Light-by-light scattering sum rule

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Based on: PLB736(2014)11; PRD90 (2014) 036004;
PRD94 (2016) 116061; PRD95 (2017) 056007;
PRD97 (2018) 036012;

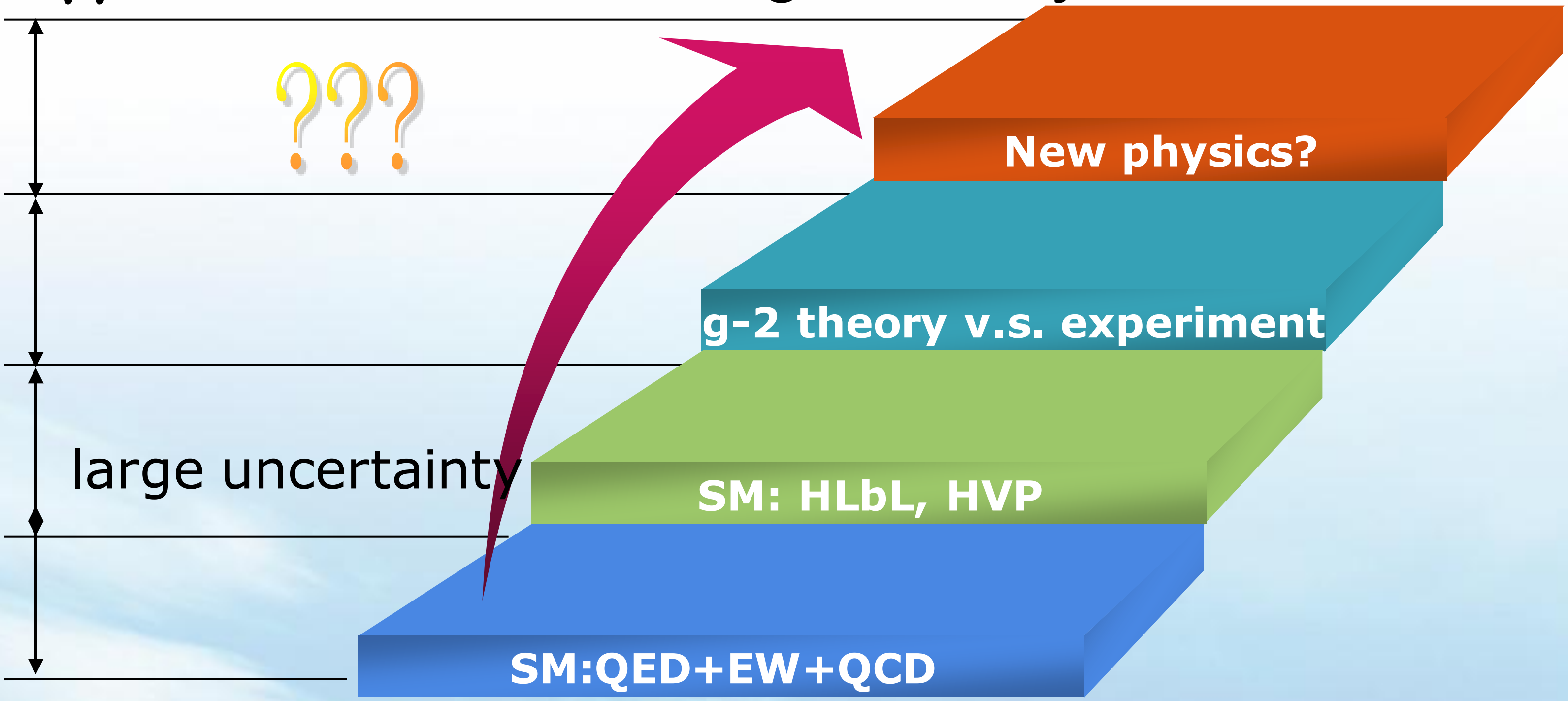
JÜLICH
FORSCHUNGSZENTRUM

Outlines

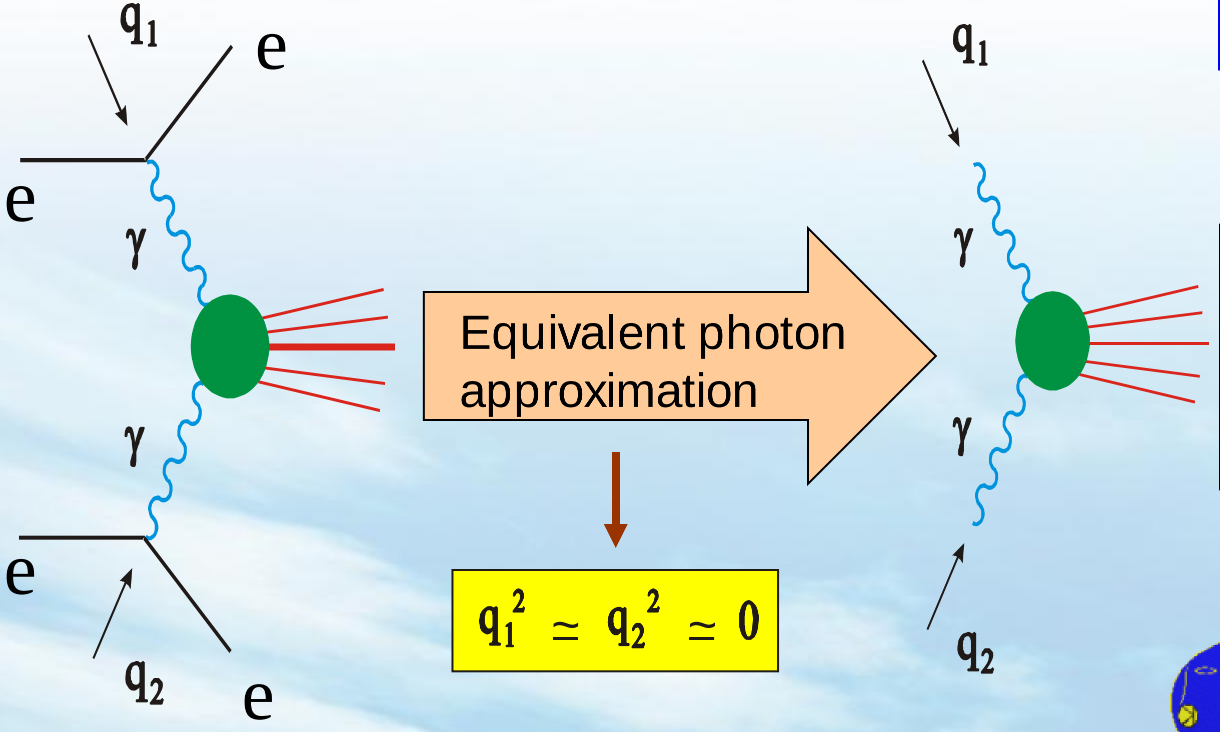
1. Introduction
2. Hadronic amplitudes
3. photon amplitudes
4. Constraints to LbL sumrule
5. Summary

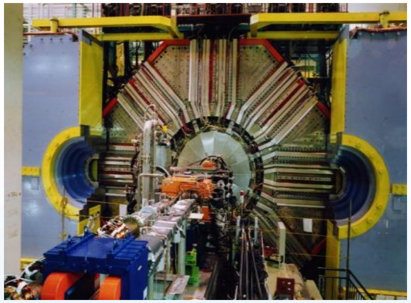
1.Introduction

- HVP, HLbL?
- $\gamma\gamma \to$MM contributes significantly to LbL sumrule

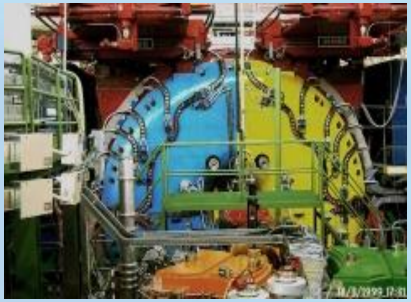


- $\gamma\gamma \to$ MM has the clean background
- $\gamma\gamma \to$ MM contributes significantly to LbL sumrule





π^+	π^0	K^+	$\bar{K}^0$
π^-	π^0	K^-	K^0



Strategy

Hadron

Fit to get $\pi\pi$ - KK coupled channel scattering amplitudes

Photon

With Final State Interaction Theorem we construct di-photon amplitudes and fit to the data

constraint

Constraints to LbL sumrule, polarizabilities. ..

2. Hadronic amplitudes

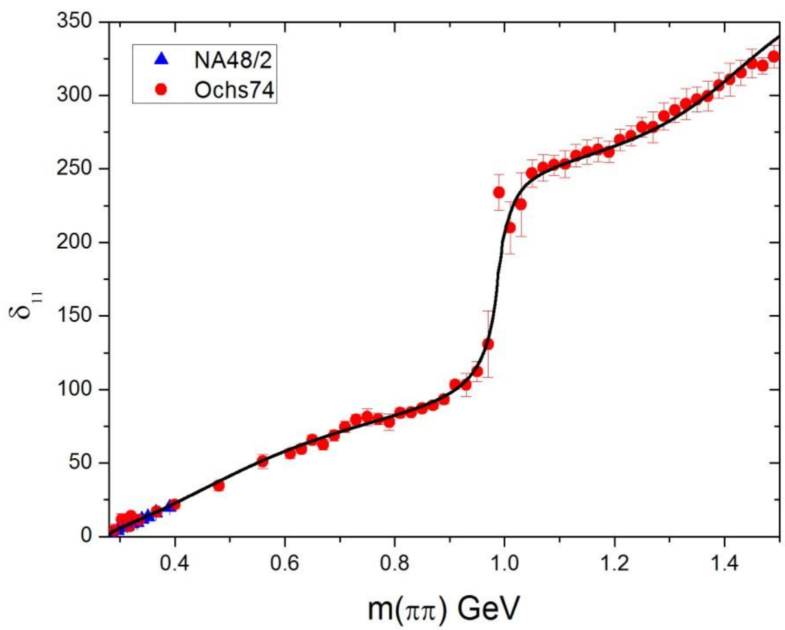
- $\pi\pi$ - KK **scattering inputs**
  - K-matrix to represent S and D partial waves
  - Data on Phase shifts and inelasticities of $\pi\pi$ - KK coupled channel scattering.
  - BABAR's Dalitz plot analysis of $D_s^+ \to (\pi^+\pi^-)\pi^+$ and $D_s^+ \to (K^+K^-)\pi^+$ process. BES's analysis on $J/\psi \to \pi^+\pi^-\phi$ and $J/\psi \to K^+K^-\phi$.
  - Dispersion analysis:
    - ❿ T-matrix of $\pi\pi$ scattering by CFDIV .
    - ❿ $\pi\pi \to KK$ amplitudes given by Roy-Steiner Equation.

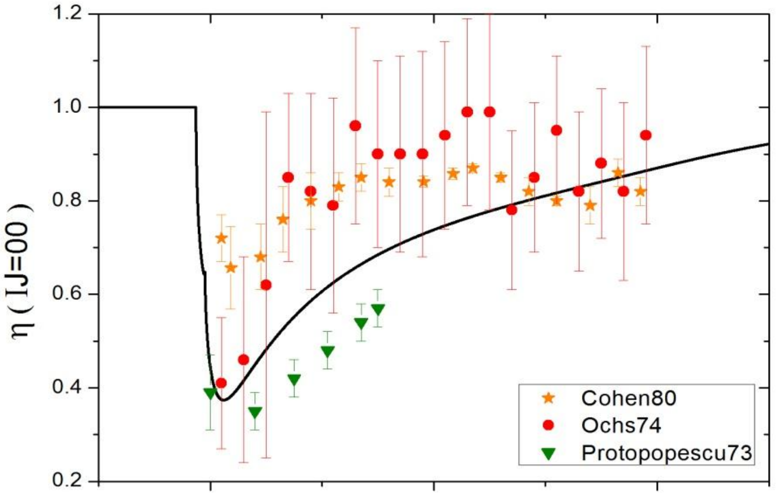
Descotes *et.al*
EPJC33 (2004) 409

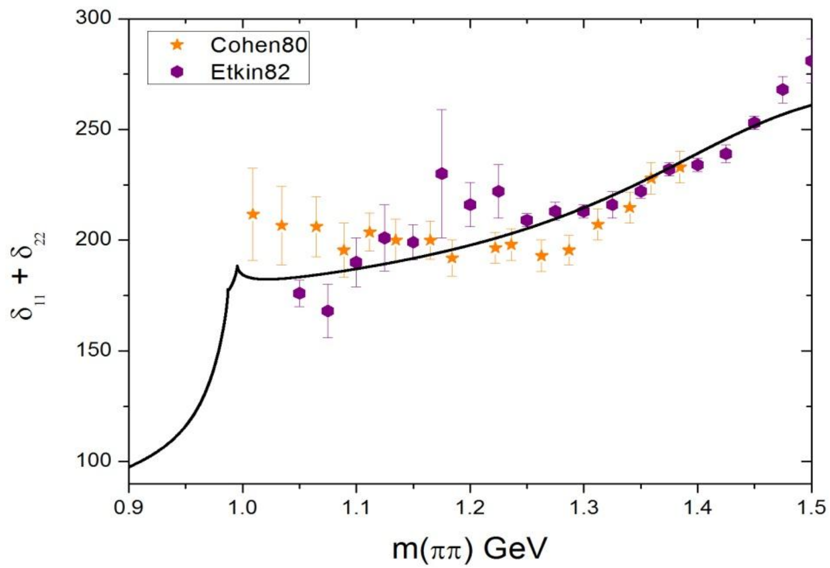
Pelaez *et al*.
PRD83 (2011) 074004

Data: phase shift and inelasticity

- $\pi\pi\rightarrow\pi\pi$,KK phase shift and inelasticity

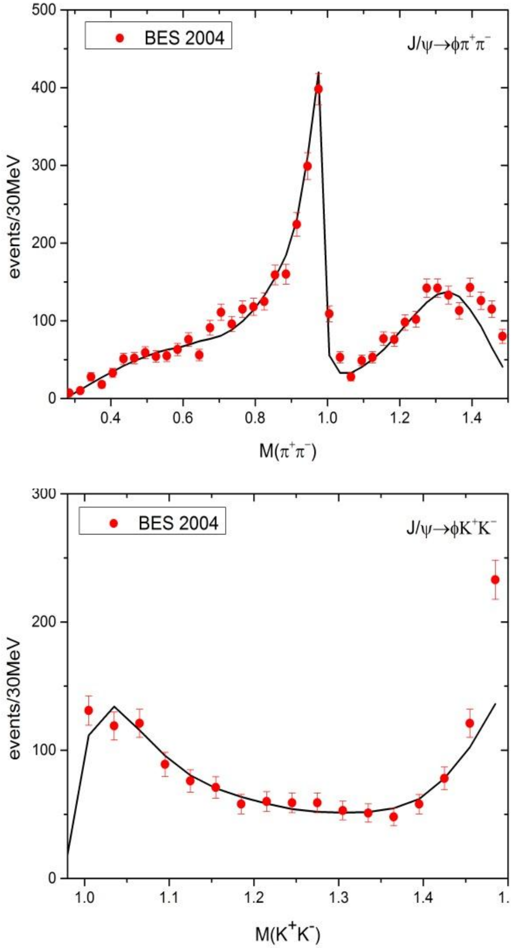


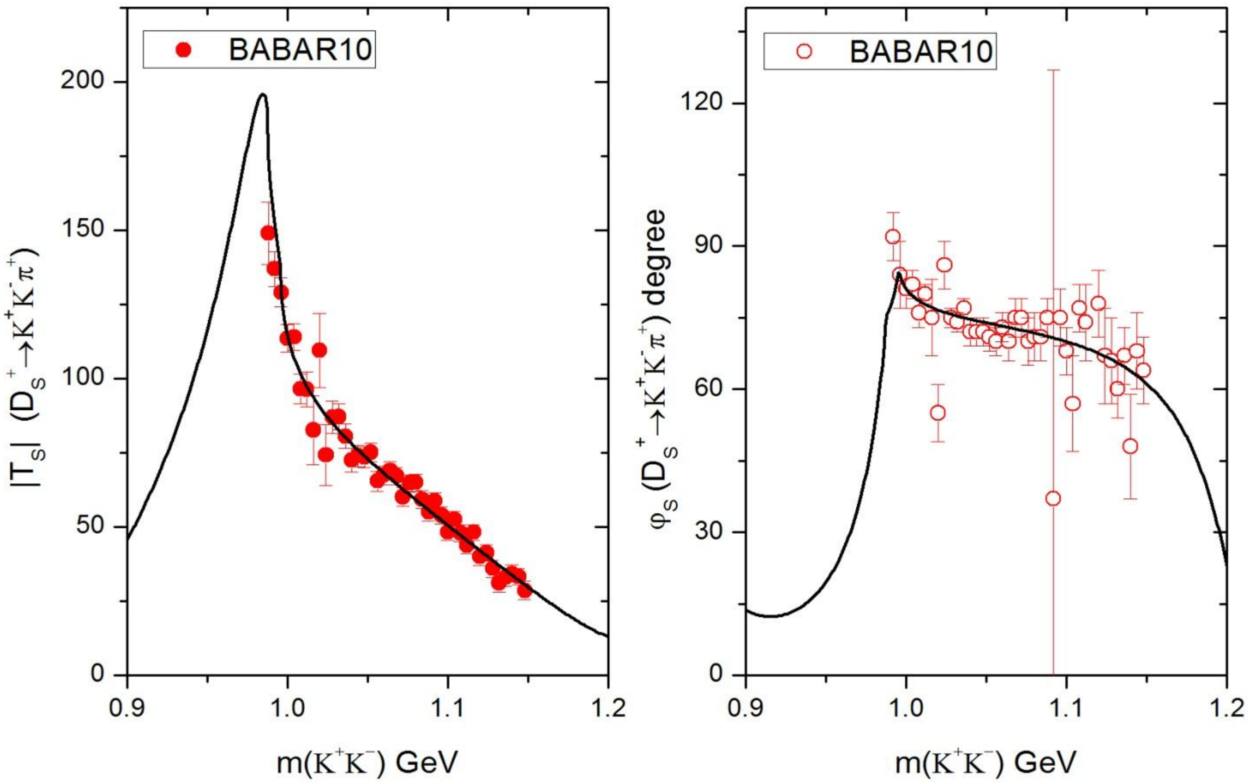




BABAR && BES

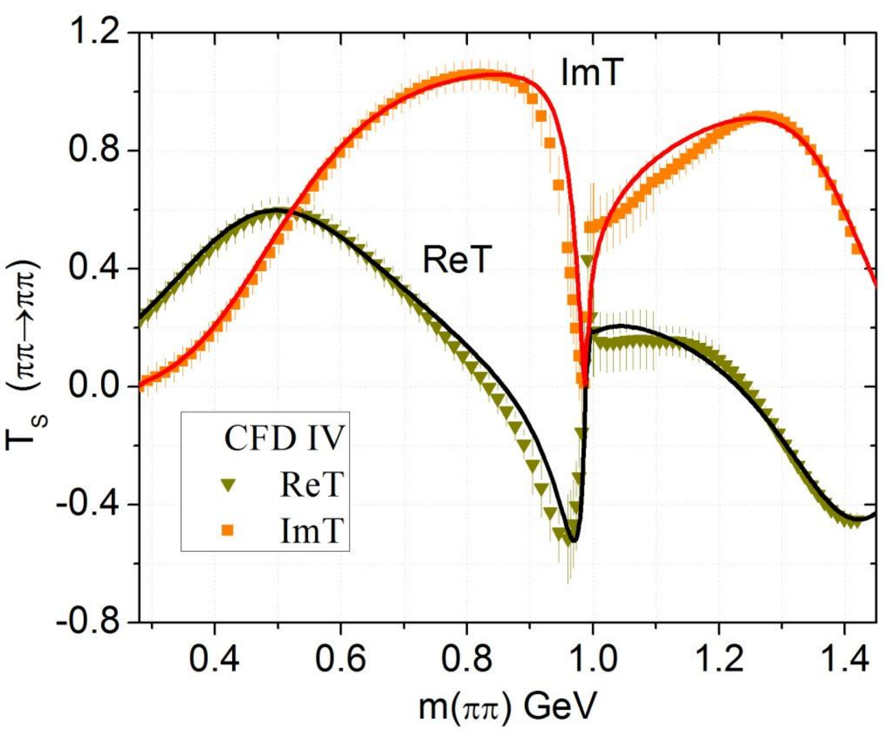
- **$\pi\pi$ - KK scattering inputs**
  - KK threshold region is important as it is around $f_0(980)$.

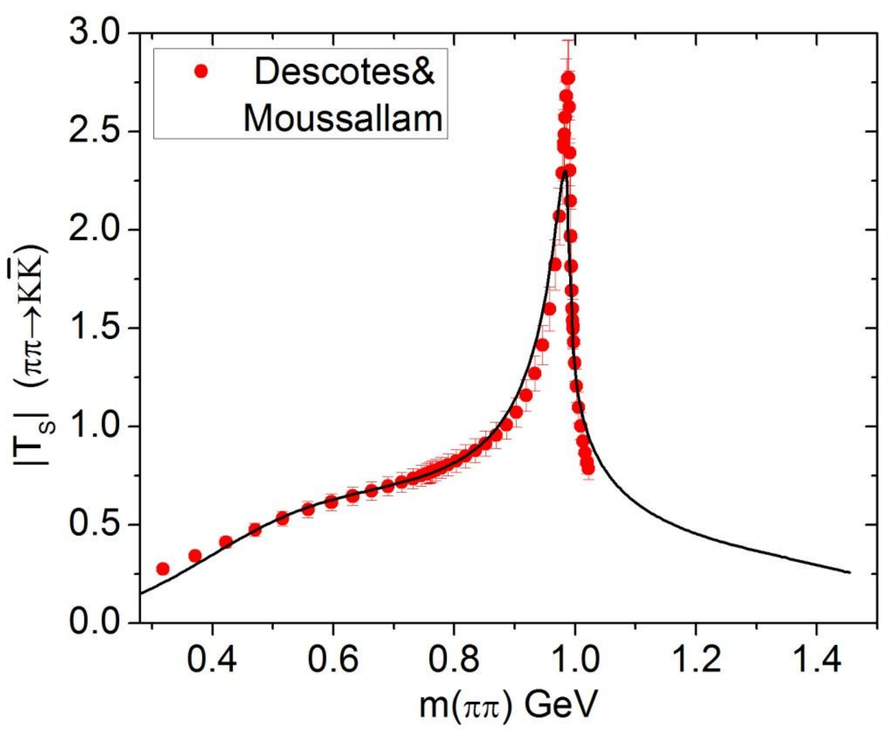




Dispersion analysis constraints

- The constraints from Roy like equation, they have taken crossing symmetry into account.



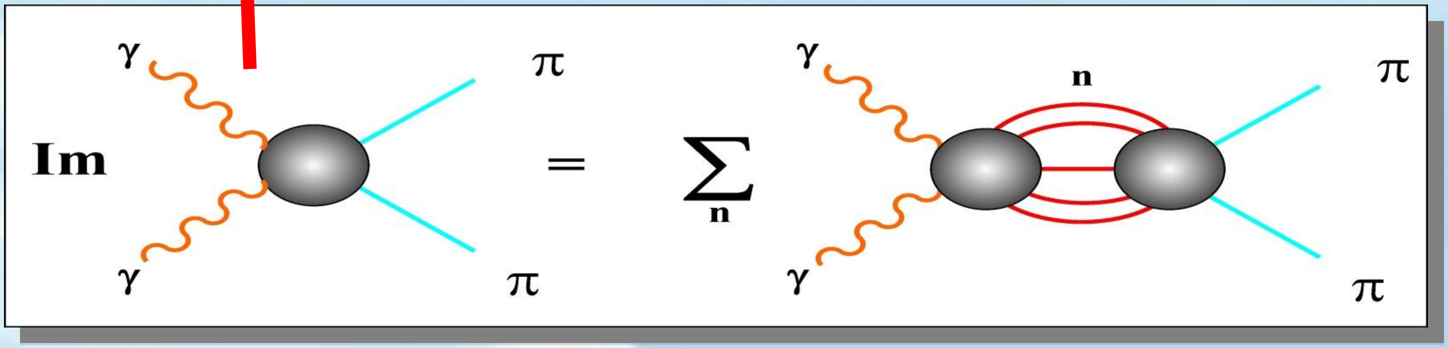


3. Photon amplitudes

- With Final State Interaction Theorem (FSIT), We can construct the amplitudes of photon-photon scattering into meson pairs:

$$\mathcal{F}^I_{J\lambda}(\gamma\gamma \to \pi\pi; s) = \alpha_1{}^I_{J\lambda}(s)\,\hat{T}^I_J(\pi\pi \to \pi\pi; s) + \alpha_2{}^I_{J\lambda}(s)\,\hat{T}^I_J(\pi\pi \to \overline{K}K; s)\,,$$

$$\mathcal{F}^I_{J\lambda}(\gamma\gamma \to \overline{K}K; s) = \alpha_1{}^I_{J\lambda}(s)\,\hat{T}^I_J(\pi\pi \to \overline{K}K; s) + \alpha_2{}^I_{J\lambda}(s)\,\hat{T}^I_J(\overline{K}K \to \overline{K}K; s)\,.$$



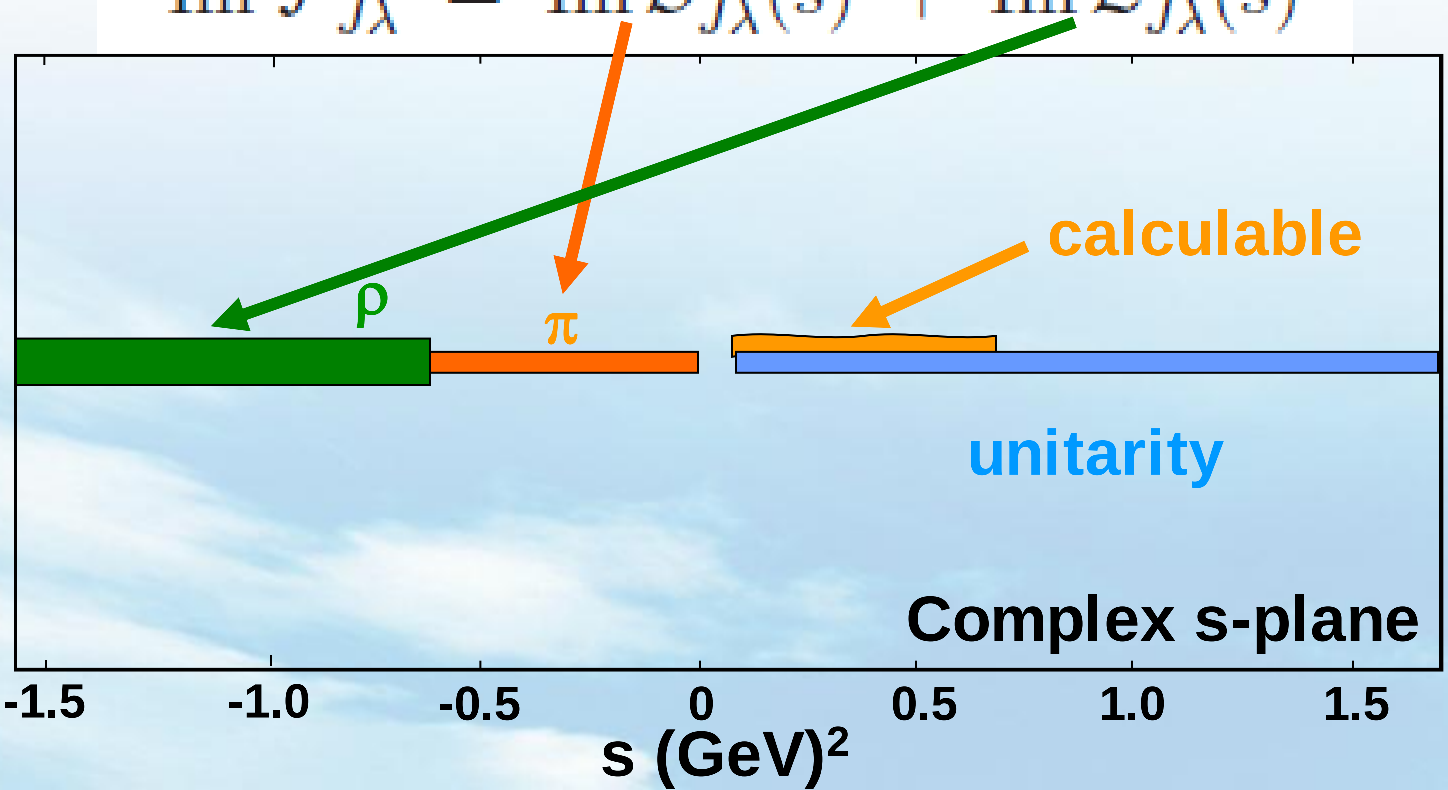
Photon-photon collision

- To constraint the di-photon amplitudes, we follow such steps:
  - We use dispersion relation to calculate the amplitudes below 0.6GeV, and give errors.
  - Fit the overall $\gamma\gamma\to\pi\pi$ and $\gamma\gamma\to K\bar{K}$ datasets, get a very narrowed patch of solutions.

Dispersion relations

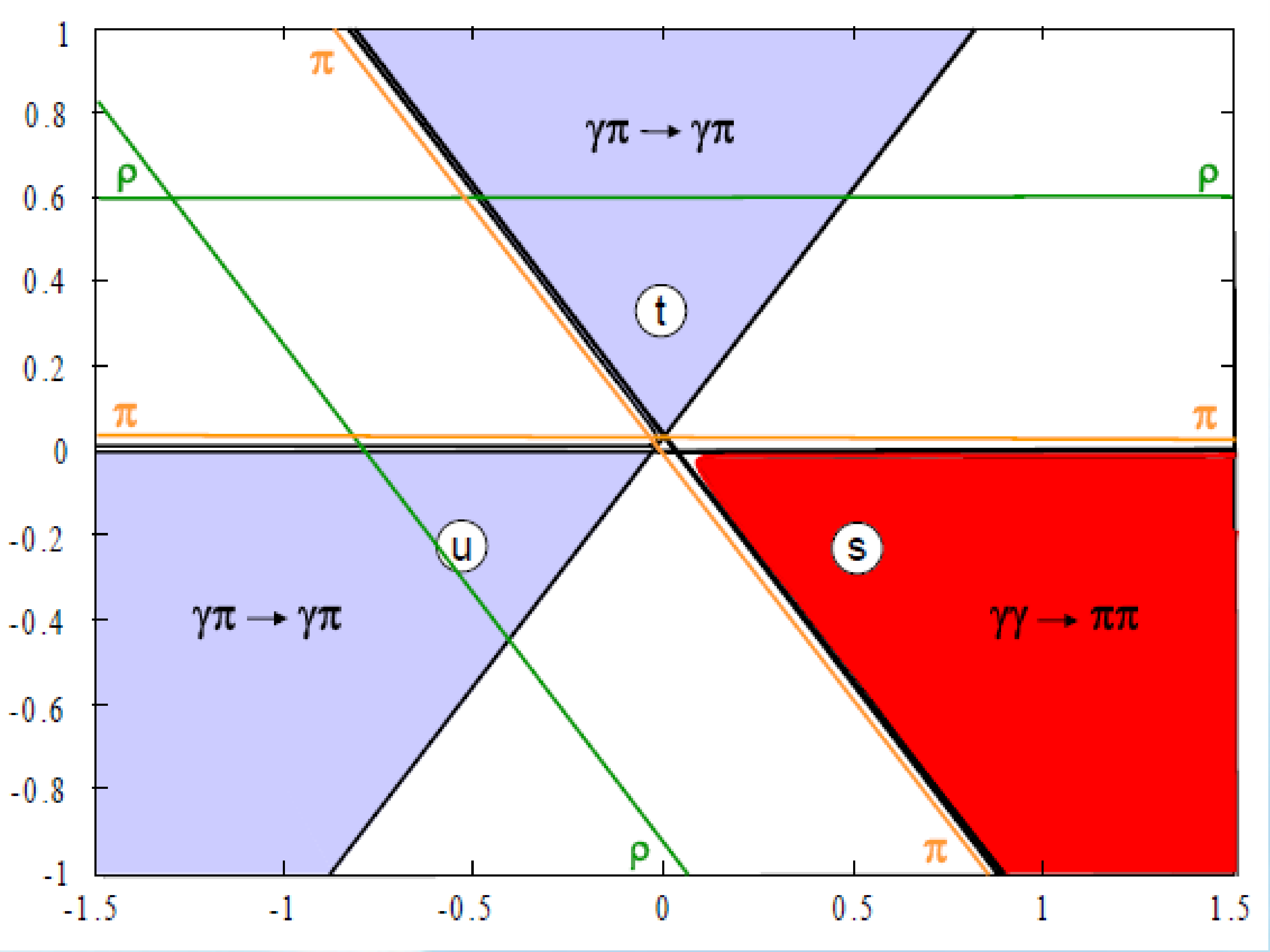
- FSIT: r.h.c is 'fixed' by that of hadronic amplitudes
- l.h.c is divided into two parts: Born term and other cross channel exchange terms. When s<0:

$$\mathrm{Im}\,\mathcal{F}^I_{J\lambda} = \mathrm{Im}\,\mathcal{B}^I_{J\lambda}(s) + \mathrm{Im}\,\mathcal{L}^I_{J\lambda}(s)$$



Vector, Axial-Vector, Tensor contributions

- LHCs of ρ, ω, a_1, b_1, h_1 give an error band of low energy amplitudes,
- Remain parts are parametrized as an effective pole 'T'.



Dispersion relations

- Low's low energy theorem tells us that:

$$\mathcal{F}^I_{J\lambda}(s) \to \mathcal{B}^I_{J\lambda}(s) \quad \text{as } s \to 0$$

- Thus we can write Dispersion relations of

$$\mathcal{F}^I_{00}(s) = \mathcal{B}^I_{00}(s) + b^I s\, \Omega^I_{00}(s) + \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right] \Omega^I_{00}(s')^{-1}}{s'^2(s'-s)} - \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_R ds' \frac{\mathcal{B}^I_{00}(s')\, \mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2(s'-s)}$$

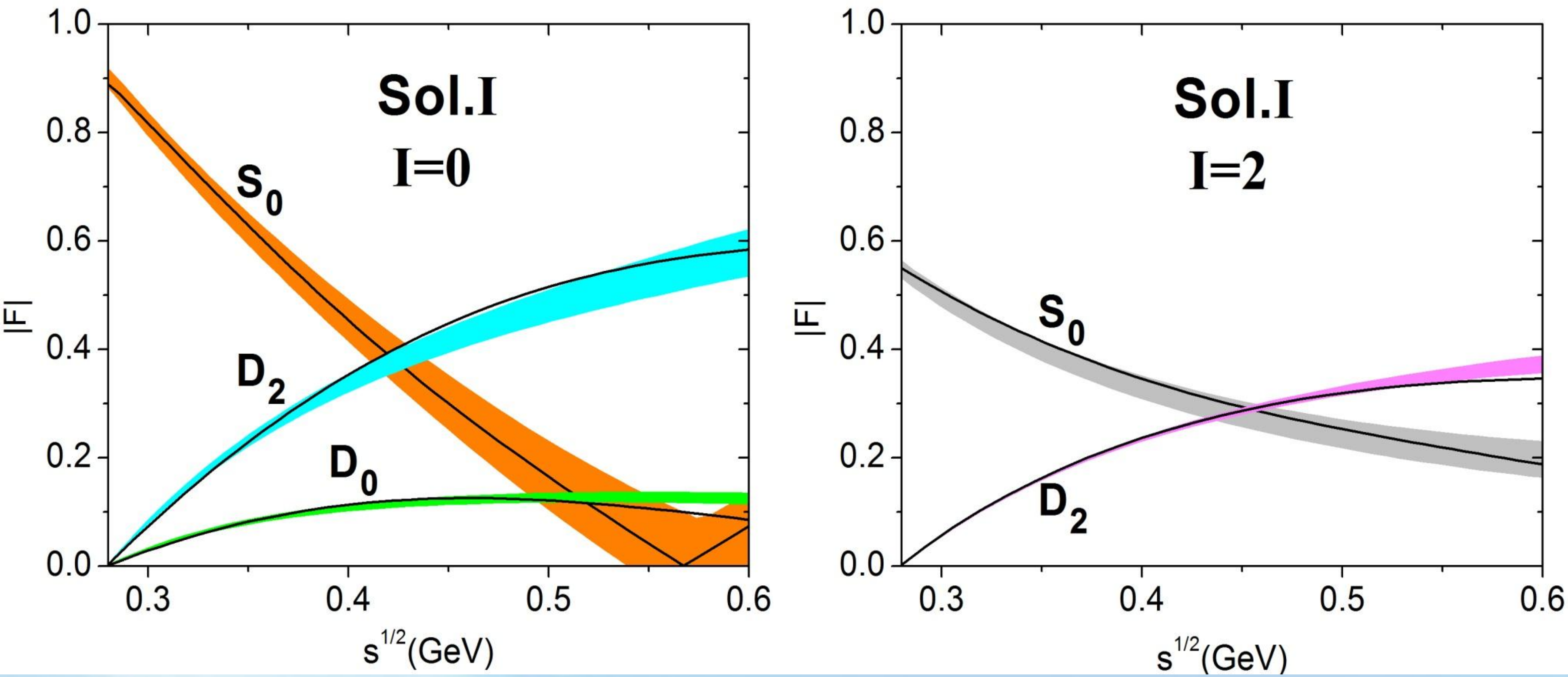
Solved by ChPT

$$\mathcal{F}^I_{J\lambda}(s) = \mathcal{B}^I_{J\lambda}(s) + \frac{s^n(s-4m_\pi^2)^{J/2}}{\pi} \Omega^I_{J\lambda}(s) \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{J\lambda}(s')\right]\, \Omega^I_{J\lambda}(s')^{-1}}{s'^n(s'-4m_\pi^2)(s'-s)} - \frac{s^n(s-4m_\pi^2)^{J/2}}{\pi} \Omega^I_{J\lambda}(s) \int_R ds' \frac{B^I_{J\lambda}(s')\, \mathrm{Im}\left[\Omega^I_{J\lambda}(s')^{-1}\right]}{s'^n(s'-4m_\pi^2)(s'-s)} .$$

Threshold behaviour

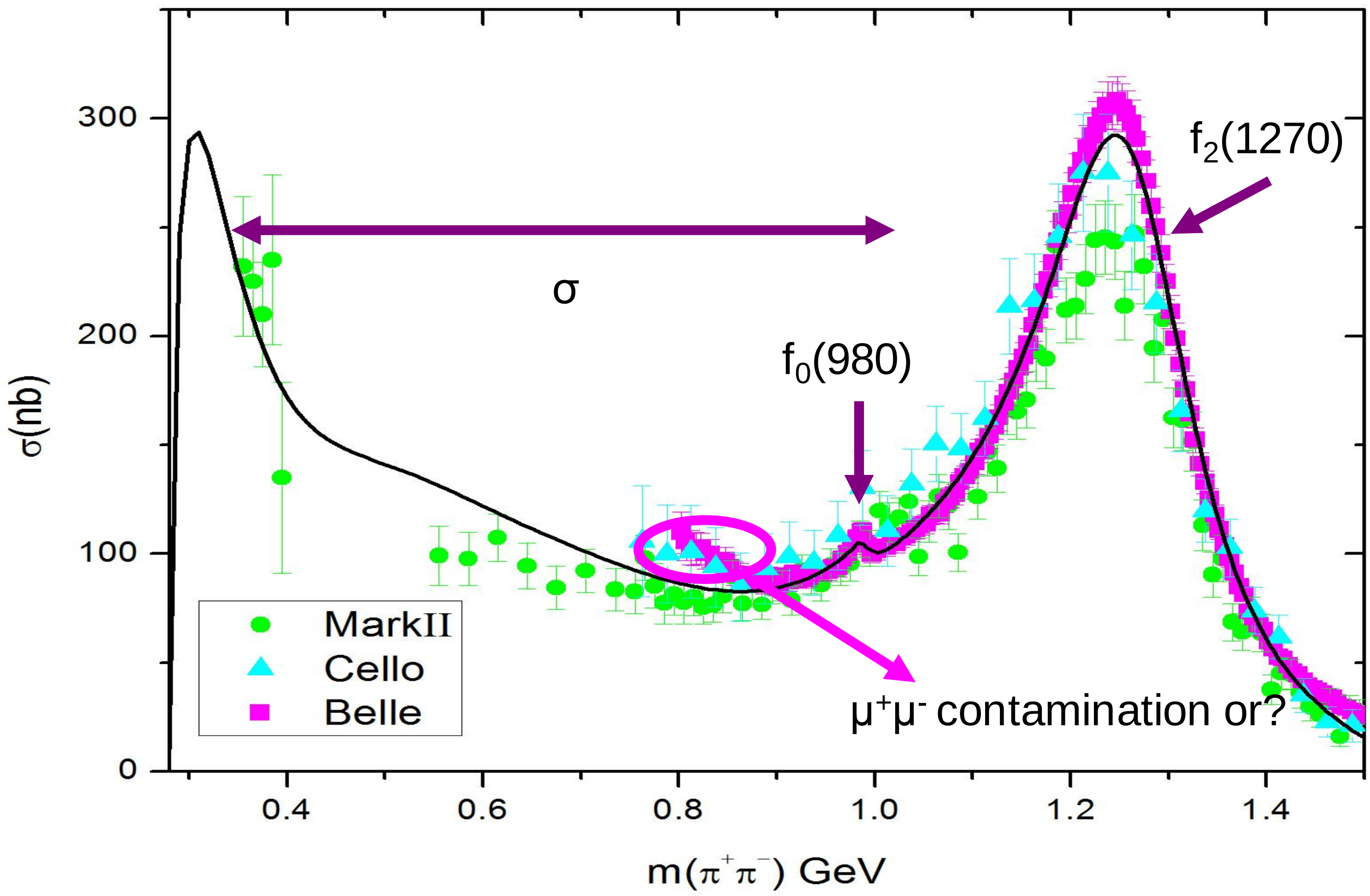
Constraints on low energy amplitudes

- Finally we have the bands given by dispersion relations:

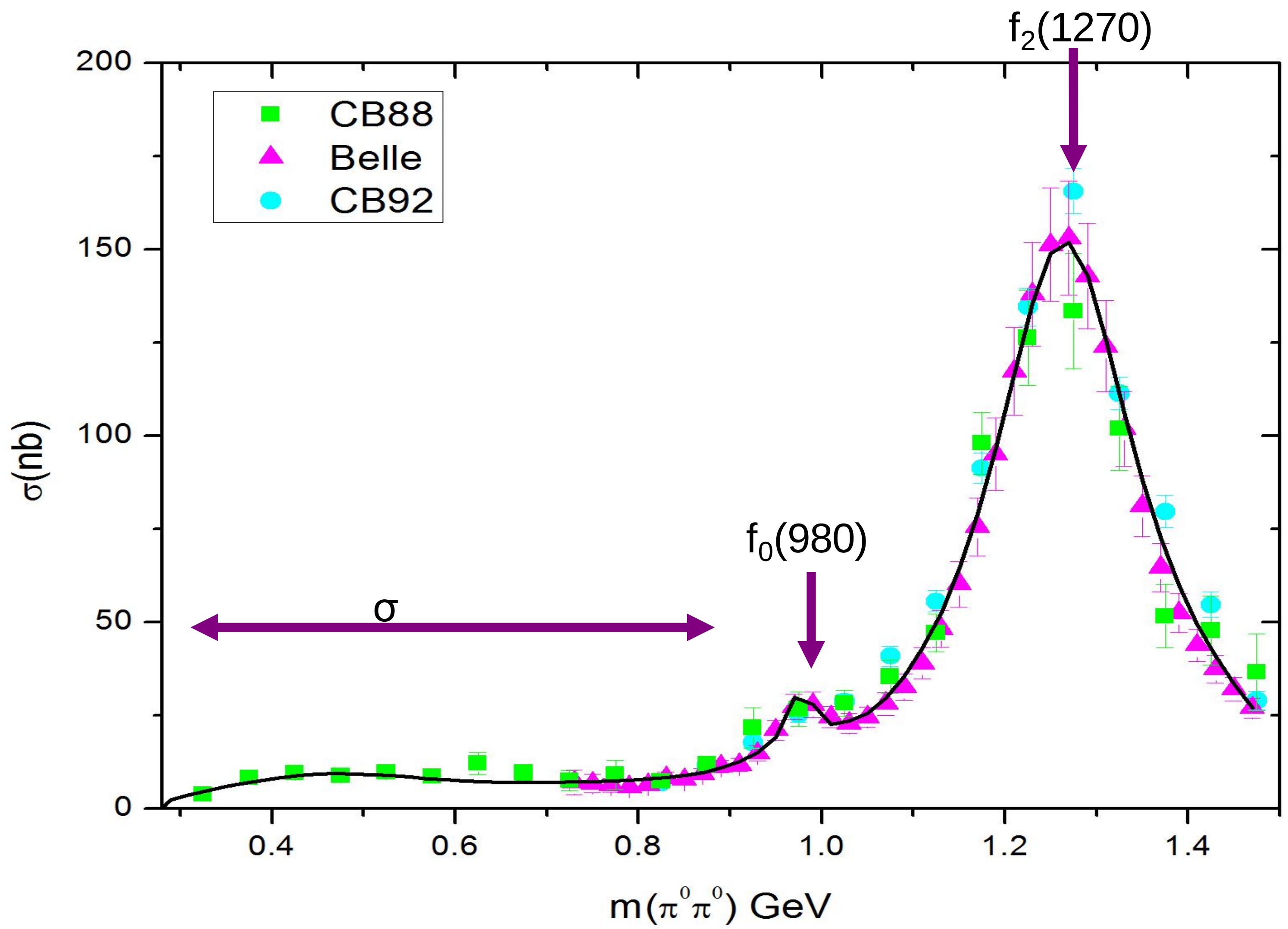


$\gamma\gamma \rightarrow \pi^+\pi^-$ integrated cross section

- With these constraints, we fit all datasets. The integrated cross sections with limited angular coverage



$\gamma\gamma \to \pi^0\pi^0$ integrated cross section



Differential cross section

- The differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\rho(s)}{128\pi^2 s} \left[|M_{+-}|^2 + |M_{++}|^2\right]$$

- And the helicity amplitudes are given by

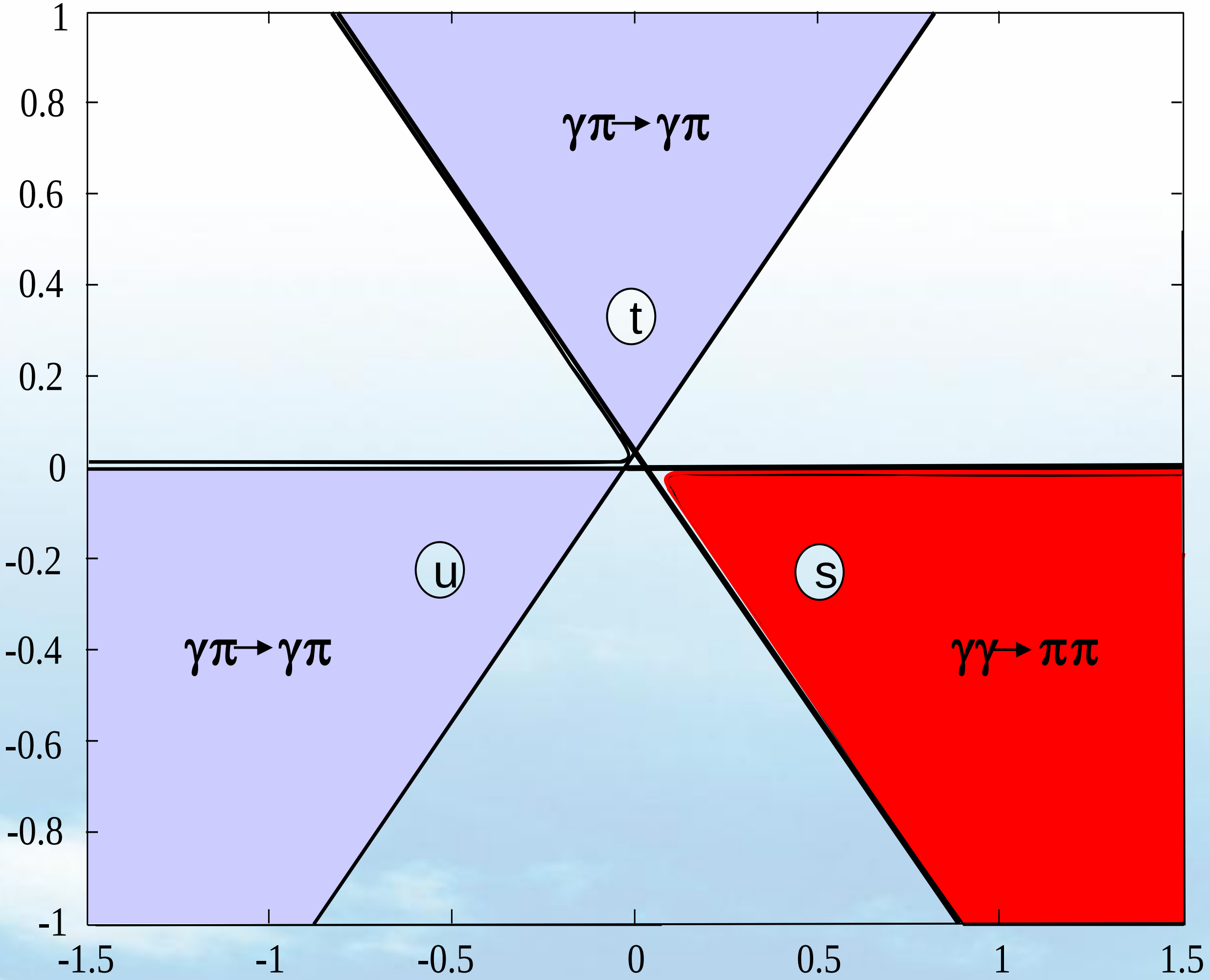
$$M_{++}(s,\theta,\phi) = e^2\sqrt{16\pi} \sum_{J\geq 0} F_{J0}(s)\, Y_{J0}(\theta,\phi) \ ,$$
$$M_{+-}(s,\theta,\phi) = e^2\sqrt{16\pi} \sum_{J\geq 2} F_{J2}(s)\, Y_{J2}(\theta,\phi) \ .$$

- For the integrated cross section, S-D_0 waves will interfere strongly through 1-1.4GeV.
- Angular distribution it is more efficient to distinguish each partial wave.

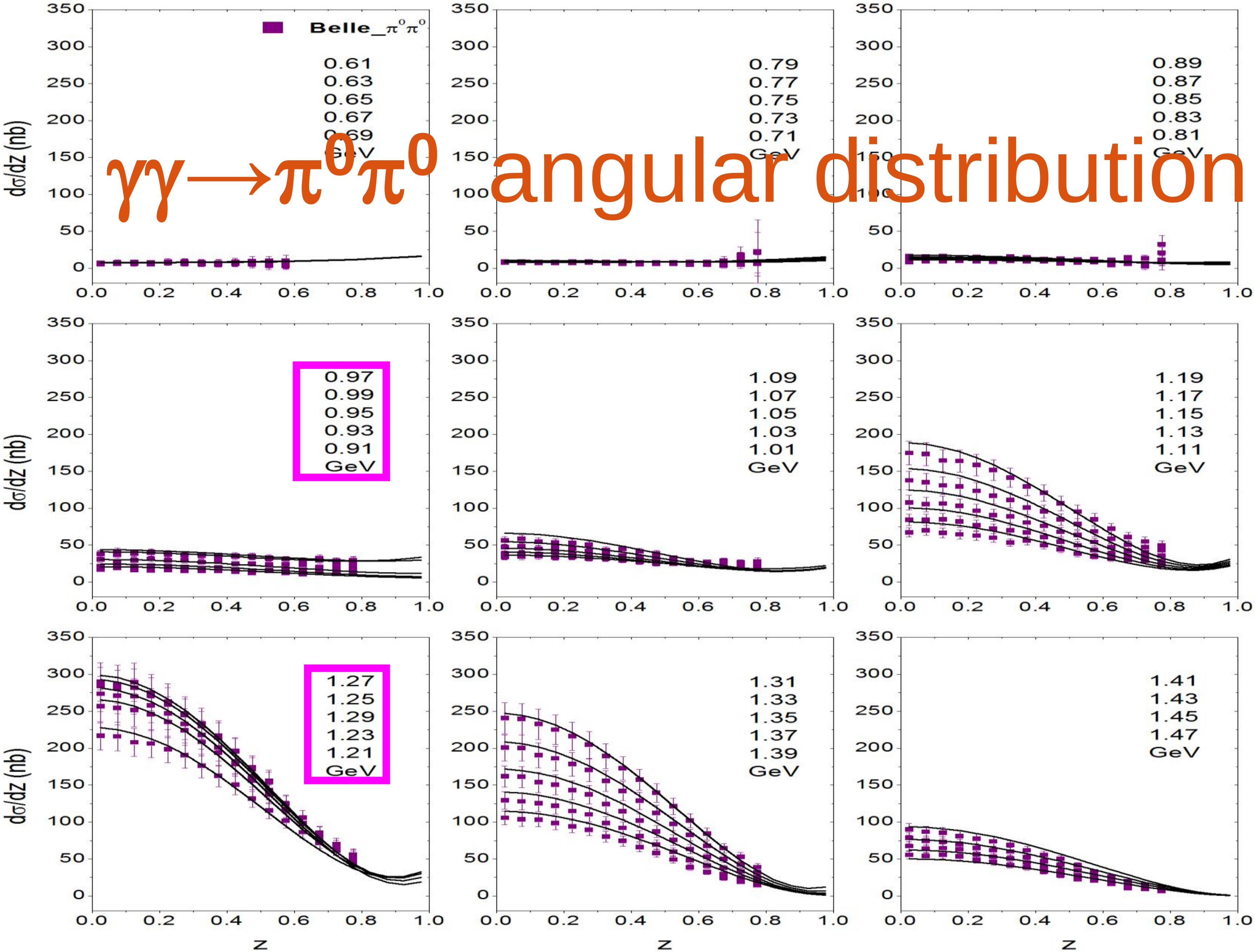
Why angular distribution?

- we can predict the full cross section if we know each partial wave.

- The angular distribution is helpful to seperate each partial wave.

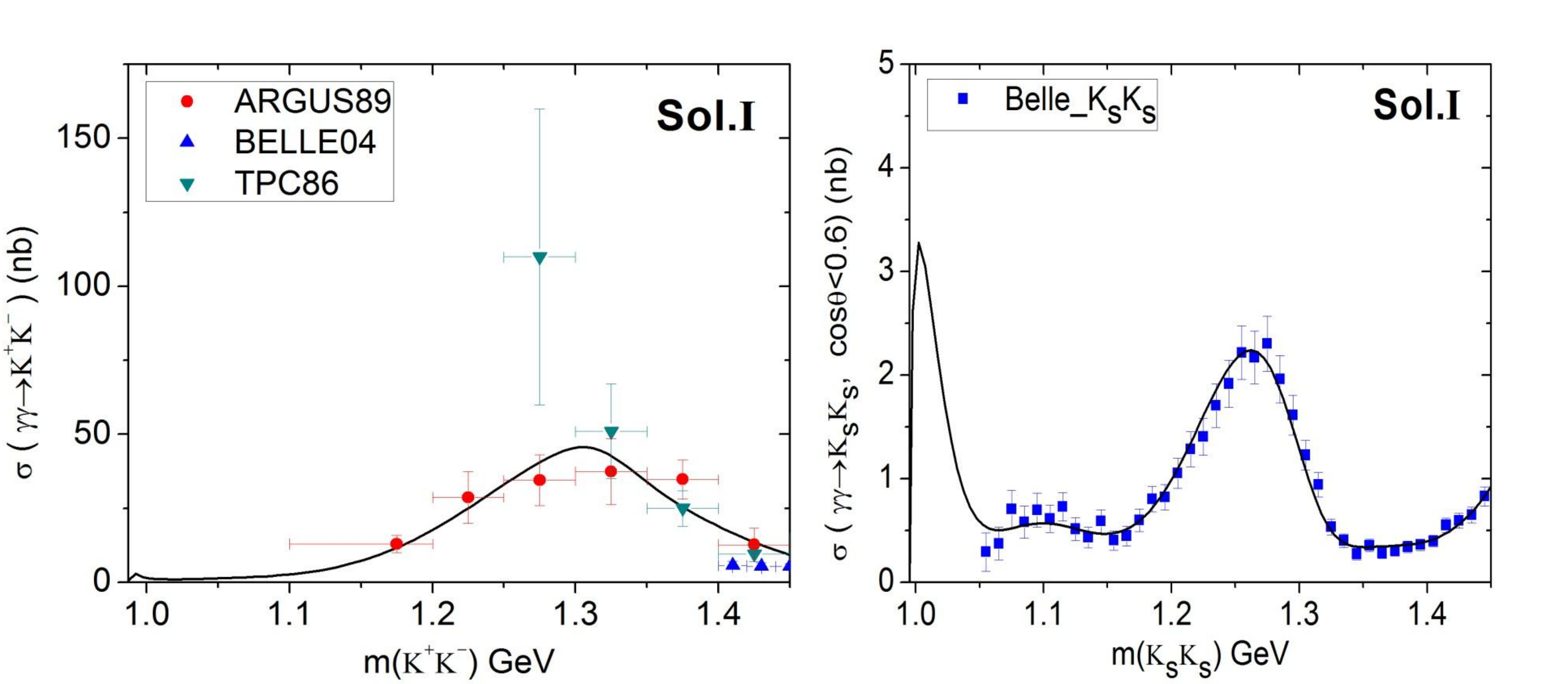


$\gamma\gamma \to \pi^0\pi^0$ angular distribution

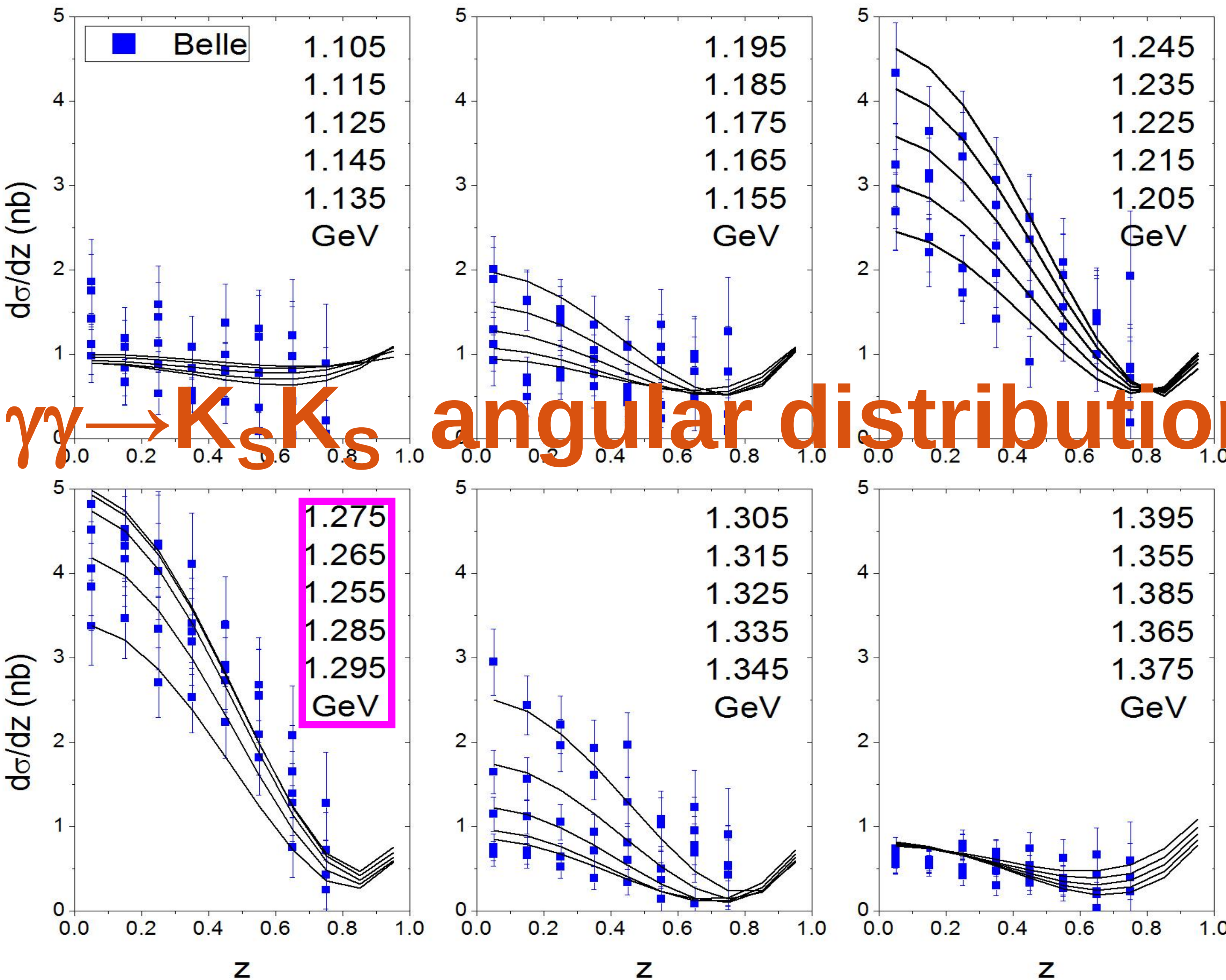


$\gamma\gamma\to$KK integrated cross section

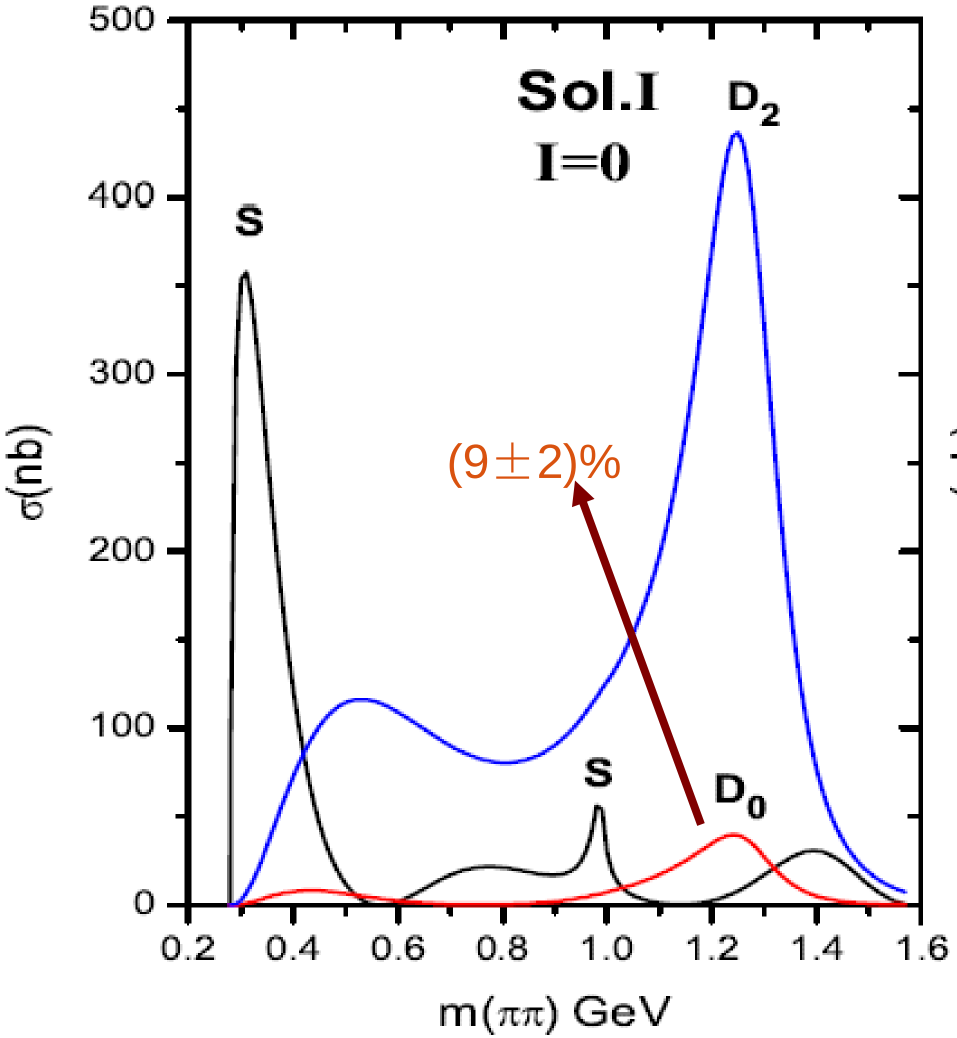
- If only fit to $\gamma\gamma\to\pi\pi$, we will get a region of solutions. $\gamma\gamma\to K\bar{K}$ data is helpful to select solutions.
- The latest $K_S K_S$ data of Belle make the accurate coupled channel analysis possible. Especially the angular distribution.

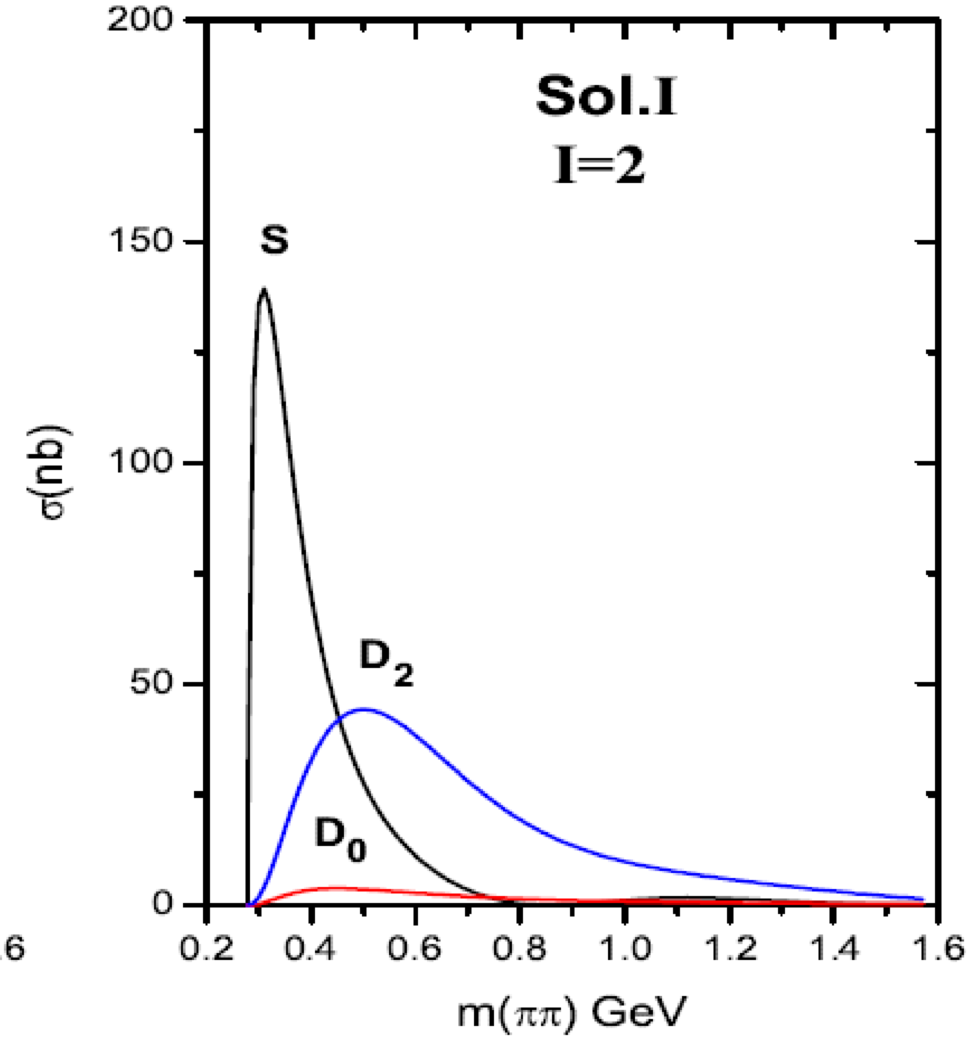


$\gamma\gamma \to K_S K_S$ angular distribution

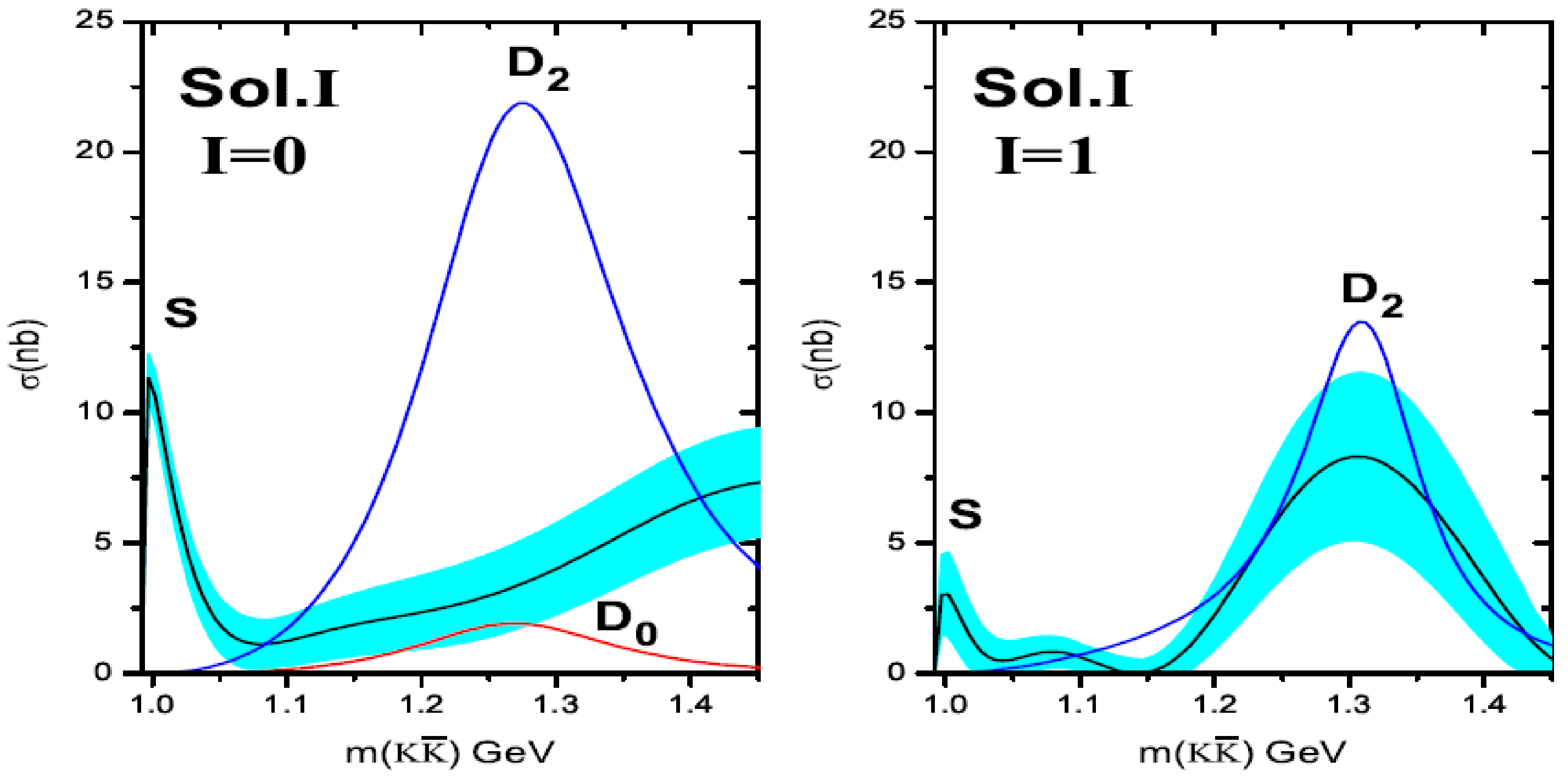


$\gamma\gamma\to\pi\pi$ individual partial waves





γγ→KK individual partial waves, I=1



4.Constraints to light-by-light sumrule

- For LbL one needs photons with virtualities from threshold up to around 2 GeV^2. Our massless photon amplitudes are boundary values when $Q^2 = 0$.
- Narrow resonance estimates from the tensor mesons are not enough.
- We can test the simplest Pascalutsa-Vanderhaeghen sumrule.

$$0 = \int_0^\infty ds \, \frac{\Delta\sigma(s)}{s} ,$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_0^\infty ds \, \frac{\sigma_{||}(s) \pm \sigma_{\perp}(s)}{s^2} .$$

$\sigma_2(s) - \sigma_0(s)$

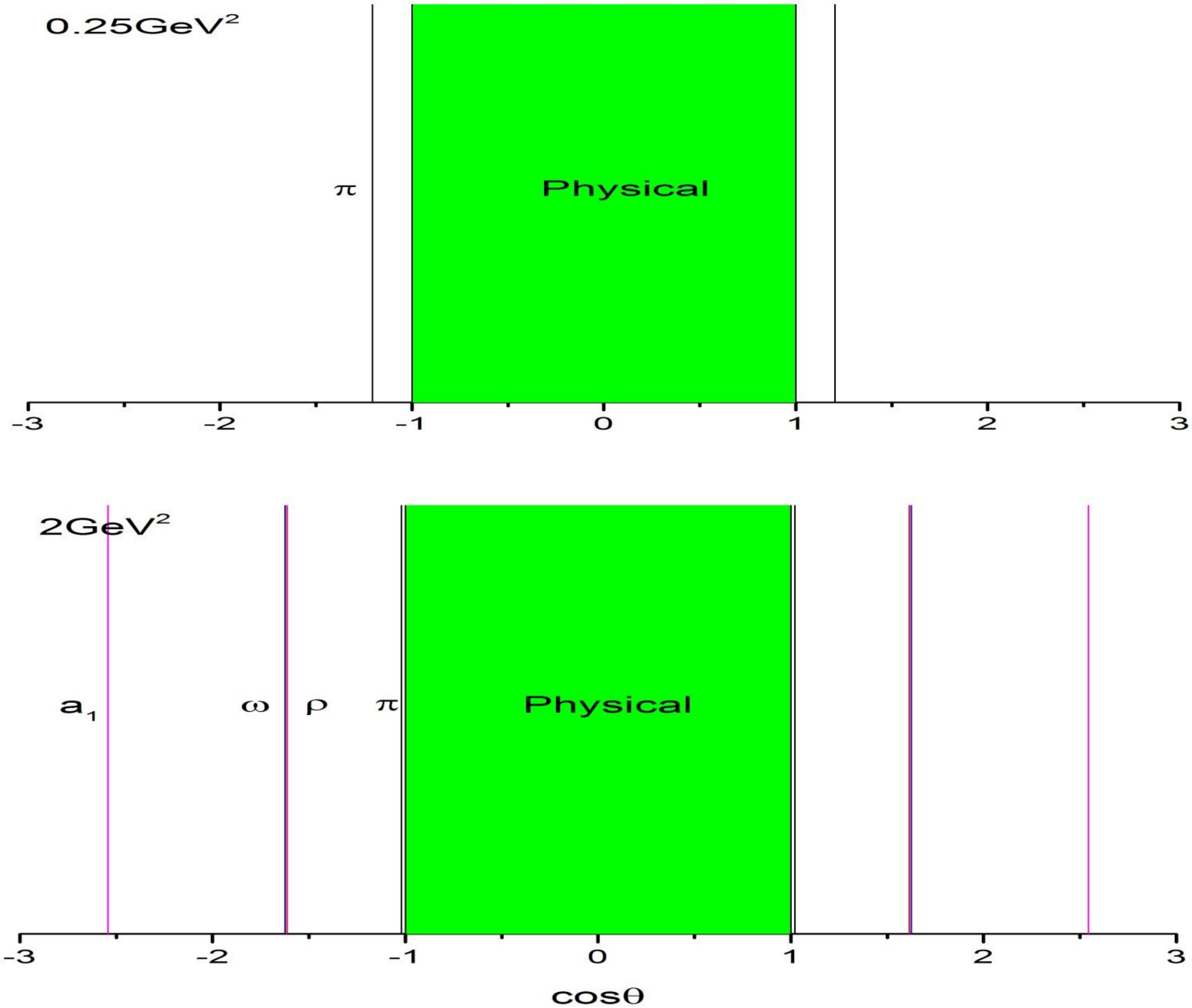
V.Pascalutsa & M.Vanderhaeghen, PRL105 (2010) 201603.

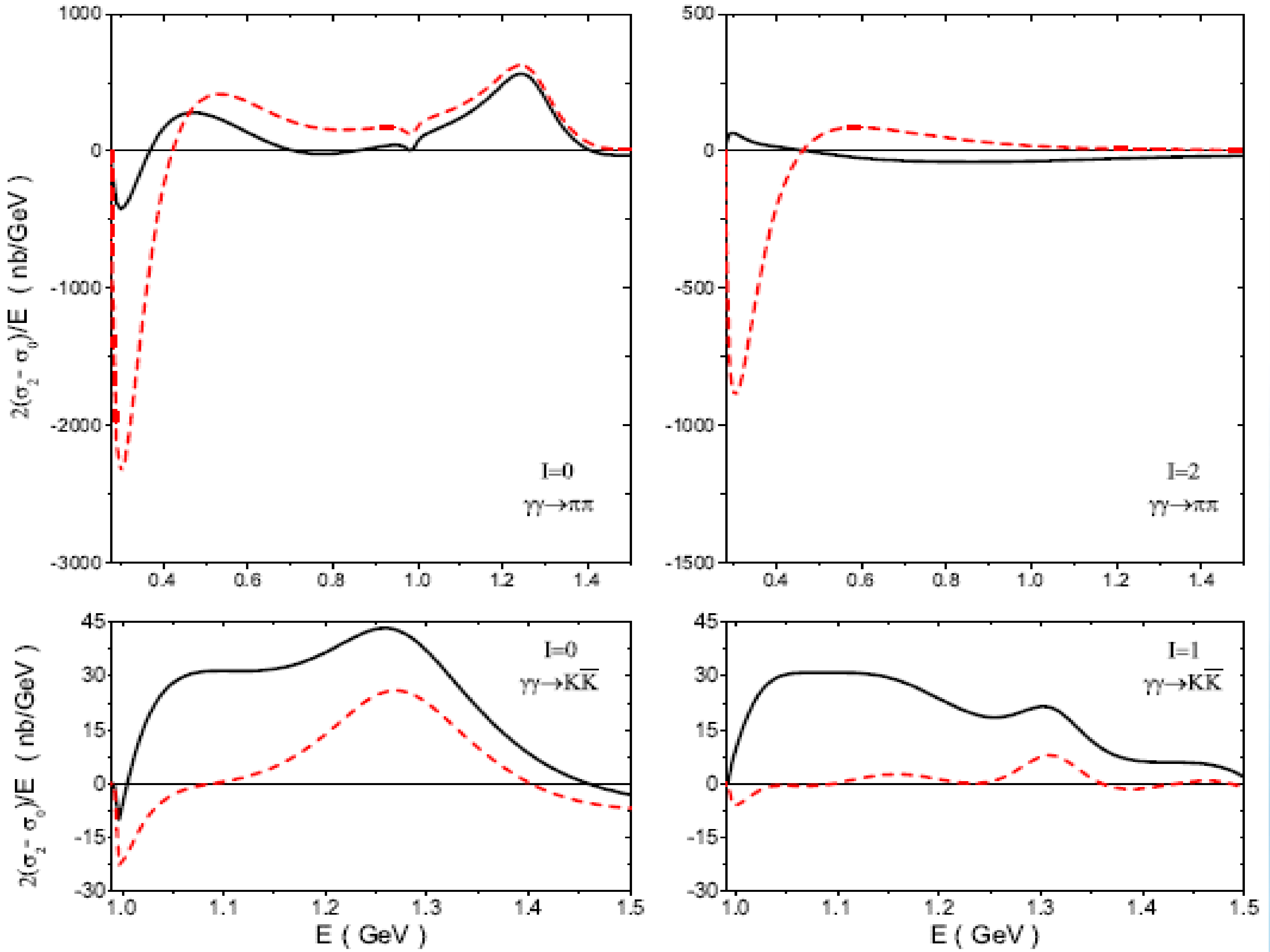
Constraints to light-by-light sumrule

- For $\gamma\gamma\rightarrow\pi\pi$ amplitudes above 2 GeV^2 we use Born terms to estimate, the uncertainty is within 10%.
- For $\gamma\gamma\rightarrow KK$ amplitudes the uncertainty is within 25%. But they have much less contribution to the PV sumrule.
- The Born term itself satisfies PV sumrule, so higher partial waves do not contribute. Finally one has:

$$\overline{\Delta}\sigma^I(s) = \sigma^I_{D2}(s) - \sigma^I_S(s) - \sigma^I_{D0}(s) - \left[\sigma^I_{D2}(s) - \sigma^I_S(s) - \sigma^I_{D0}(s)\right]_{Born}$$

Constraints to light-by-light sumrule





$2(\sigma_2-\sigma_0)/E$ (nb/GeV)
I=0
γγ→ππ
I=2
γγ→ππ
I=0
γγ→K$\bar{K}$
I=1
γγ→K$\bar{K}$
E (GeV)
E (GeV)

Constraints to light-by-light sumrule

- The contribution to PV sumrule is certainly not zero.

evaluation of $\Delta^I(4m_\pi^2, \infty, Z=1)$	$I=0$	$I=1$	$I=2$
$\gamma\gamma \to \pi^0$ [6] (nb)	-	-190.9±4.0	-
$\gamma\gamma \to \eta, \eta'$ [6] (nb)	-497.7±19.3	-	-
$\gamma\gamma \to a_2(1320)$ [6] (nb)	-	*135.0±12±25* †	-
$\gamma\gamma \to \pi\pi$ (nb)	308.0±41.5	-	-44.2±6.1
$\gamma\gamma \to \overline{K}K$ (nb)	23.7±7.5	18.1±4.9	-
SUM (nb)	-166.0±46.4	-37.8±28.4	-44.2±6.1

Constraints to light-by-light sumrule

- 4π channel's contribution is roughly of 150–200 nb in the I = 0 mode and 50 nb in the I = 2 mode.
- We have no decomposition information about the amplitudes of multi-particles' channel.

$$\mathcal{R}(s_1, s_2; \text{channel}) = \frac{\Delta(s_1, s_2, Z = 1; \text{channel})}{\Sigma(s_1, s_2, Z_{\text{exp}}; \text{channel})}$$

contribution to PV sumrule

total cross section

Channel	Publication	E_1 (GeV)	E_2 (GeV)	Σ (nb)	$\mathcal{R}(Born)$
$\pi^+\pi^-$ $(Z = 0.6)$	[16]	2.4	4.1	0.44 ± 0.01	1.61
K^+K^- $(Z = 0.6)$	[16]	2.4	4.1	0.39 ± 0.01	1.29
$\pi^0\pi^0$ $(Z = 0.8)$	[17]	1.44	3.3	8.8 ± 0.2	1.18
$\pi^0\pi^0\pi^0$	[18]	1.525	2.425	5.8 ± 0.8	1.55
$\pi^+\pi^-\pi^0$ (non-res.)	[19]	0.8	2.1	23.0 ± 1.3	1.39
$K_sK^\pm\pi^\mp$	[20]	1.4	4.2	9.7 ± 1.6	
$\pi^+\pi^-\pi^+\pi^-$	[21]	1.1	2.5	$215 \pm 11 \pm 21$	1.49
$\pi^+\pi^-\pi^+\pi^-$	[22]	1.0	3.2	$153 \pm 5 \pm 39$	1.48
$\pi^+\pi^-\pi^0\pi^0$	[23]	0.8	3.4	$103 \pm 4 \pm 14$	1.42

Pascalutsa-Vanderhaeghen light-by-light sumrule

- 4π is likely the largest contribution to be added below 2.5 GeV to make the PV sumrule for both I=0,2 zero.
- Experiments on 4π production would be rather helpful, for example $\rho^+\rho^-$, $\rho^0\rho^0$ production from two untagged photon.

Local hidden gauge approach+FSI?

BESIII(BEPCII)? Belle(KEKB)?

Pion polarizabilities

Polarizabilities may play important role on LbL sumrule

K.T.Engel et.al.
PRD86 (2012)
037502

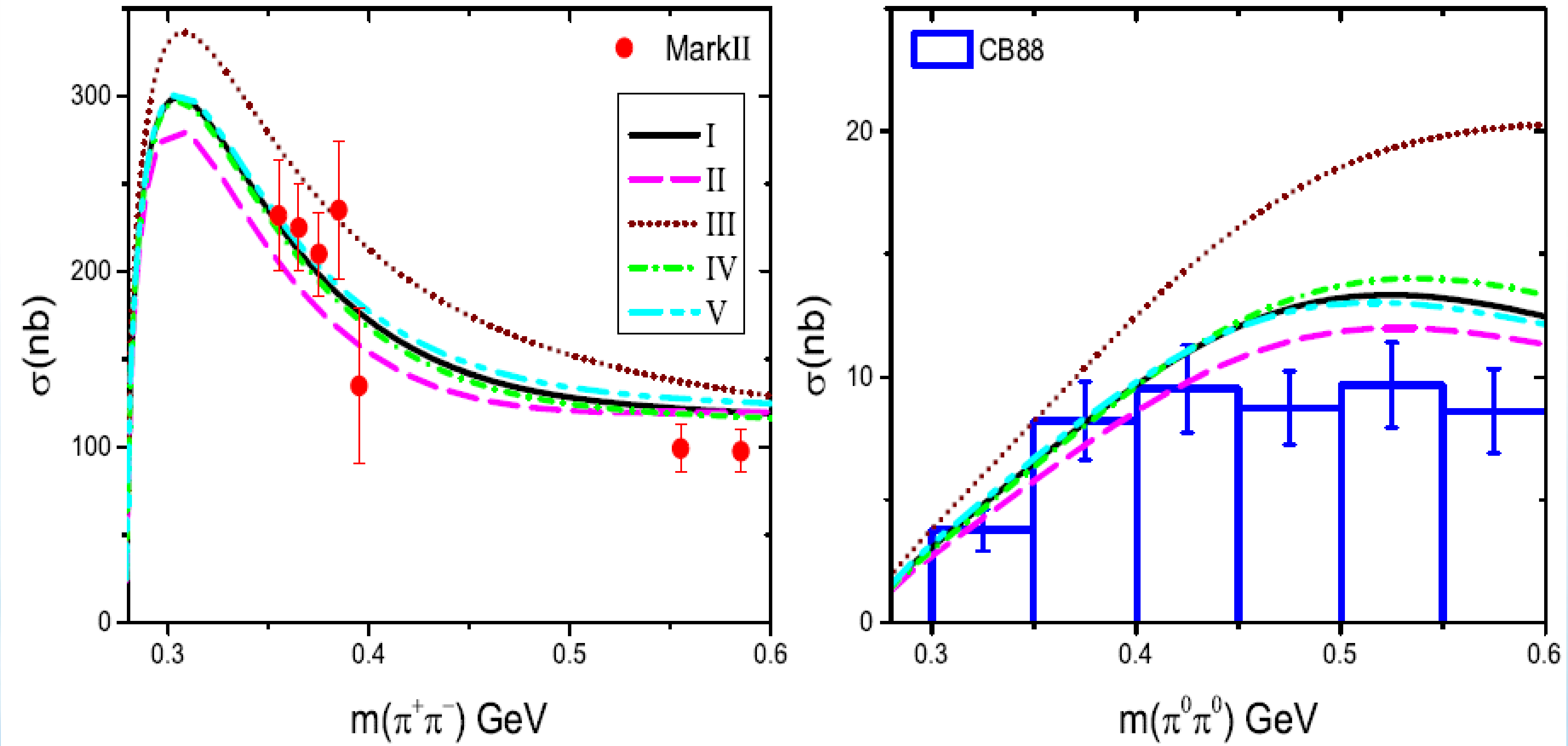
Amplitudes

polarizability: by extrapolation at s=0

?

cross section

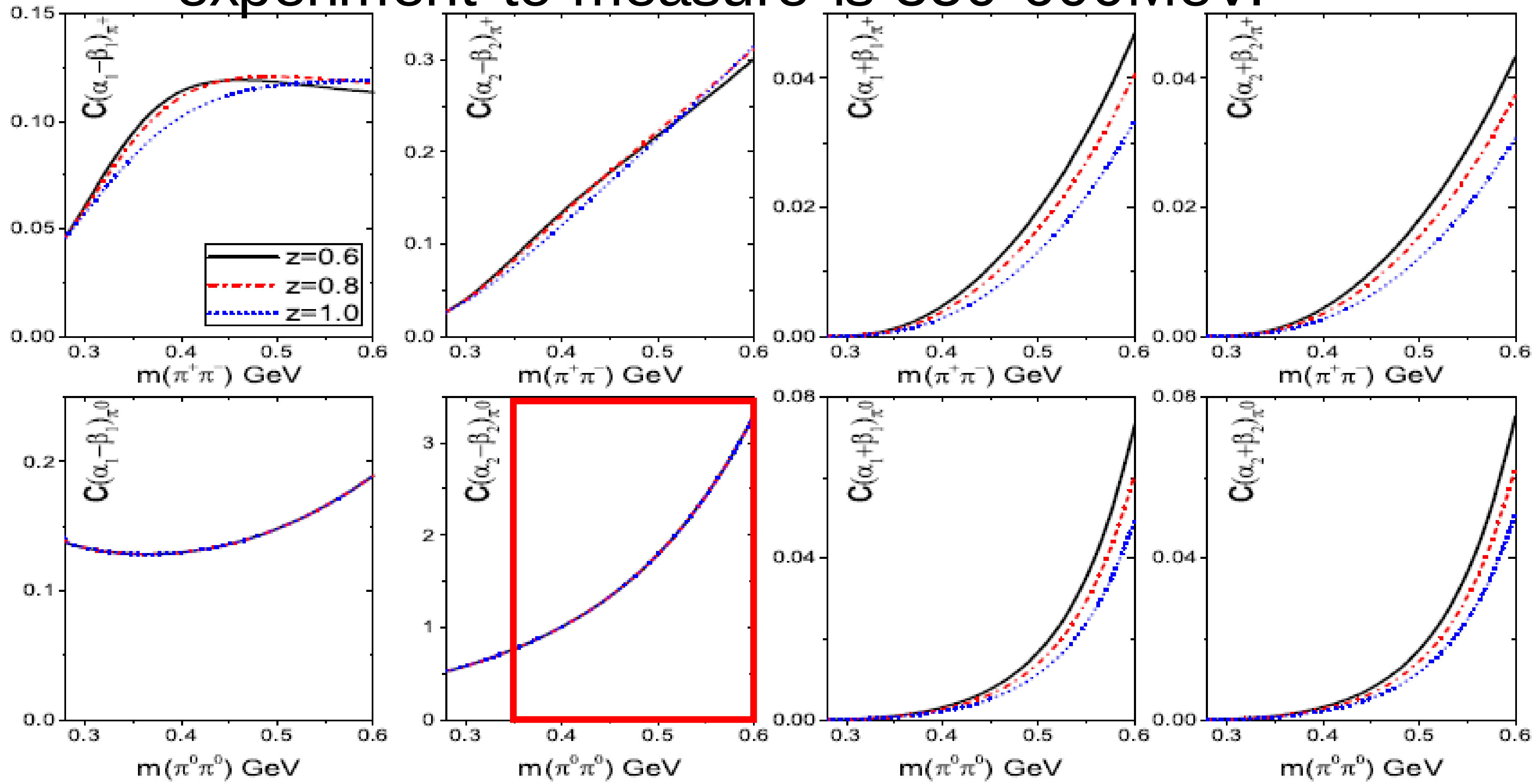
Polarizabilities



Model III, $(\alpha_1-\beta_1)_{\pi+}$ = 11.6, has been exclude by CB's data

Correlation functions

- The correlations between pion polarizabilities and $\gamma\gamma\to\pi\pi$ cross sections: the best region for experiment to measure is 350-600MeV.



Polarizabilities

Polarizabilities may play important role on LbL sumrule

Polarizabilities $\lambda = 0$	Model I	Model II	Model III	Model IV	Model V	ChPT + Resonance Model
$(\alpha_1 - \beta_1)_{\pi^+}$	**4.0 ± 1.2 ± 1.4**	**0.0**	**11.6**	**4.0**	**4.0**	5.7±1.0
$(\alpha_2 - \beta_2)_{\pi^+}$	15.7±1.1	13.0±1.1	20.9±1.1	13.2±3.4	18.1±2.5	16.2[21.6]
$(\alpha_1 - \beta_1)_{\pi^0}$	-0.9±0.2	-0.8±0.1	-1.1±0.2	-0.8±0.2	-1.0±0.2	-1.9±0.2
$(\alpha_2 - \beta_2)_{\pi^0}$	20.6±0.8	17.8±0.8	26.0±0.8	18.6±2.4	22.4±1.8	37.6±3.3
$\lambda = 2$						
$(\alpha_1 + \beta_1)_{\pi^+}$	0.26±0.07	0.26±0.07	0.26±0.07	0.17±0.51	0.42±0.22	0.16[0.16]
$(\alpha_2 + \beta_2)_{\pi^+}$	-1.4±0.5	-1.4±0.5	-1.4±0.5	-0.9±3.5	-2.4±1.5	-0.001
$(\alpha_1 + \beta_1)_{\pi^0}$	0.60±0.06	0.60±0.06	0.60±0.06	-0.04±0.52	0.90±0.17	1.1±3.3
$(\alpha_2 + \beta_2)_{\pi^0}$	-3.7±0.4	-3.7±0.4	-3.7±0.4	0.4±3.4	-5.5±1.1	0.04

fixed by Adler zero and $(\alpha_1-\beta_1)_{\pi+} = 4.0$

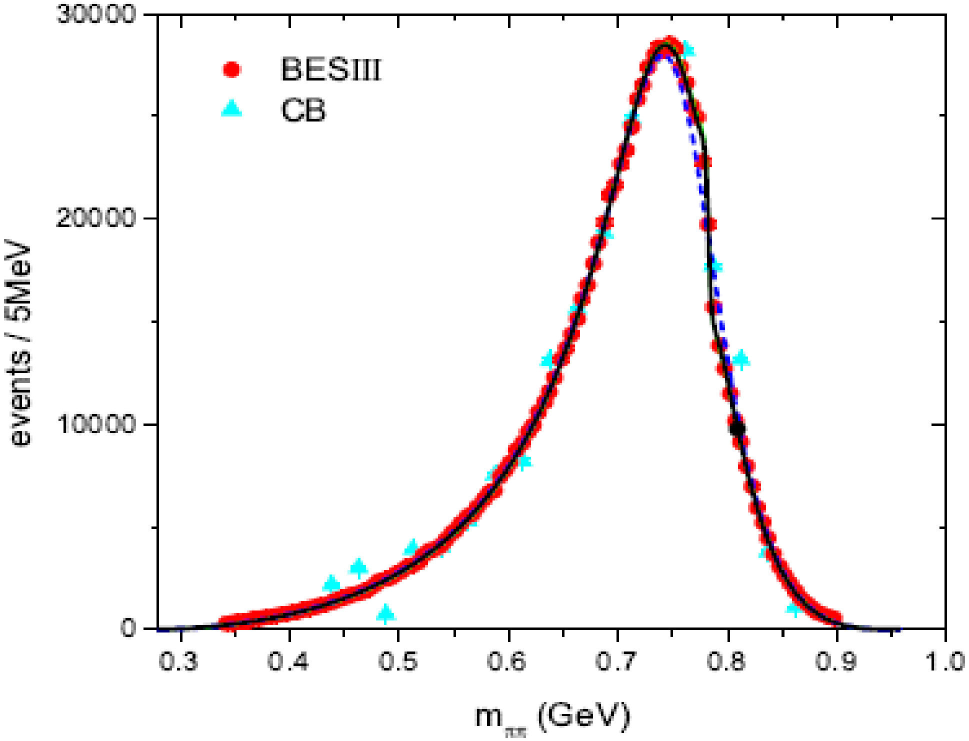
easiest one to be measured by experiment

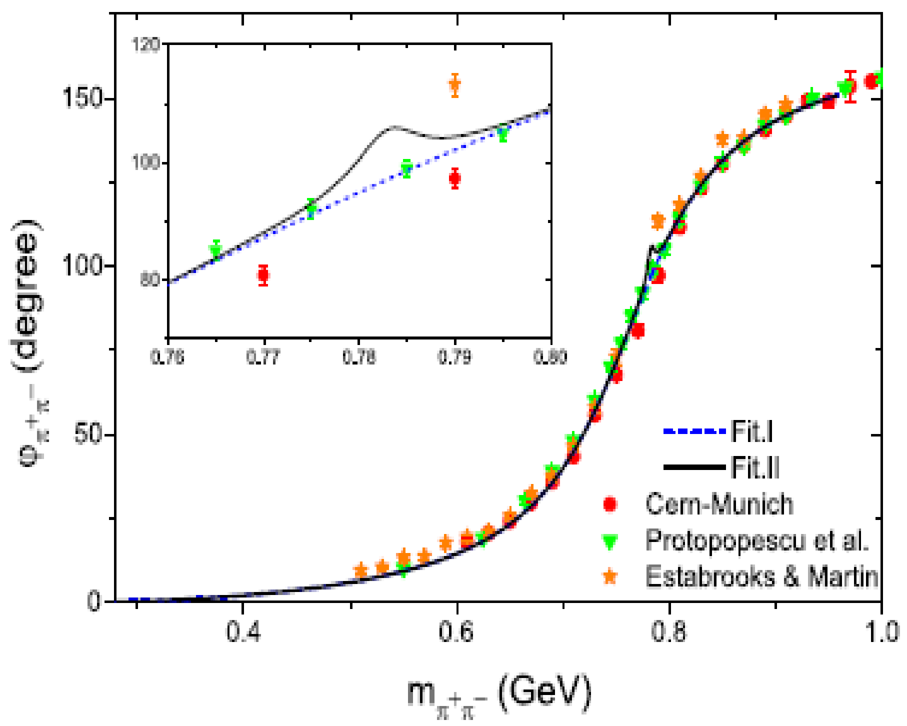
Virtual photon?

- Belle has new data on $\gamma^*\gamma \rightarrow \pi^0\pi^0$, BESIII also has plans to measure single-tagged photon.
- We focus on the simplest case, $\eta' \rightarrow \pi\pi\gamma$, through intermediate states $\pi\pi$, one obtains the form factor of $\eta' \rightarrow \gamma^*\gamma$.
- BESIII just published their high statistics data on $\eta' \rightarrow \pi\pi\gamma$.

Amplitude analysis

- Events distribution
- $\pi^+\pi^-$ P-wave phase-shift should take into consideration of isospin violation → $\gamma^*\gamma \rightarrow \pi^+\pi^-$





- With the anomalous decay amplitudes, one can extract the information of resonances.
- This amplitude can be used to constrain transition form factor of η’.
- It can also be used in the estimation of the PV sum rule, with one virtual photon.

6. Summary

Amplitudes

Including all new datasets and analyticity, unitarity, crossing symmetry, we perfom an amplitude analysis on photon photon collision.

LbL constraint

Our individual amplitudes are the boundary of LbL amplitudes when virtual photons are changed into real photons.

PV sumrule

We test PV sum rules for real photon case. 4π is likely the largest contribution to be added below 2.5 GeV.

polarizability

We predict pion polarizabilities. They may also play an important role in LbL scattering.