HLbL contribution to $(g-2)_\mu$ on the lattice: finite-volume effects

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Finite-volume and infinite-volume formulations

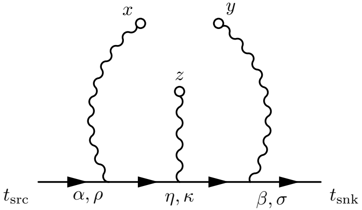
- a_μ^{HLbL} in finite-volume QCD and QED:
  - PRD93(2016)014503 (RBC/UKQCD): Connected diagram with $m_\pi = 171$ MeV; $a_\mu^{\mathrm{HLbL}} = 13.21(68) \times 10^{-10}$
  - PRL118(2017)022005 (RBC/UKQCD): Connected and leading disconnected diagram with $m_\pi = 139$ MeV; $a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$ (potentially large finite-volume systematics)

  Strategy: extrapolate away $1/L^n$ ($n \geq 2$) errors. Can we use effective theory (Bijnens' talk) to remove leading terms?

- a_μ^{HLbL} in finite-volume QCD and infinite-volume QED:
  - Method proposed and successfully tested against the lepton-loop analytic result: arXiv:1510.08384 (Mainz), arXiv:1609.08454 (Mainz)
  - Similar method plus subtraction scheme to reduce systematic errors; successfully tested against lepton-loop analytic result: PRD96(2017)034515 (RBC/UKQCD)

  Strategy: FV errors exponentially suppressed but still may be significant, effect on noise?

Finite-volume QED prescription (PRD93(2016)014503)



- The $\mathrm{QED_L}$ prescription uses the photon propagator

$$G_{\rm L}^{\mu\nu}(x) = \frac{\delta^{\mu\nu}}{V} \sum_k{}' \frac{1}{\hat{k}^2} e^{ikx} \,, \tag{1}$$

  where $\hat{k}^2 = \sum_\mu 4\sin^2(k_\mu/2)$ and $V = \prod_\mu L_\mu$ with lattice dimensions L_μ. The sum is over all momenta with components $k_\mu = 2\pi n_\mu / L_\mu$ with $n_\mu \in [0, \ldots, L_\mu - 1]$ and the restriction that $k_0^2 + k_1^2 + k_2^2 \neq 0$.
- For fixed x and y can get result for all z in $\mathcal{O}(V \log V)$ time using convolutions starting at $t_{\rm src}$ and $t_{\rm snk}$; has statistical advantage for leading disconnected diagram (M^2 trick is cheap Taku's talk)

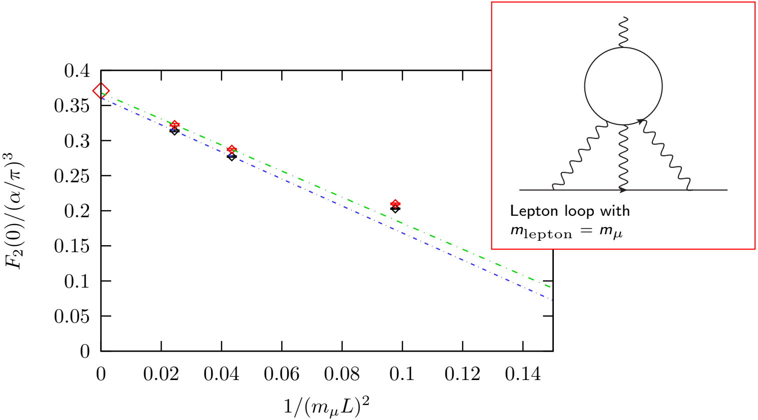
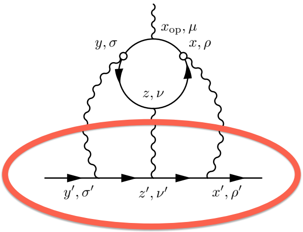


Figure 11. Results for $F_2(0)$ from QED connected light-by-light scattering. These results have been extrapolated to the $a^2 \to 0$ limit using two methods. The upper points use the quadratic fit to all three lattice spacings shown in Fig. 10 while the lower point uses a linear fit to the two left most points in that figure. Here we extrapolate to infinite volume using the linear fits shown to the two, left-most of the three points in each case.

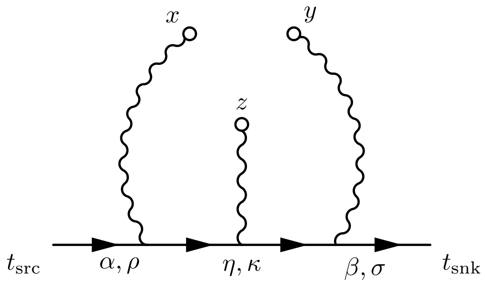
Infinite-volume QED prescription (QED$_\infty$)



Remove power-law like finite-volume errors by computing the muon-photon part of the diagram in infinite volume (C.L. talk at lattice 2015 and Green et al. 2015, PRL115(2015)222003; Asmussen et al. 2016, PoS,LATTICE2016 164)

Now completed PRD96(2017)034515 with improved weighting function.

Details:



We define

$$i^3\mathcal{G}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \mathfrak{G}_{\sigma,\kappa,\rho}(y,z,x) + \text{other 4 permutations}\,.$$

and add the Hermitian conjugate with permuted indices (does not alter F_2 but makes this kernel infrared finite)

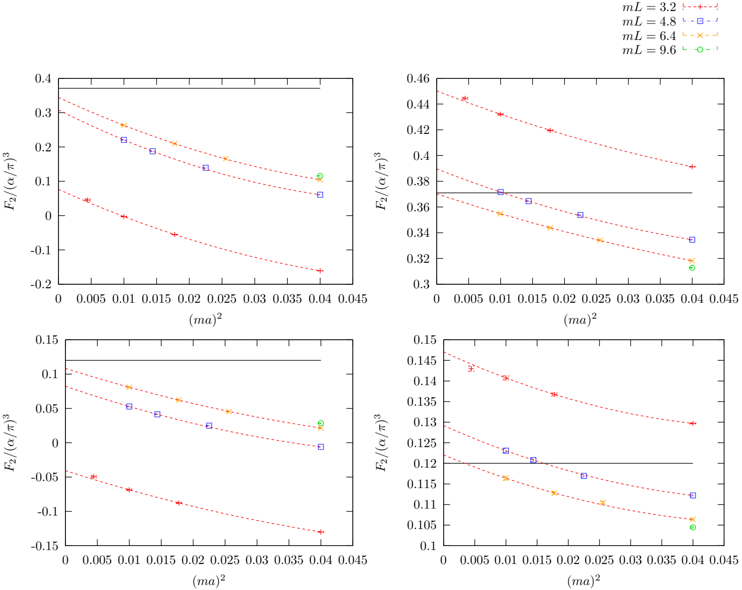
$$\mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \frac{1}{2}\mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \frac{1}{2}[\mathfrak{G}_{\kappa,\sigma,\rho}(z,y,x)]^\dagger$$

For $m_{\rm line} = 1$ this yields the kernel

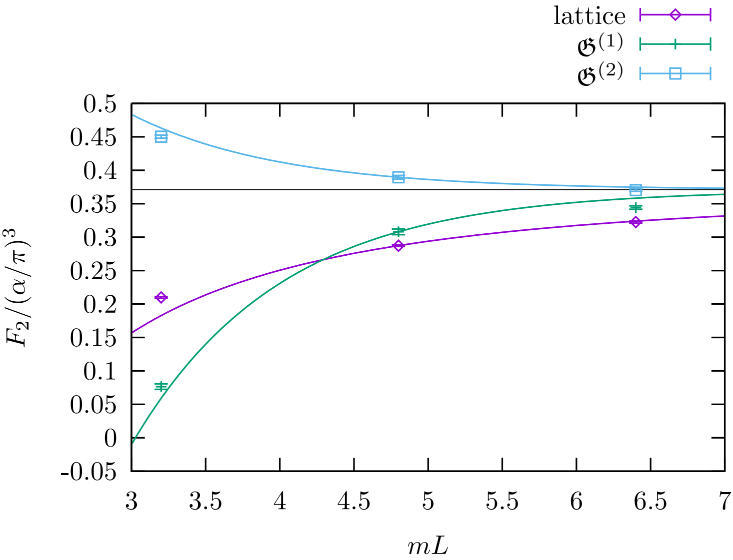
$$\mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) = \frac{\gamma_0+1}{2} i\gamma_\sigma \left(-\not{\partial}_y + \gamma_0 + 1\right) i\gamma_\kappa \left(\not{\partial}_x + \gamma_0 + 1\right) i\gamma_\rho \frac{\gamma_0+1}{2} \times \frac{1}{4\pi^2}\int d^4\eta \frac{1}{(\eta-z)^2} f(\eta-y) f(x-\eta).$$

Due to current conservation, we can also devise a subtraction scheme that we found suppresses significantly finite-volume and discretization errors (demonstrated in the lepton loop case)

$$\mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(x,y,z) = \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,y) + \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,y)$$



$m_{\text{loop}} = m_\mu$ (top), $m_{\text{loop}} = 2m_\mu$ (bottom)
Without subtraction (left), with subtraction (right)



"Lattice" here refers to the finite-volume QED method

Lattice QCD ensembles for the quark loop

48I ($48^3 \times 96$), $L = 5.47$ fm, $a^{-1} = 1.730$ GeV, $m_\pi = 139$ MeV
64I ($64^3 \times 128$), $L = 5.35$ fm, $a^{-1} = 2.359$ GeV, $m_\pi = 139$ MeV
24D ($24^3 \times 64$), $L = 4.67$ fm, $a^{-1} = 1.015$ GeV, $m_\pi = 141$ MeV
32D ($32^3 \times 64$), $L = 6.22$ fm, $a^{-1} = 1.015$ GeV, $m_\pi = 141$ MeV
48D ($48^3 \times 64$), $L = 9.33$ fm, $a^{-1} = 1.015$ GeV, $m_\pi = 141$ MeV

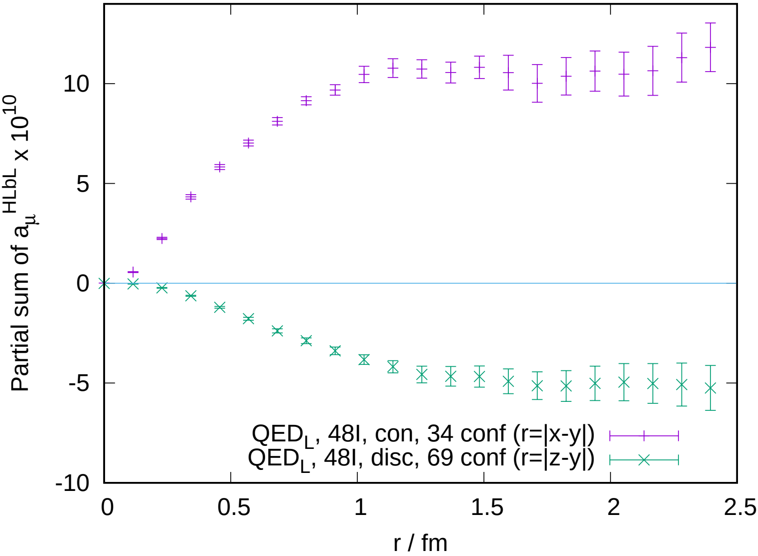
All of them are chirally symmetric domain-wall configurations with 2+1 flavors.

So far studied: QED$_L$ on 24D, 48I, and 64I; QED$_\infty$ on 24D, 48D, and 48I.

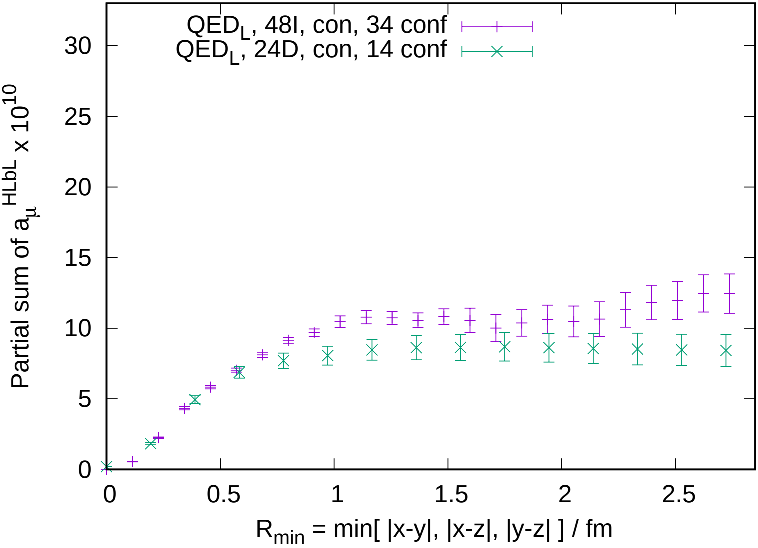
For QED$_\infty$ for now only show connected diagram since disconnected QED$_\infty$ analysis is still too premature.

Technical progress to handle large volumes: Multi-Grid Lanczos
arXiv:1710.06884

QED$_L$ result from PRL118(2017)022005:

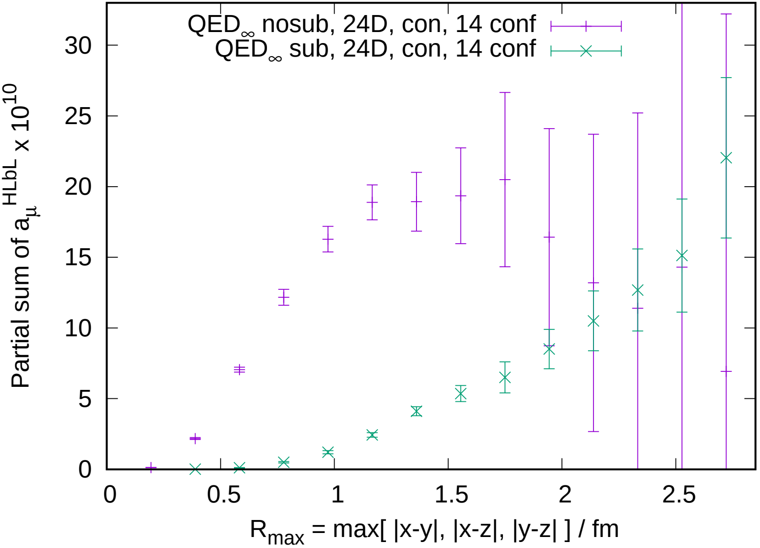


Connected diagram in QED_L on 24D and 48I lattices:



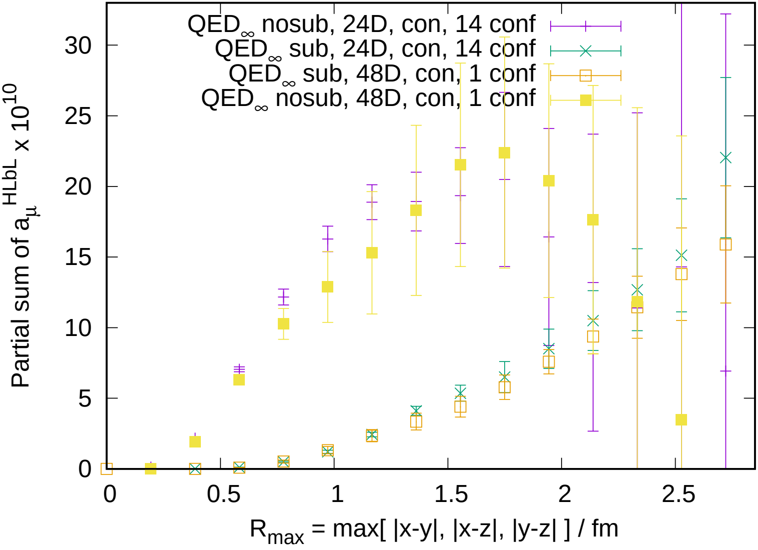
Short-distance: disc errors maybe small, Long-distance: effect of different volume? ($L_{24D} = 4.67$ fm, $L_{48I} = 5.47$ fm)

Connected diagram in QED_∞ on 24D:



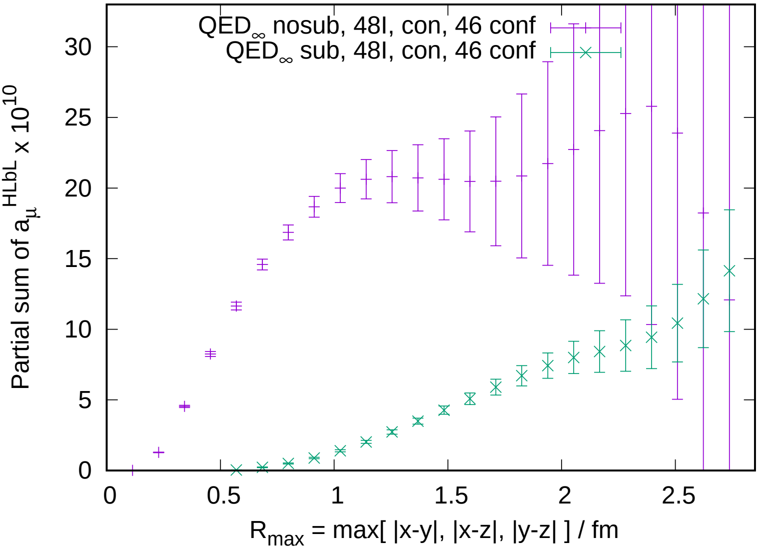
Subtraction that helped reduce discretization and volume errors in lepton-loop case has significant effect on on-set of plateau in this plot. LD noise large.

Connected diagram in QED_∞ on 24D and 48D:

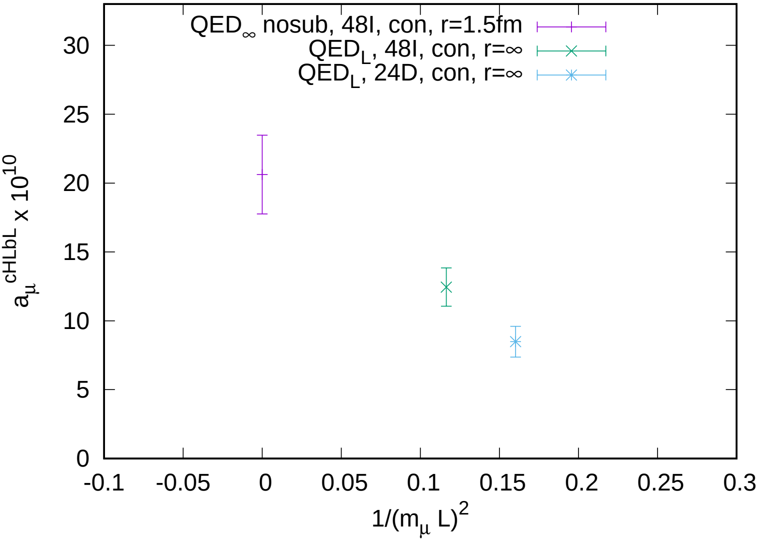


With current poor statistics no sign of QCD FV effect. May be hidden at very long distances? Perform study of long-distance π^0-pole contribution: talk by N. Christ

Connected diagram in QED_∞ on 48I:

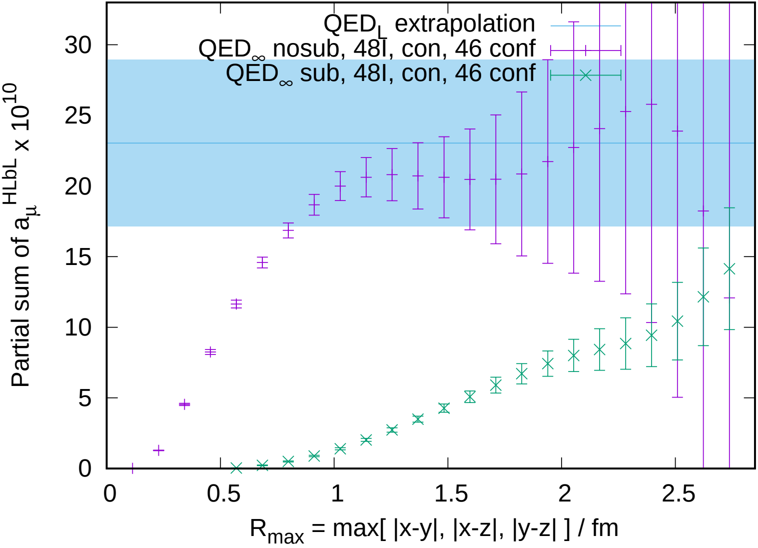


Consistency of QED_L and QED_∞:



This plot is preliminary and needs to be refined with a proper continuum limit since 24D and 48I have different lattice cutoff.

Consistency of QED_L and QED_∞:



What is left to be done:

- Improve statistics, complete runs on 32D and 48D ensembles
- Disconnected diagrams in QED_∞ (almost done)
- Take infinite-volume limit in both QED_L and QED_∞ and check for consistency

Thank you