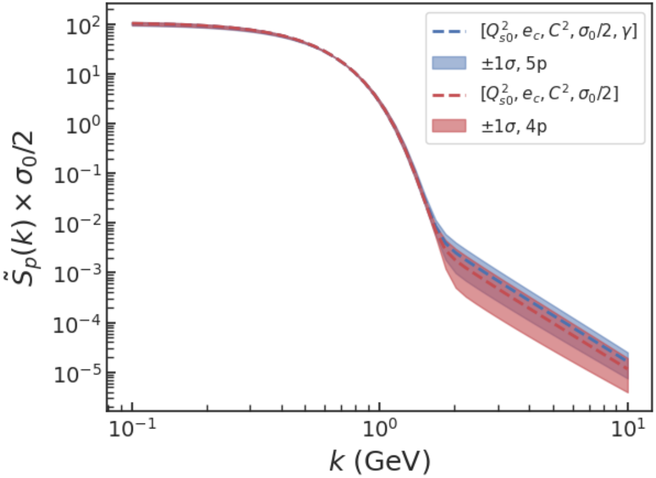

Summary

- Posterior distributions of quantities parametrizing the initial condition of the BK evolution has been extracted using Bayesian inference constrained against HERA data
- Correlations between parameters observed from the posteriors
- First time one obtains uncertainty estimates for the BK initial condition
- Theoretical predictions for observables now provides mean estimates and uncertainties.

Further work

- Extension to other functional forms of the initial condition
- NLO fits to further probe saturation

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