Finite temperature generalization of QCD potentials

Víctor López Pardo

IGFAE, Universidade de Santiago de Compostela, Spain

UConn-NSF summer school on saturation and EIC
Univesity of Connecticut, CT
August 5, 2023

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Introduction

Our objective is to study the interaction of particles bound by the strong force inside a QCD hot and dense medium

Ideal setup for the open quantum systems (OQS) formalism. We need a potential ("model" of the force between quarks), e.g. the Cornell potential:

$$V_{\mathrm{Cornell}}(r) = -\frac{\tilde{\alpha}_s}{r} + \sigma r,$$

with $\tilde{\alpha}_s = C_F g^2/(4\pi)$. We will look for different heavy quark model potentials using potential non-relativistic QCD (pNRQCD).

How is the potential between quarks affected by the presence of a medium? Is there a general formula? Permittivity.

Apply results to the $X(3872)$ inside quark-gluon plasma (QGP): rough estimation of the decay rate and of the survival probability of the $X(3872)$.

Introduction

The Born-Oppenheimer (BO) approximation allows to decouple the dynamics of the light components (light quarks or gluonic excitations) from the heavy quark dynamics.

As the potential can be obtained with Wilson loops, results can be derived from both hard-thermal loop (HTL) perturbation theory and lattice QCD.

In position space, using HTL perturbation theory, the inverse of the permittivity is:

$$\varepsilon^{-1}(r, m_D) = \frac{\delta(r)}{4\pi r^2} - \frac{m_D^2 e^{-m_D r}}{4\pi r} - i\frac{m_D T}{4\sqrt{\pi} r} G_{1,3}^{2,1}\left(\begin{matrix} -\frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{4} m_D^2 r^2 \right).$$

It is important to notice that the permittivity is valid in the vacuum case $m_D = T = 0$, since $\varepsilon^{-1}(r, 0) = \delta(r)/(4\pi r^2)$.

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The general formula I

Starting with a vacuum potential between two static (anti)quarks $V_{\text{vac}}(\mathbf{r})$ and using the convolution theorem, we have that:

$$V(\mathbf{r}, m_D) = (V_{\text{vac}} * \varepsilon^{-1})(\mathbf{r}, m_D),$$

with $\varepsilon(\mathbf{r}, m_D)$ the permittivity, being m_D the Debye or screening mass.

Instead of performing the convolution, we will come with en equation for $V(\mathbf{r}, m_D)$ and solve it [D. Lafferty, and A. Rothkopf (2020)].

We start from the Gauss's law for a unit charge, which states the identity:

$$\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right) = 4\pi\delta(\mathbf{r}),$$

where $\hat{\mathbf{r}}/r^2$ plays the role of a chromoelectric field. In general, it holds that:

$$\nabla \cdot \left(\frac{-\nabla V_{\text{vac}}(r)}{r^2 E(r)} \right) = 4\pi\delta(\mathbf{r}),$$

with $E(r) = -\,\partial V_{\text{vac}}(r)/\partial r\,.$

The general formula II

This can be written in components as:

$$\left(\frac{1}{r^2 E(r)^2}\frac{\partial E(r)}{\partial r}\frac{\partial}{\partial r} - \frac{1}{r^2 E(r)}\frac{\partial^2}{\partial r^2}\right) V_{\text{vac}}(r) = \mathcal{G}(r) V_{\text{vac}}(r) = 4\pi\delta(\mathbf{r}),$$

where the application of this linear operator to $V(r, m_D) = (V_{\text{vac}} * \varepsilon^{-1})(r, m_D)$ leads to the general equation:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-1}{E(r)}\frac{\partial V(r, m_D)}{\partial r}\right) = 4\pi\varepsilon^{-1}(r, m_D).$$

This formally solves as:

$$V(r, m_D) = C + bV_{\text{vac}}(r) + 4\pi \int^{r} \mathrm{d}r' \frac{\partial V_{\text{vac}}(r')}{\partial r'} \int^{r'} \mathrm{d}r'' r''^2 \varepsilon^{-1}(r'', m_D),$$

where we considered $E(r) = -\,\partial V_{\text{vac}}(r)/\partial r$.

The general formula III

We need to determine C and b using the fact that $V_{\text{vac}}(r) = V(r,0)$ and also enforce a smooth transition for $m_D \to 0$.
Then, the general formula can then be written as [V. López-Pardo, N. Armesto, M. A. Escobedo and E. G. Ferreiro (in preparation)]:

$$V(r,m_D) = C(m_D) + 4\pi \int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} \int^{r'} \mathrm{d}r'' r''^2 \varepsilon^{-1}(r'',m_D),$$

with the prescription that $C(m_D)$ must ensure that $V(r,m_D) = V_{\text{vac}}(r)$ up to $\mathcal{O}(m_D)$.
Using the previous permittivity and writing $V_{\text{vac}}(r) = V(r,0)$, we have that:

$$\mathrm{Re}[V(r,m_D)] = C(m_D) + \int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} e^{-m_D r'}(m_D r' + 1),$$

$$\mathrm{Im}[V(r,m_D)] = -\int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} \frac{\sqrt{\pi} m_D T}{2} r'^2 G^{2,1}_{1,3}\left(\left. \begin{matrix} \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, -1 \end{matrix} \,\right|\, \frac{1}{4} m_D^2 r'^2 \right).$$

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The vacuum potential

Let's apply the method to the $X(3872)$ ($m_{X(3872)} = 3871.65(06)$ MeV, $I^G(J^{PC}) = 0^+(1^{++})$)

We have to postulate a vacuum potential for the $X(3872)$. Once we have the vacuum potential, we can compute its in-medium counterpart and obtain some results.

We present here a model for the vacuum potential, but any other sufficiently well-reasoned model is equally valid.

We will rely on lattice QCD results for hybrids to find a suitable model: hybrid Π_u^- ($c\bar{c}$, $m_{\Pi_u^-} = 4184$ MeV y $J^{PC} = 1^{++}$) potential [S. Capitani, O. Philipsen, C. Reisinger, C. Riehl, and M. Wagner (2019)]:

$$V_{\text{vac}}(r) = \frac{A_{-1}}{r} + A_0 + A_2 r^2,$$

with A_{-1}, A_0 y A_2 constants.

The in-medium potential

By substituting the previous vacuum potential into our general formula and performing the vacuum limit prescription, we get:

$$\mathrm{Re}[V(r,m_D)] = A_{-1}\left(m_D + \frac{e^{-m_D r}}{r}\right) + A_0 + $$
$$+ A_2\left[\frac{6}{m_D^2}(1-e^{-m_D r}) - \left(2r^2 + \frac{6r}{m_D}\right)e^{-m_D r}\right],$$
$$\mathrm{Im}[V(r,m_D)] = \frac{\sqrt{\pi}\,T}{m_D}\frac{A_{-1}}{r}G^{2,1}_{1,3}\left(\begin{array}{c} \frac{3}{2} \\ \frac{3}{2},\frac{3}{2},0 \end{array}\middle|\,\frac{1}{4}m_D^2 r^2\right) + $$
$$+ 4\frac{\sqrt{\pi}\,T}{m_D^3}A_2\,G^{2,1}_{1,3}\left(\begin{array}{c} \frac{3}{2} \\ \frac{5}{2},\frac{5}{2},0 \end{array}\middle|\,\frac{1}{4}m_D^2 r^2\right),$$

where the imaginary part will be problematic due to divergencies.

In-medium potential for different Debye masses

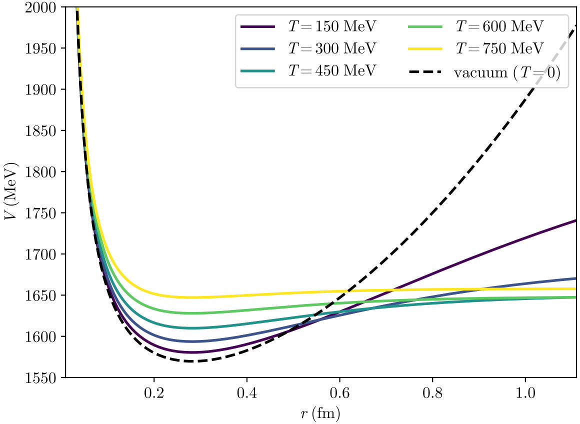

Figure:
$A_{-1} = 17.778$ MeV $\cdot$ fm,
$A_0 = 1475.23$ MeV and
$A_2 = 394.742$ MeV $\cdot$ fm^{-2}.

The octet imaginary part

It can be shown that the imaginary part we derived corresponds to the decay rate of a singlet state ($\Gamma^s(r, m_D)$), so we need to obtain the octet counterpart ($\Gamma^o(r, m_D)$), which can be done easily in the $N_c \to \infty$ limit:

$$\Gamma^o(m_D) = \lim_{r \to \infty} \Gamma^s(r, m_D) = \lim_{r \to \infty} \mathrm{Im}[V(r, m_D)],$$

where we dropped the radial dependency because the decay rate only depends on the temperature (or Debye mass) in this approximation. Since the imaginary part diverges as $r \to \infty$ we need to introduce a regularization (which corresponds to take into account the string breaking effect). Once we do so we arrive to:

$$\Gamma^o(T) = A_{-1} T + \frac{A_2 T}{m_D^3} \frac{6\pi}{\Delta},$$

where we know that $m_D = m_D(T)$, and Δ is a regularization constant to be determined.

Decay rate of the $X(3872)$

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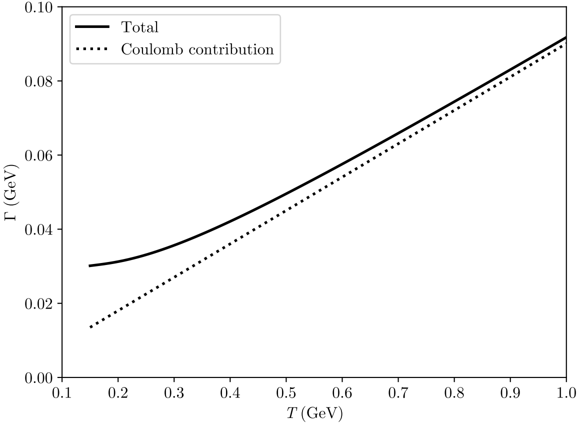

Figure: $\Gamma(T)$ with $\Delta \approx 6\pi$. It is lower than expected, similar to the decay rate of the $\Upsilon(1S)$.

The survival probability

We can compute the survival probability:

$$S(t) = \exp\left[-\int_{t_0}^{t} \mathrm{d}\tau \Gamma(T(\tau))\right].$$

If we consider $m_D \propto T$, we can model $\Gamma(T) = A_{-1}T + b_2/T^2$, where b_2 can be determined through experimental data. Using Bjorken evolution ($tT^3 = \text{const.}$) we can arrive to an expression of the survival probability:

$$S(t) = \exp\left\{-\frac{3t_0 A_{-1} T(t_0)}{2}\left[\left(\frac{t}{t_0}\right)^{2/3} - 1\right]\right\}\times$$
$$\times \exp\left\{-\frac{3t_0 b_2}{5T^2(t_0)}\left[\left(\frac{t}{t_0}\right)^{5/3} - 1\right]\right\}.$$

The $X(3872)$ survival probability

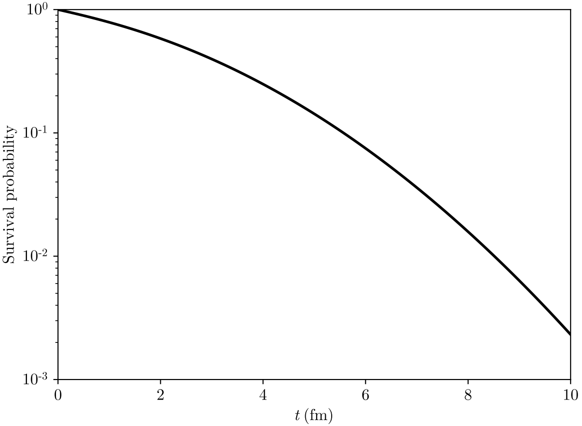

Figure: The time evolution of the survival probability. The time is in fm ($c = 1$), and 10 fm is around 3 ys.

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Conclusions

- The general method developed here can be applied to a wide range of particles.
- Useful to predict melting temperatures, lifetimes – when dealing with the imaginary contribution – or resonances, among others.
- We need a solid argument to justify the form of the vacuum potential for a hadron.
- We have powerful tool to extrapolate the results to the case of these particles in a medium.
- Preliminary results for the $X(3872)$ are promising.

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Thank you very much!

victorlopez.pardo@usc.es

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Mesonic molecules vs. tetraquarks

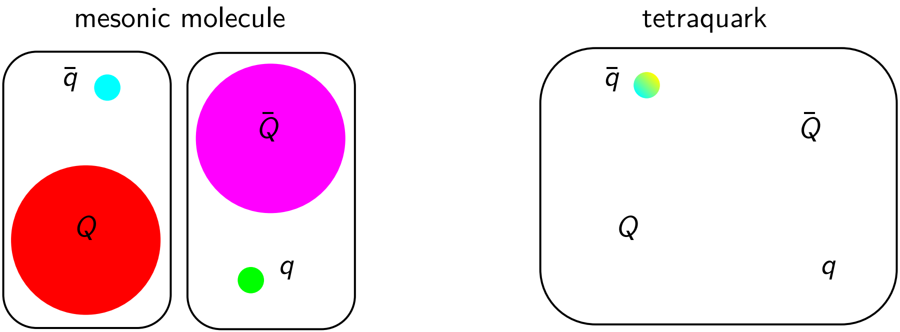

Figure: On this schematic representation the colour neutrality is represented by the black lines surrounding the groups of (anti)quarks and the lower case q and capital Q are reserved for light and heavy quarks, respectively.

Debye screening

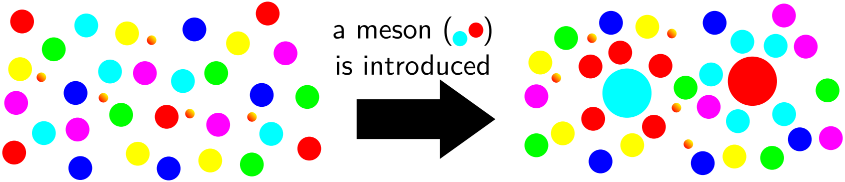

Figure: The medium size circles represent quarks and antiquarks, while the small double-coloured circles are meant to be gluons; the big circles on the right are the quark and antiquark that conformed the meson of interest.

The $X(3872)$ meson

The $X(3872)$ (also $\chi_{c1}(3872)$) is an exotic meson candidate first observed by Belle. Since it does not fit inside the quark model it is proposed as a tetraquark. Relevant information:

- Mass: $m_{X(3872)} = 3871.65(06)$ MeV.
- Composition: $c\bar{c}$ + two light quarks, probably $c\bar{c}u\bar{u}$.
- Decay modes: $X(3872) \to D^0\bar{D}^0$, $X(3872) \to \bar{D}^{*0}D^0$, $X(3872) \to \pi^+\pi^- J/\psi$...
- Quantum numbers: $I^G(J^{PC}) = 0^+(1^{++})$

Evidence of $X(3872)$ production inside QGP in January, 2022.

Heavy hybrids and tetraquarks

Similarities between (heavy) hybrids and tetraquarks:

- Contain a heavy quark-antiquark pair.
- The potential within the pair is modified by colour-charged light particles.

Colour charge differs between gluons (8 representation) and quarks (3 representation), but we can assume that a light quark-antiquark pair behaves in a similar way to a set of gluons:

$$3 \otimes \overline{3} = 8 \oplus 1, \qquad 8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1.$$

Debye mass versus T

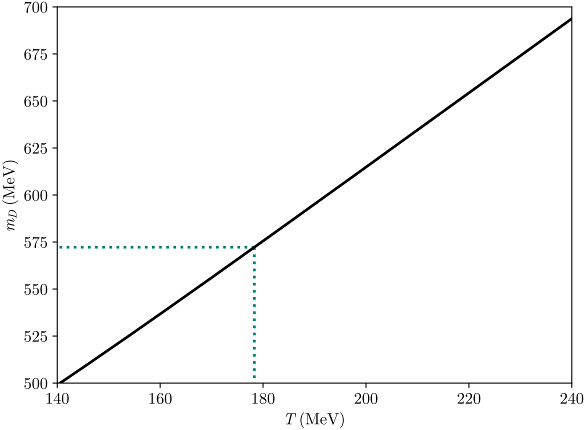

Figure:

$$m_D^2 \simeq \frac{4\pi^2 T^2}{3\ln\left(7.547\, T/\Lambda_{\overline{\rm MS}}\right)}$$

with $\Lambda_{\overline{\rm MS}} \approx 375$ MeV.

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