MODELS AND HLBL: DISCONNECTED CONTRIBUTIONS AND FIRST STEPS TOWARDS FINITE VOLUME CORRECTIONS

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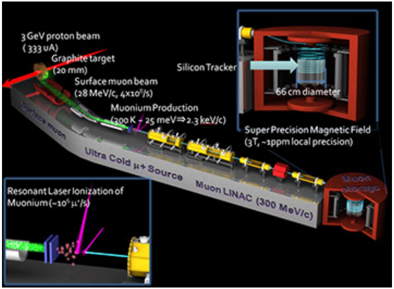
Why do we do this?

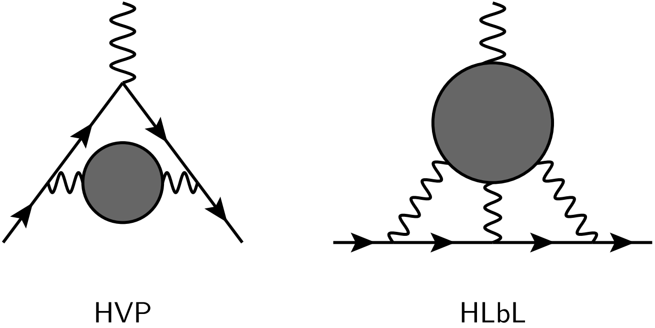
The muon $a_\mu = \dfrac{g-2}{2}$ will be measured more precisely



J-PARC

Fermilab

Hadronic contributions



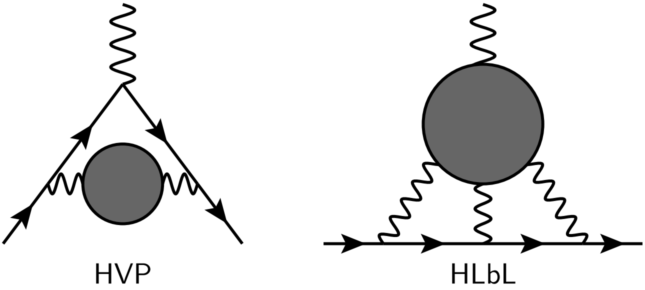
- The blobs are hadronic contributions
- There are higher order contributions of both types (with photons outside the blobs)
- Extra photons inside the blobs more tricky (not needed at the moment for HLbL)

To ChPT or not to ChPT

- ChPT = Effective field theory describing the lowest order pseudo-scalar representation
- or the (pseudo) Goldstone bosons from spontaneous breaking of chiral symmetry.
- Describes pions, kaons and etas at low-energies
- It's an effective field theory: new parameters or LECs at each new order
- Recent review of LECs:

  JB, Ecker,Ann.Rev.Nucl.Part.Sci. 64 (2014) 149 [arXiv:1405.6488]
- a_μ is a very low-energy quantity, why not just calculate it in ChPT?

To ChPT or not to ChPT



- Fill the blobs with pions and kaons
- Lowest order for both HVP and HLbL:
  pure pion loop (or scalar QED): well defined answer
- NLO: the blob is nicely finite
  but not after the muon/photon integrations
- Needs a counterterm (NLO LEC) that is the muon $g-2$

To ChPT or not to ChPT

- So need more than ChPT
- Experiment
- Dispersion relations
- lattice QCD
- Models: my talks at Q-park 2017, Capri 2015 and 2017
- I will give some general comments/overview and then restrict to some new results
- ChPT can be used to put constraints, help understanding results and estimate not evaluated parts,...

Why models?

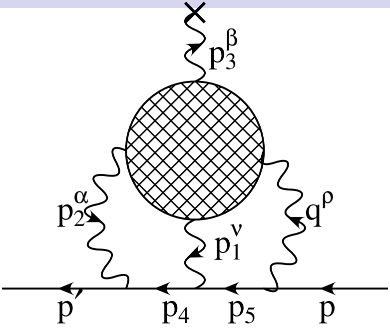
- Pro:
  - Can calculate with them (important in the past)
  - Can use them to understand features of better/more exact calculations
  - Can use them to estimate contributions from regions the other methods do not include
  - Can use them together with better methods to produce better models
- Con:
  - They are not the underlying theory or reality (experiment)
  - hard to estimate errors (guesstimates)
  - Beware: just model quark is different from QCD quark
  - Beware: model pion might not be quite the real pion
- Reminder:
  - HVP: high precision needed
  - HLbL: "just a bit" better than at present, but need to make sure the error estimate is not way off

Requirements

Requirements for models: Do as well you can

- Constrain as much as possible from experiment
  - measured states
  - measured form-factors
  - mesaured relevant scattering processes
- Constrain as much as possible from theory
  - include QCD short-distance constraints
  - include long distance constraints from ChPT
- Use common sense
  - Vary model parameters
  - Is your model general enough to describe what you want to describe
  - Different regions treated differently: is there some consistency
- As well as you can should improve with time

HLbL: the main object to calculate



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

A separation proposal: a start

E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD," Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

- Use ChPT p counting and large N_c
- p^4, order 1: pion-loop
- p^8, order N_c: quark-loop and heavier meson exchanges
- p^6, order N_c: pion exchange

Does not fully solve the problem
only short-distance part of quark-loop is really p^8
but it's a start

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Implemented by two groups in the 1990s:

- Hayakawa, Kinoshita, Sanda: meson models, pion loop using hidden local symmetry, quark-loop with VMD, calculation in Minkowski space (HKS)
- JB, Pallante, Prades: Try using as much as possible a consistent model-approach, ENJL, calculation in Euclidean space (BPP)

Papers: BPP and HKS

- JB, E. Pallante and J. Prades
  - "Comment on the pion pole part of the light-by-light contribution to the muon g-2," Nucl. Phys. B **626** (2002) 410 [arXiv:hep-ph/0112255].
  - "Analysis of the Hadronic Light-by-Light Contributions to the Muon $g-2$," Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388].
  - "Hadronic light by light contributions to the muon g-2 in the large N_c limit," Phys. Rev. Lett. **75** (1995) 1447 [Erratum-ibid. **75** (1995) 3781] [arXiv:hep-ph/9505251].
- Hayakawa, Kinoshita, (Sanda)
  - "Pseudoscalar pole terms in the hadronic light by light scattering contribution to muon g - 2," Phys. Rev. **D57** (1998) 465-477. [hep-ph/9708227], Erratum-ibid.D66 (2002) 019902[hep-ph/0112102].
  - "Hadronic light by light scattering contribution to muon g-2," Phys. Rev. **D54** (1996) 3137-3153. [hep-ph/9601310].
  - "Hadronic light by light scattering effect on muon g-2," Phys. Rev. Lett. **75** (1995) 790-793. [hep-ph/9503463].

Some main observations: Pseudo-scalar exchange

- The largest constribution is π^0 (and η, η') exchange/pole
- Beware: pole/exchange not quite the same
- Most evaluations are in reasonable agreement
- I will use it for an estimate of disconnected/connected on the lattice
- Point-like VMD: π^0, η and η' give $a_\mu^P = (5.58, 1.38, 1.04) \cdot 10^{-10}$.
- η' large due to charge factors
- Will be discussed in many more talks here (Stoffer, Kubis, Roig, Hashimoto, Gerardin, Chrits, Tu,. . .)
- My expectations:
  VMD cuts off too fast, short distance constraints on formfactors: final number somewhat higher
- My impression: Generally under control, improve precision

Some main observations: Pion-loop

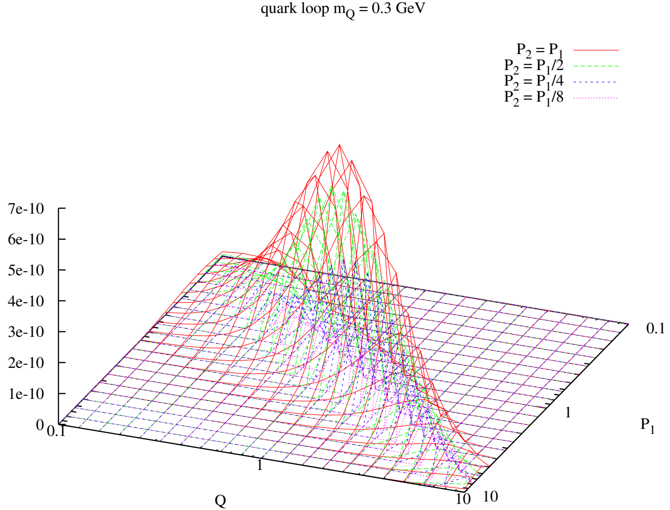
- The pion loop can be sizable but a large difference between the two evaluations $a_\mu = (-1.9(BPP), -0.5(HKS)) \cdot 10^{-10}$.
- Kaon loop is very small
- Pure pion loop, larger numbers proposed Engel, Ramsey-Musolf
- Solved: short-distance constraints lead to BPP more correct, some change when including a_1 for polarizability
  JB,Relefors,Zahiri-Abyaneh, 1208.3548,1208.2554,1308.2575,1510.05796
  JB, Relefors, JHEP **1609** (2016) 113 [arXiv:1608.01454 [hep-ph]].
  $a_\mu^{\pi-\text{loop}} = (-2.0 \pm 0.5)\ 10^{-10}$
- Remaining: Rescatterring and scalar part
  - Related: dispersive by Colangelo et al. (Stoffer)
  - Related: (Danilkin)
- My impression: Generally under control

Some main observations: Everything else

Cut-off Λ (GeV)	$a_\mu \times 10^7$ Electron Loop	$a_\mu \times 10^9$ Muon Loop	$a_\mu \times 10^9$ Constituent Quark Loop (q=e)
0.5	2.41(8)	2.41(3)	0.395(4)
0.7	2.60(10)	3.09(7)	0.705(9)
1.0	2.59(7)	3.76(9)	1.10(2)
2.0	2.60(6)	4.54(9)	1.81(5)
4.0	2.75(9)	4.60(11)	2.27(7)
8.0	2.57(6)	4.84(13)	2.58(7)
Known Results	2.6252(4)	4.65	2.37(16)

- M_Q : 300 MeV (known fully analytically)
- Slow convergence:
  - electron: all at 500 MeV
  - Muon: only half at 500 MeV, at 1 GeV still 20% missing
  - 300 MeV quark: at 1 GeV still 50% missing
  - Real charges: about $5 \cdot 10^{-10}$, above 1 GeV about $2.5 \cdot 10^{-10}$

Pure quark loop: momentum area



Most from $P_1 \approx P_2 \approx Q$, sizable large momentum part

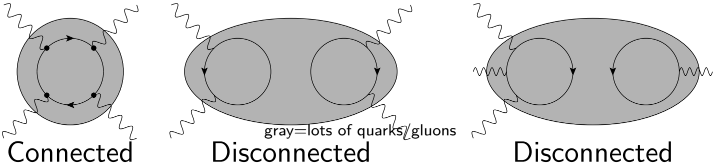
Other estimates of the remainder

- BPP: ENJL quark-loop, short-distance quark-loop and a_1: $2.6 \cdot 10^{-10}$
- BPP: scalar: goes to dispersive pionloop $-0.7 \cdot 10^{-10}$
- Melnikov-Vainshtein: QCD short-distance constraint gives and extra $2.1 \cdot 10^{-10}$ for π^0 exchange
- Melnikov-Vainshtein a_1 probably wrong, all others axials about $2.6 \cdot 10^{-10}$
- Other exchanges come with varying size and typically (much) below $\pm 0.3 \cdot 10^{-10}$
- Guesstimate: remainder will be $(2 - 3) \cdot 10^{-10}$ but need bounds on the errors
- All estimates with larger numbers: use a light quark mass below 1 GeV

Overview of models: conclusions

- Present:
  - Pseudo-scalar exchange: $(9\text{–}10) \cdot 10^{-10}$
  - Pion-loop+scalar: $-(2\text{–}3) \cdot 10^{-10}$
  - Remainder: $(2\text{–}4) \cdot 10^{-10}$
- Future:
  - Better approaches are taking over
  - Lattice: we already saw many new numbers
  - Dispersive: one π, η, η' and two pions/kaons will be under control and : errors small enough
  - Future for models: are there large other contributions?
- Present for models: so far no sign we were way off

Disconnected/Connected



- Use the breakup of contributions from previous slide
- Pseudo-scalar exchange: large effects and cancellations
- Pion-loop: reasonable effects and additive
- Rest: small

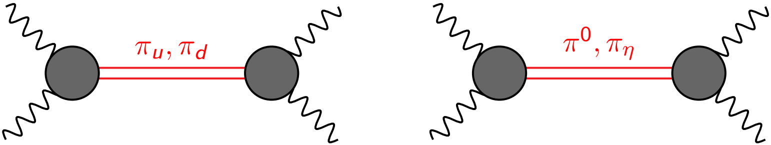
Disconnected/connected: pion loop and remainder

- Pion-loop
  - π, ρ, a_1 well described at large N_c
  - Full VMD or similar dispersive will be connected
  - Rescattering is large N_c suppressed
    estimate via scalar exchange or
    dispersive estimate without polarizability
  - connected $-2.0 \cdot 10^{-10}$
  - disconnected $-0.7 \cdot 10^{-10}$
  - about 25% of total and additive
- Remainder
  - Short-distance: disconnected needs two-gluon exchange suppressed by α_S^2 and loop factors
  - Resonance exchanges: nonet is usually a decent first approximation
  - So expect: disconnected smaller part of the total

Disconnected/connected: pseudo-scalar exchange

- Connected diagrams only:
  - the gluon exchanges responsible for $U(1)_A$ breaking are not included at all (anomaly is via $G\widetilde{G}$)
  - η' becomes light, mainly $(\bar{u}u + \bar{d}d)/\sqrt{2}$
    Call it π_η which has the same mass as the pion
  - Or the two-light states are π_u $(\bar{u}u)$ and π_d $(\bar{d}d)$
  - η becomes mainly $\bar{s}s$ and much heavier than the pion (and thus small contribution)
- Assume that couplings are not affected (not too bad experimentally)

Disconnected/Connected



- Two flavour case only: up and down quarks (three flavour not more difficult, just more numbers)
- Meson couplings to two-photons is via quark-loop
- Look at charge factors for Connected
  - As "quark-loop": $q_u^4 + q_d^4 = \frac{17}{81}$
  - As π_u, π_d: $q_u^2 q_u^2 + q_d^2 q_d^2 = \frac{17}{81}$
  - As π^0, π_η: $\left(\frac{q_u^2 - q_d^2}{\sqrt{2}}\right)^2 + \left(\frac{q_u^2 + q_d^2}{\sqrt{2}}\right)^2 = \frac{9}{162} + \frac{25}{162} = \frac{17}{81}$
- Include $U(1)_A$ breaking: π_η heavy
  - π^0: $\left(\frac{q_u^2 - q_d^2}{\sqrt{2}}\right)^2 = \frac{9}{162}$

Disconnected/Connected

- So in this limit:
  - Two-flavour case
  - $U(1)_A$ breaking makes π_η infinitely heavy
  - Full result dominated by pseudo-scalar exchange
  - $U(1)_A$ breaking does not affect couplings
  - $$\begin{aligned} \text{Connected:} &\quad \frac{34}{162} \\ \text{Disconnected:} &\quad -\frac{25}{162} \\ \text{Sum:} &\quad \frac{9}{162} \end{aligned}$$
- All assumptions get corrections but final conclusion stays

The disconnected contribution is expected to be large and of opposite sign with significant cancellations

- Argument used to go from large-N_c to π^0, η, η' in JB, Pallante, Prades, Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388]
- This form: JB, Relefors, JHEP **1609** (2016) 113 [arXiv:1608.01454]

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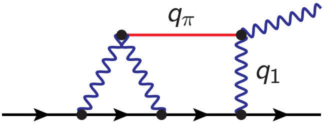
Disconnected/Connected

- Let's do numerics and factors for the full VMD case
- Full result: $(5.58 + 1.38 + 1.04 = 8.0) \cdot 10^{-10}$
- Connected: $((5.58(1 + (25/9)) + 0.2) = 22) \cdot 10^{-10}$
- thus disconnected: $-14 \cdot 10^{-10}$
- The other disconnected parts are basically in the error on this number
- disconnected a little over half of the connected and other sign

Finite volume corrections

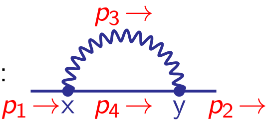
- Not much known analytically (at least to me)
- Photons are massless: possibly large corrections
- Idea: put photons (and muons) in larger/infinite volume
- Expect main correction from the π^0-exchange then, lightest hadronic state
- But: q_π quantized then q_1 as well: photon volume corrections back?



- Need to go back and understand momentum conservation

Simplest case: infinite volume

- Simplest diagram, pointlike couplings:



- Infinite volume (and states normalized accordingly)

$$\int d^d x \int d^d y e^{-ip_1\cdot x} e^{ip_2\cdot y} \int \frac{d^d p_3}{(2\pi)^d} \frac{e^{ip_3\cdot(x-y)}}{p_3^2} \int \frac{d^d p_4}{(2\pi)^d} \frac{e^{ip_4\cdot(x-y)}}{p_4^2 - m^2}$$

- doing the x and y integrals gives
  $(2\pi)^d\delta^d(p_1 - p_3 - p_4)(2\pi)^d\delta^d(p_3 + p_4 - p_2)$
- Final

$$(2\pi)^d\delta^d(p_1 - p_2) \int \frac{d^d p_3}{(2\pi)^d} \frac{1}{p_3^2} \frac{1}{(p_3 - p_1)^2 - m^2}$$

Simplest case: finite volume for all

- Time extension infinite, spatial finite extent
- x and y integrals over infinite time but finite spatial
- Propagators: $\vec{x}$ to all $\vec{y} + \vec{n}_y L$ (periodic b.c.)
- States now normalized differently
- $n_y = (0, \vec{n}_y)$

$$\int_{\text{box}} d^d x \int_{\text{box}} d^d y \frac{e^{-ip_1\cdot x}}{\sqrt{L^3}} \frac{e^{ip_2\cdot y}}{\sqrt{L^3}}$$

$$\sum_{\vec{n}_y} \int \frac{d^d p_3}{(2\pi)^d} \frac{e^{ip_3\cdot(x-y-n_y L)}}{p_3^2} \sum_{\vec{m}_y} \int \frac{d^d p_4}{(2\pi)^d} \frac{e^{ip_4\cdot(x-y-m_y L)}}{p_4^2 - m^2}$$

- Sums make the spatial momentum integrals discrete

Simplest case: finite volume for all

- Shift $\vec{m}_y = \vec{n}_y + \vec{l}_y$ and use $e^{ip_2\cdot y} = e^{ip_2\cdot(y+n_yL)}$

$$\int_{\text{box}} d^dx \sum_{\vec{n}_y} \int_{\text{box}} d^dy \frac{e^{-ip_1\cdot x}}{\sqrt{L^3}} \frac{e^{ip_2\cdot(y+n_yL)}}{\sqrt{L^3}}$$

$$\int \frac{d^dp_3}{(2\pi)^d} \frac{e^{ip_3\cdot(x-y-n_yL)}}{p_3^2} \sum_{\vec{l}_y} \int \frac{d^dp_4}{(2\pi)^d} \frac{e^{ip_4\cdot(x-y-l_yL-n_yL)}}{p_4^2-m^2}$$

- $\sum_{\vec{n}_y} \int_{\text{box}} d^dyf(y+n_yL) = \int d^dyf(y) \Rightarrow (2\pi)^d\delta^d(p_3+p_4-p_2)$
- Time integral over x^0 gives $2\pi\delta(E_1-E_2)$
- Spatial integral over $\vec{x}$ ($\vec{p}_1, \vec{p}_2, \vec{p}_4$ all discrete): $L^3\delta^{d-1}_{\vec{p}_1,\vec{p}_2}$
- $(2\pi)\delta(E_1-E_2)\delta^{d-1}_{\vec{p}_1,\vec{p}_2} \sum_{\vec{l}_y} \int \frac{d^dp_3}{(2\pi)^d} \frac{e^{-ip_3\cdot l_y}}{p_3^2((p_3-p_1)^2-m^2)}$
- Discrete sum over p_3, remove spatial zero mode QED_L

Simplest case: finite volume for massive only

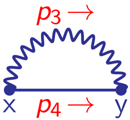
- Only the massive propagator needs the periodicity

$$\int_{\text{box}} d^d x \int_{\text{box}} d^d y \frac{e^{-ip_1\cdot x}}{\sqrt{L^3}} \frac{e^{ip_2\cdot y}}{\sqrt{L^3}}$$
$$\int \frac{d^d p_3}{(2\pi)^d} \frac{e^{ip_3\cdot(x-y)}}{p_3^2} \sum_{\vec{m}_y} \int \frac{d^d p_4}{(2\pi)^d} \frac{e^{ip_4\cdot(x-y-m_y L)}}{p_4^2 - m^2}$$

- Trick to get one delta function no longer works
- Time integrals x, y give $(2\pi)^2\delta(E_1 - E_2)\delta(E_1 - E_3 - E_4)$
- $\Delta(z) = \sin(z)/z$ ($\vec{x}, \vec{y}$ $-L/2$ to $L/2$ gives this)
- $(2\pi)^2\delta(E_1 - E_2) \int \frac{d^d p_3}{(2\pi)^d} L^3 \sum_{\vec{m}_y} \int \frac{d^d p_4}{(2\pi)^d} \frac{e^{-ip_4\cdot m_y L}}{p_3^2(p_4^2 - m^2)}$

$$\delta(E_1 - E_3 - E_4) \prod_{i=1,2,3} \Delta\left(\frac{(p_1^i - p_3^i - p_4^i)L}{2}\right) \Delta\left(\frac{(p_2^i - p_3^i - p_4^i)L}{2}\right)$$

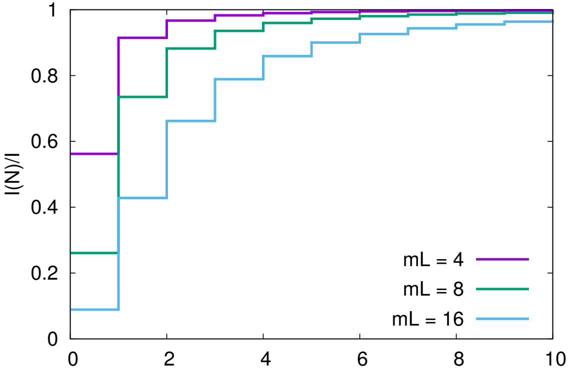
Simplest case: finite volume for massive only

- no momentum conservation
- $\Delta(pL/2)$ is peaked with a width of order $2\pi/L$ in p
- $\Delta(pL/2)$ has a long tail
- No easy way to get a feeling for its behaviour
- No way (for me so far) to check $1/L$ behaviour analytically
- Study an even simpler case numerically
- $p_1 = p_2 = 0$ and take a derivative w.r.t. m^2
- I.e.: $\frac{1}{i}\int \frac{d^d p_3}{p_3^2(p_3^2-m^2)^2}$ or $\frac{\partial}{\partial m^2}\Bigg($  $\Bigg)$

- Infrared and ultraviolet finite
- Infinite volume: $\frac{-1}{16\pi^2 m^2}$

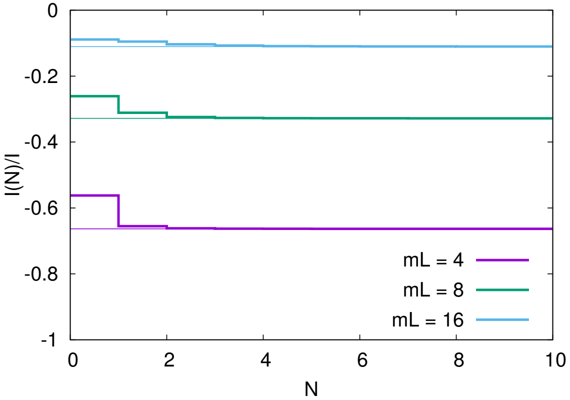
Absolute simplest: infinite volume

- Do the p_3^0 integral anaytically for all cases
- Make a grid of $p_4^i = \frac{2\pi}{L}$ up to $\pm N$
  (actually done by adding new layers on the cube)
- For infinite volume: set $p_3^i = p_4^i + q^i$ and integrate q^i from $-\pi/L$ to π/L



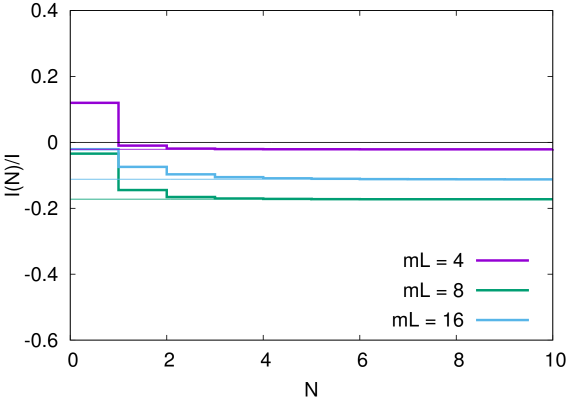
QED_L drops the first one: large corrections

Absolute simplest: QED_L



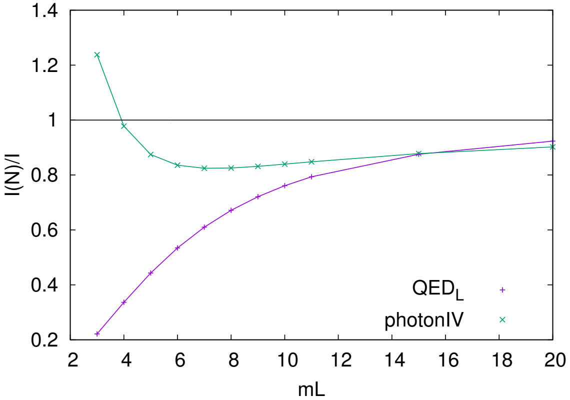
- photon in IV: corrections increases from $mL = 4$ to $mL = 8$
- Then goes back down again

Absolute simplest: photonIV



- QED_L: main correction is from dropping the first bin
- The thin lines are the exact QED_L answers

QED_L and photonIV: comparison



- QED_L bad at small L: main cause zero-bin is large
- But at larger L picture more unclear

Finite volume: conclusions

- QED_L bad at small L: main cause zero-bin is large
- But at larger L picture more unclear
- Work on other ways in progress
- Finite volume corrections are very quantity dependent
  - Mass electromagnetic corrections: large
    Hayakawa-Uno,BMW
  - HVP electromagnetic corrections: small
    JB, Boyle, Hermansson Truedsson, Janowski, Juettner, Portelli

Conclusions

See the subconclusions presented after each part