

Discretization errors in Light-by-Light scattering calculations

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Outline I

1 Introduction

- The continuum limit in finite volume QED
- Towards the continuum limit in finite volume QCD
- towards the continuum limit in QED_∞

2 Summary/Outlook

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Continuum limit in 2+1 flavor lattice QCD

- Scaling trajectory: choose set of physical obs. to keep fixed as $a \rightarrow 0$
- Our choice: pion mass, kaon mass, and Omega baryon mass to set a [Arthur et al., 2013]
 $am_{ud}(\beta), am_s(\beta)$ to keep $m_\pi/m_\Omega, m_K/m_\Omega$ fixed to their physical values as $\beta = 6/g^2 \rightarrow \infty$
- Other obs. will differ from their physical values by $O(a^n)$ along trajectory
- Using Domain wall fermions (DWF), $n = 2$ due to chiral symmetry of DWF

β	a^{-1} GeV	f_π (MeV)	f_K (MeV)
2.13	1.730 (4)	131.13 (14)	156.39 (16)
2.25	2.359 (7)	130.92 (24)	156.94 (24)
∞	0	130.19 (89)	155.51 (83)

[Blum et al., 2014]

2+1 flavor, physical point QCD ensembles (RBC/UKQCD) [Blum et al., 2014]

- Gluons: Iwasaki (I) gauge action (RG improved, plaq+rectangle) and I-DSDR (ID)
- quarks: Möbius-DWF

	$24^3 \times 64$ ID	$48^3 \times 96$ ID	$32^3 \times 64$ ID	$48^3 \times 96$ I	$64^3 \times 128$ I
a^{-1} (GeV)	1.0	1.0	1.4	1.73	2.36
a (fm)	0.2	0.2	0.14	0.114	0.084
L (fm)	4.8	9.6	4.5	5.47	5.38
L_s	24	24	12	24	12
m_π (MeV)	139	140	140	139	135

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1 Introduction

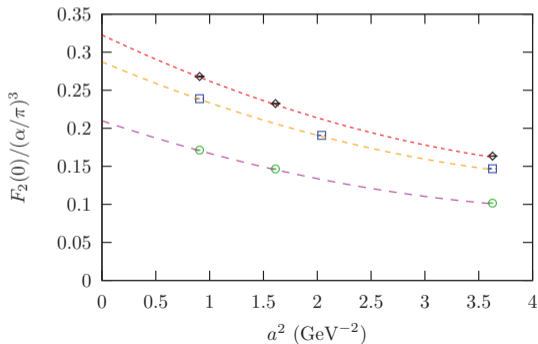
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Continuum limit of a_μ^{LbL} in QED_L [Blum et al., 2016]

- Photons: Feynman gauge, QED_L : omit all modes with $\vec{q} = 0$ [Hayakawa and Uno, 2008]
- muons: $L_s = \infty$ free domain-wall fermions (DWF), $m_\mu = 106$ MeV



L/fm	$F_2(0)/(\alpha/\pi)^3$
5.9	$0.2030(8) - 0.0357(6)(a/\text{GeV})^2$
8.9	$0.2773(9) - 0.0432(5)(a/\text{GeV})^2$
11.9	$0.3138(12) - 0.0515(9)(a/\text{GeV})^2$

- 15-20% error at 1 GeV^{-2}
- $a^2 \sim 0.17 - 0.33 \text{ GeV}^{-2}$
typical range for QCD calculations,
- 1 GeV^{-2} FV study

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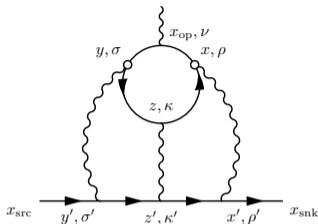
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Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]

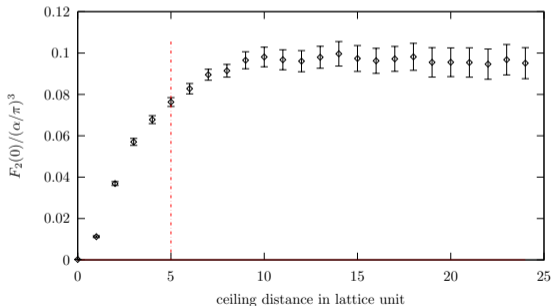
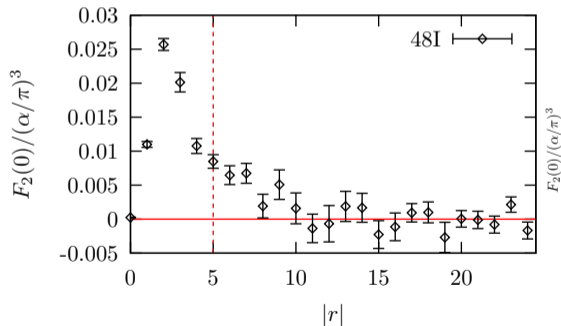


$$\mathcal{F}_\nu^C(\vec{q}; x, y, z, x_{\text{op}}) = (-ie)^6 \mathcal{G}_{\rho, \sigma, \kappa}(\vec{q}; x, y, z) \mathcal{H}_{\rho, \sigma, \kappa, \nu}^C(x, y, z, x_{\text{op}})$$

$$\begin{aligned}
 & i^4 \mathcal{H}_{\rho, \sigma, \kappa, \nu}^C(x, y, z, x_{\text{op}}) \\
 = & \sum_{q=u, d, s} \frac{(e_q/e)^4}{6} \langle \text{tr} [-i \gamma_\rho S_q(x, z) i \gamma_\kappa S_q(z, y) i \gamma_\sigma S_q(y, x_{\text{op}}) i \gamma_\nu S_q(x_{\text{op}}, x)] \rangle_{\text{QCD}} + 5 \text{ permutations} \\
 & i^3 \mathcal{G}_{\rho, \sigma, \kappa}(\vec{q}; x, y, z) \\
 = & e^{\sqrt{m^2 + \vec{q}^2}/4(t_{\text{snk}} - t_{\text{src}})} \sum_{x', y', z'} G_{\rho, \rho'}(x, x') G_{\sigma, \sigma'}(y, y') G_{\kappa, \kappa'}(z, z') \\
 & \times \sum_{\vec{x}_{\text{snk}}, \vec{x}_{\text{src}}} e^{-i\vec{q}/2 \cdot (\vec{x}_{\text{snk}} + \vec{x}_{\text{src}})} S(x_{\text{snk}}, x') i \gamma_{\rho'} S(x', z') i \gamma_{\kappa'} S(z', y') i \gamma_{\sigma'} S(y', x_{\text{src}}) + 5 \text{ permutations}
 \end{aligned}$$

Physical point cHLbL contribution, 48^3 , 1.73 GeV lattice [Blum et al., 2017a]

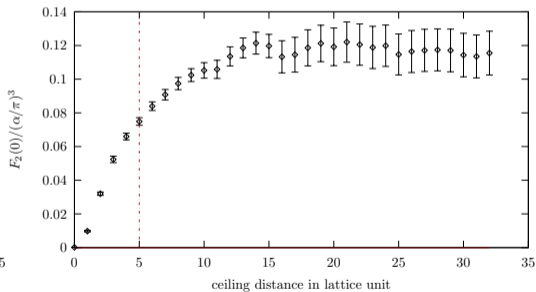
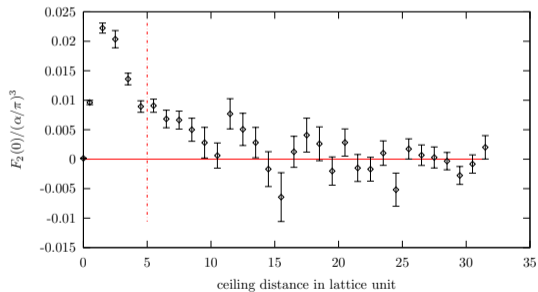
- Measurements on 65 configurations, separated by 20 trajectories
- ignore strange quark contribution (down by 1/17 plus mass suppressed)
- exponentially suppressed with distance
- most of contribution by about 1 fm



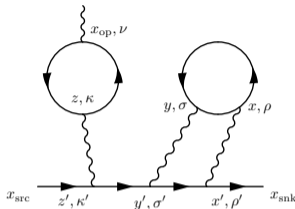
$$a_\mu^{\text{cHLbL}} = 11.60 \pm 0.96 \times 10^{-10}$$

Physical point cHLbL contribution, 64^3 , 2.36 GeV lattice (preliminary)

- Measurements as before, but 43 configurations
- exponentially suppressed with distance
- most of contribution by about 1 fm



Leading disconnected contribution



$$\frac{F_2^{\text{dHLbL}}(0)}{m} \frac{(\sigma_{s',s})_i}{2} = \sum_{r,x} \sum_{x_{\text{op}}} \frac{1}{2} \epsilon_{i,j,k} (x_{\text{op}})_j \cdot i \bar{u}_{s'}(\vec{0}) \mathcal{F}_k^D(x, y=r, z=0, x_{\text{op}}) u_s(\vec{0})$$

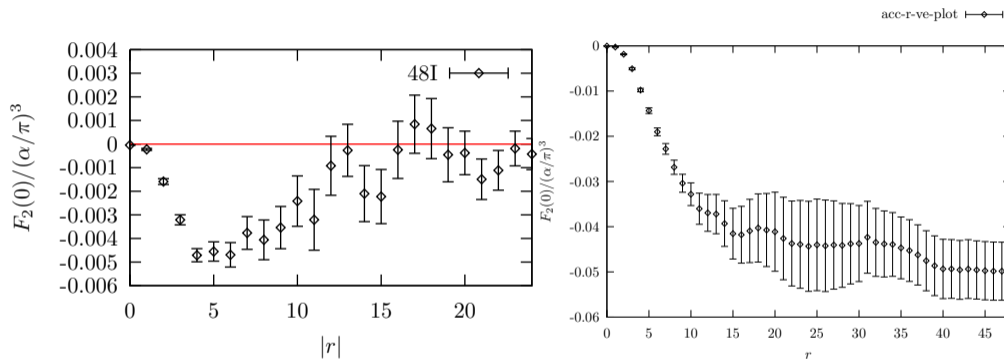
$$\mathcal{H}_{\rho,\sigma,\kappa,\nu}^D(x, y, z, x_{\text{op}}) = \left\langle \frac{1}{2} \Pi_{\nu,\kappa}(x_{\text{op}}, z) [\Pi_{\rho,\sigma}(x, y) - \Pi_{\rho,\sigma}^{\text{avg}}(x-y)] \right\rangle_{\text{QCD}}$$

$$\sum_{x_{\text{op}}} \frac{1}{2} \epsilon_{i,j,k} (x_{\text{op}})_j \langle \Pi_{\rho,\sigma}(x_{\text{op}}, 0) \rangle_{\text{QCD}} = \sum_{x_{\text{op}}} \frac{1}{2} \epsilon_{i,j,k} (-x_{\text{op}})_j \langle \Pi_{\rho,\sigma}(-x_{\text{op}}, 0) \rangle_{\text{QCD}} = 0$$

- Because of parity, the expectation value for the (moment of) left loop averages to zero.
- $[\Pi_{\rho,\sigma}(x, y) - \Pi_{\rho,\sigma}^{\text{avg}}(x-y)]$ is only a noise reduction technique. $\Pi_{\rho,\sigma}^{\text{avg}}(x-y)$ should remain constant through out the entire calculation.

Physical point dHLbL contribution, 48^3 , 1.73 GeV lattice [Blum et al., 2017a]

- strange contributes less than 5 %

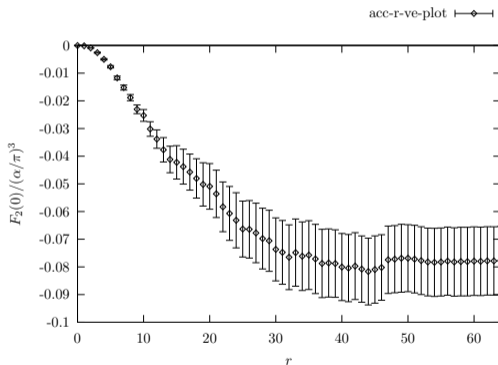
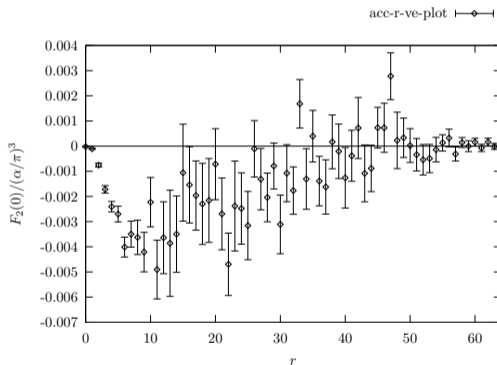


$$a_{\mu}^{\text{dHLbL}} = -6.25 \pm 0.80 \times 10^{-10}$$

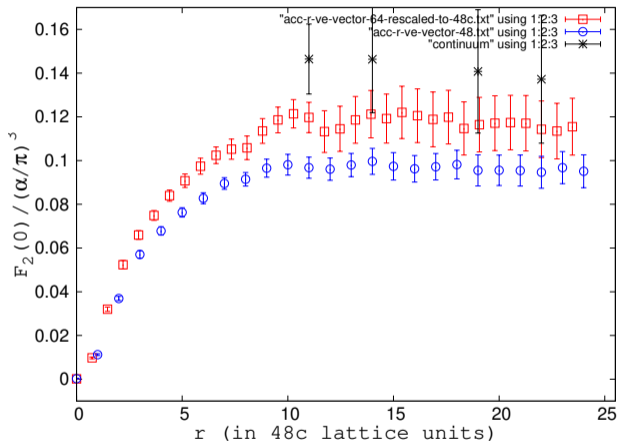
$$a_{\mu}^{\text{cHLbL}} + a_{\mu}^{\text{dHLbL}} = 5.35 \pm 1.35 \times 10^{-10}$$

Physical point dHLbL contribution, 64^3 , 2.36 GeV lattice (preliminary)

- 39 configurations

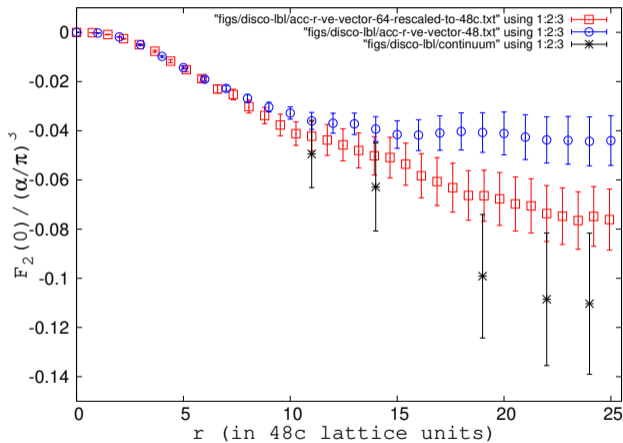


cHLBL: simple continuum limit (preliminary)



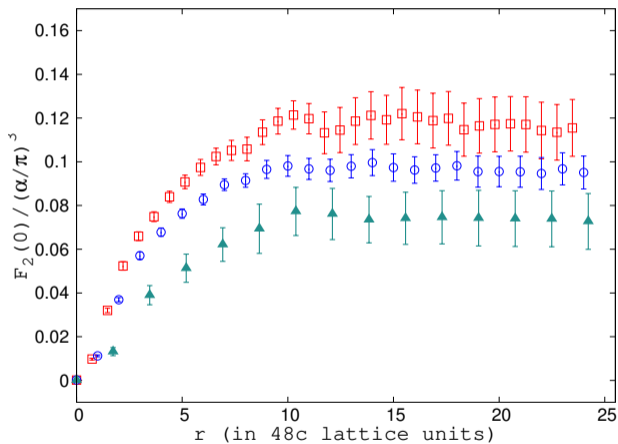
- linear in $a^2 \rightarrow 0$
- Collecting more statistics for finer ensemble (43 \rightarrow 65)

dHLbL: lattice spacing effect (preliminary)



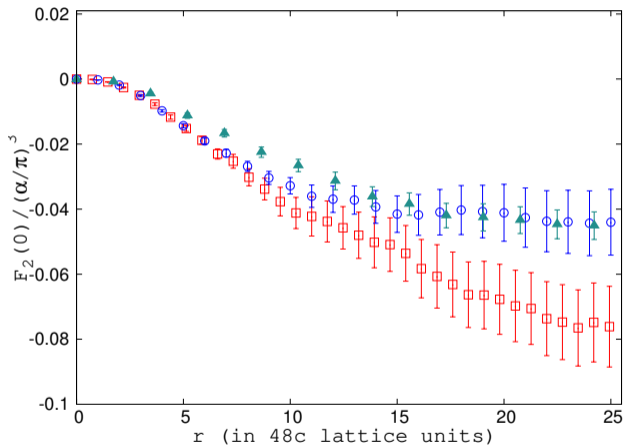
- linear in $a^2 \rightarrow 0$
- Effects tend to cancel with connected piece
- Collecting more statistics (39 \rightarrow 65)

cHLbL: lattice spacing effect (preliminary)



- Add new 24^3 , 1 GeV, ID ensemble
- I and ID slightly different, but disc. errors similar
- Collecting more statistics (9 configs)

dHLbL: lattice spacing effect (preliminary)



- Add new 24^3 , 1 GeV, ID ensemble
- $O(a^4)$ may be significant
- *c.f.* $f_\pi = 130.55(11)$, $f_K = 158.15(13)$
- Similar behavior for HVP
- Starting finer ID ensemble @1.4 GeV

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QED_∞ weighting function

[Green et al., 2015, Asmussen et al., 2016, Lehner and Izubuchi, 2015, Jin et al., 2015, Blum et al., 2017b]

$$= \mathfrak{G}_{\rho, \sigma, \kappa}(x, y, z) + 5 \text{ perms.}$$

- Note Hermitian part gives same F_2 but is infrared finite,

$$\mathfrak{G}_{\rho, \sigma, \kappa}^{(1)}(x, y, z) = \frac{1}{2} \mathfrak{G}_{\rho, \sigma, \kappa}(x, y, z) + \frac{1}{2} \mathfrak{G}_{\rho, \sigma, \kappa}(x, y, z)^\dagger$$

- In units of the muon mass m_μ ,

$$\begin{aligned} \mathfrak{G}_{\sigma, \kappa, \rho}^{(1)}(y, z, x) &= \frac{\gamma_0 + 1}{2} i\gamma_\sigma (-\not{\partial}_y + \gamma_0 + 1) i\gamma_\kappa (\not{\partial}_x + \gamma_0 + 1) i\gamma_\rho \frac{\gamma_0 + 1}{2} \\ &\times \frac{1}{4\pi^2} \int d^4\eta \frac{1}{(\eta - z)^2} f(\eta - y) f(x - \eta) \end{aligned}$$

- Current conservation implies $\sum_x \text{Loop}_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\text{op}}) = 0$ ($V \rightarrow \infty$ and $a \rightarrow 0$)
- Subtract terms that vanish as $V \rightarrow \infty$ and $a \rightarrow 0$

$$\mathfrak{G}_{\rho,\sigma,\kappa}^{(2)}(x, y, z) = \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(x, y, z) - \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(y, y, z) - \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(x, y, y) + \mathfrak{G}_{\rho,\sigma,\kappa}^{(1)}(y, y, y)$$

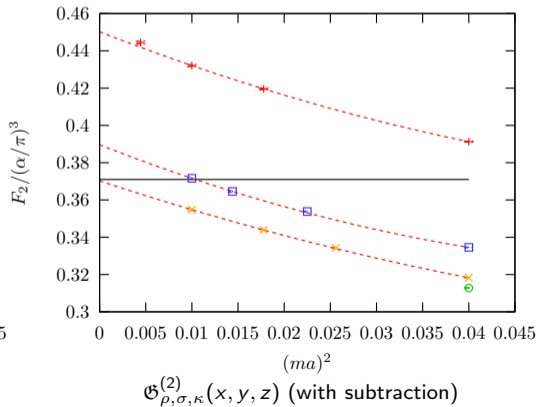
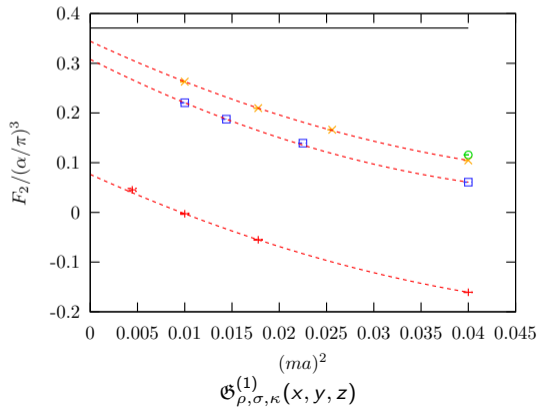
$$\mathfrak{G}_{\rho,\sigma,\kappa}^{(2)}(z, z, x) = 0 \quad \text{so short distance } O(a^2) \text{ effects suppressed.}$$

QED_∞ results- pure QED, lattice-spacing error [Blum et al., 2017b]

- discretization error \sim independent of volume
- significantly smaller with subtraction, sign flips!

$mL = 3.2$ - - - + - - -
 $mL = 4.8$ - - - □ - - -
 $mL = 6.4$ - - - × - - -
 $mL = 9.6$ - - - ○ - - -

$mL = 3.2$ - - - + - - -
 $mL = 4.8$ - - - □ - - -
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Hadronic Light-by-Light Continuum Limit Summary/Outlook

- Continuum limit under reasonably good control for cHLbL
- For dHLbL, need to improve statistics
 - more configs
 - add full volume LMA (*c.f.* HVP)
- and understand better the scaling violations on ID ensembles
 - 1.4 GeV ensemble underway
- For QED_∞ use subtracted weighting function for smaller a^2 errors

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