Light-by-light scattering sum rule overview and pion loop contribution

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SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

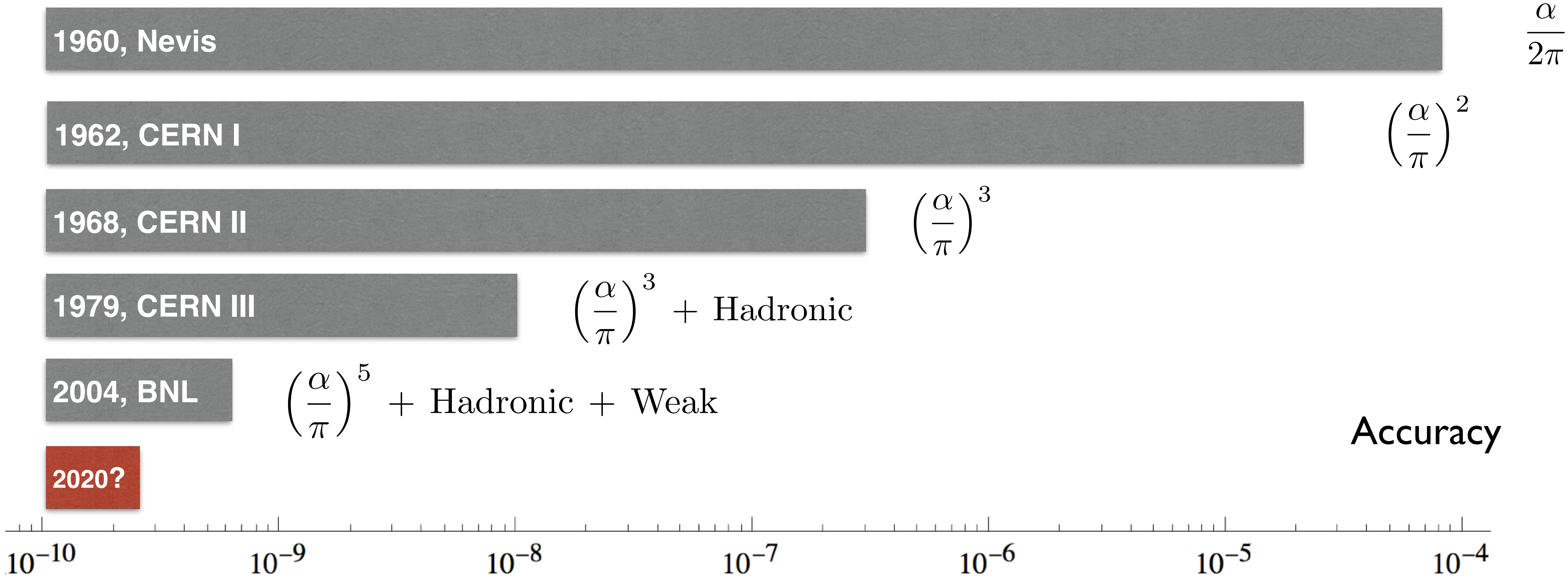
History of achieved accuracy

- Magnetic moment of the muon

$$\vec{\mu} = \frac{Q}{2m}\, g\, \vec{S}$$

- anomalous part

$$a_\mu = \frac{(g-2)_\mu}{2}$$



$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

$$a_\mu^{SM} = (11\,659\,182.8 \pm 4.9) \times 10^{-10}$$

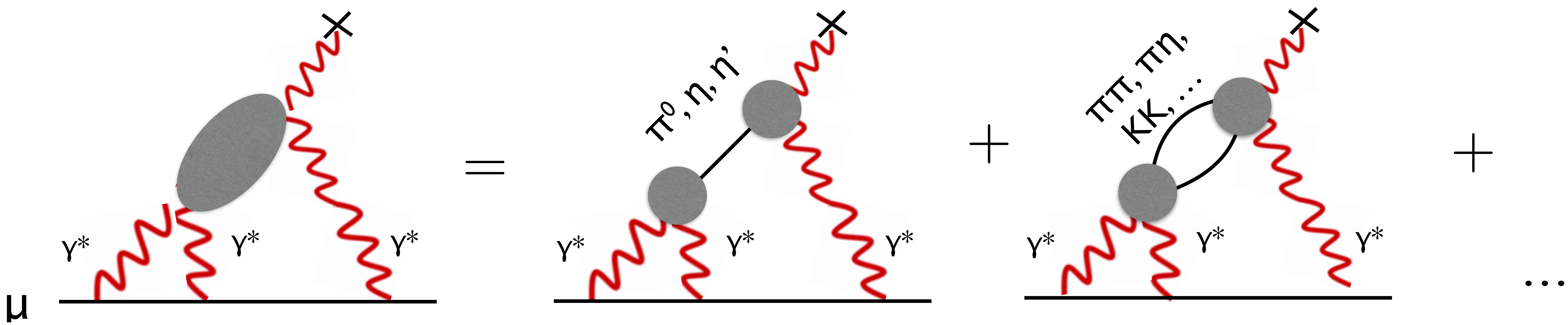
$$a_\mu^{exp} - a_\mu^{SM} = (26.1 \pm 4.9_{th} \pm 6.3_{exp}) \times 10^{-10}$$

3 - 4 σ deviation!

? 1.6_{exp}

FNAL, J-PARC

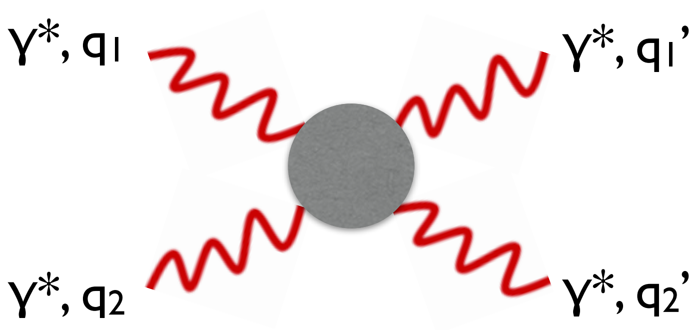
HLbL contributions to (g-2) in units 10^{-10}



Authors	π^0, η, η'	$\pi\pi$, KK	scalars	axial vectors	quark loops	Total
BPaP(96)	8.5(1.3)	-1.9(1.3)	-0.68(0.20)	0.25(0.10)	2.1(03)	8.3(3.2)
HKS(96)	8.3(0.6)	-0.5(0.8)	—	0.17(0.17)	1.0(1.1)	9.0(1.5)
KnN(02)	8.3(1.2)	—	—	—	—	8.0(4.0)
MV(04)	11.4(1.0)	—	—	2.2(0.5)	—	13.6(2.5)
PdRV(09)	11.4(1.3)	-1.9(1.9)	-0.7(0.7)	1.5(1.0)	0.23	**10.5(2.6)**
N/JN(09)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	2.2(0.5)	2.1(0.3)	11.6(3.9)
J(15)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	**0.75(0.27)**	2.1(0.3)	**10.2(3.9)**

B=Bjnens, Pa=Pallante, P=Prades, H=Hayakawa, K=Kinoshita, S=Sanda, Kn=Knecht, N=Nyffeler, M=Melnikov, V=Vainshtein, dR=de Rafael, J=Jegerlehner

Light by light scattering



$$\lambda_i = \pm 1, 0$$
$$q_i^2 = -Q_i^2$$

Helicity amplitudes

$$M_{\lambda_1' \lambda_2' \lambda_1 \lambda_2} = M^{\mu\nu\alpha\beta} \epsilon^*_\mu(\lambda_1')\, \epsilon^*_\nu(\lambda_2')\, \epsilon_\alpha(\lambda_1)\, \epsilon_\beta(\lambda_2)$$

Forward scattering $q_1 = q_1',\ q_2 = q_2'$

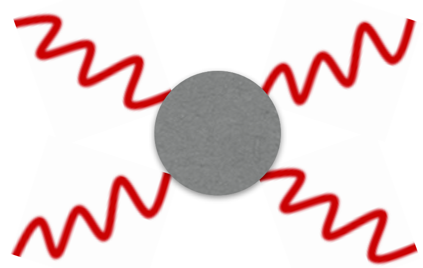
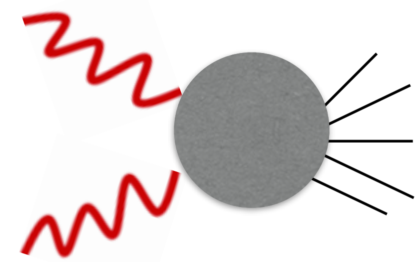
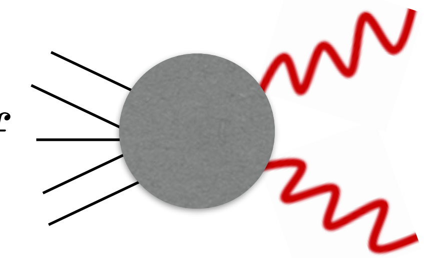
$$s = (q_1 + q_2)^2$$
$$t = (q_1 - q_1')^2 = 0$$

P and T symmetry: **81** → **8** independent amplitudes

$$M_{++,++},\ M_{+-,+-},\ M_{++,--}$$
$$M_{00,00},\ M_{+0,+0},\ M_{0+,0+}$$
$$M_{++,00},\ M_{0+,-0}$$

Light by light scattering

Unitarity

2 Im  $= \sum_f \int d\Pi_f$  f f 

For the forward scattering (optical theorem):

$$\mathrm{Im}\, M_{++,++} = 2\sqrt{X}\,\sigma_0$$
$$\mathrm{Im}\, M_{+-,+-} = 2\sqrt{X}\,\sigma_2$$
$$\mathrm{Im}\, M_{++,--} = 2\sqrt{X}\,(\sigma_{\parallel} - \sigma_{\perp})$$
$$\ldots$$

X - flux factor

Observables in: $e^+e^- \to e^+e^- f$

Analyticity (fixed *t* dispersion relation)

$$M_{++,--}(\nu) = \int_{\nu_0}^{\infty} \frac{d\nu'}{\pi} \frac{2\,\nu'\,\mathrm{Im}M_{++,--}(\nu')}{\nu'^2 - \nu^2 + i\,0}\,, \qquad \nu = \frac{s-u}{4}$$

...

(modulo subtractions)

Matching around $\nu = 0$ to the LbL Lagrangian

Euler-Heisenberg (1936)

$$\mathcal{L} = c_1 (F_{\mu\nu}F^{\mu\nu})^2 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \ldots$$

yield a number of **constraints on cross sections**

Light by light sum rules

Three super convergence relations

Gerasimov, Moulin (1975), Brodsky, Schmidt (1995)

Pascalutsa, Vanderhaeghen (2010)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_2 - \sigma_0\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^{a}\right]_{Q_2^2=0}$$

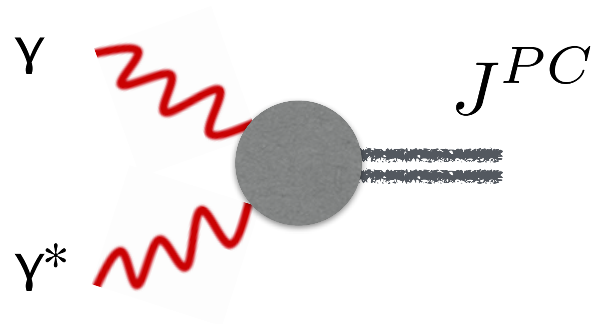
$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s, Q_1^2, Q_2^2)}{Q_1 Q_2}\right]_{Q_2^2=0}$$

Pascalutsa, Pauk Vanderhaeghen (2012), (2014)

These sum rules have been tested in perturbative QFT both at tree-level and one loop level:

- ✓ scalar QED
- ✓ spinor QED
- ✓ ϕ^4 theory
- ✓ ϕ^4 theory + resum.

Light by light sum rules: Meson production



C=+1: $J^{PC}=0^{-+}, 0^{++}, 1^{++}, 2^{++}, \ldots.$

$Q^2 \neq 0$

Landau-Yang theorem

Narrow width approximation

$$\sigma\left(\gamma^*\gamma \to J^P(\Lambda)\right) = \delta(s-m^2)\, 8\pi^2 \frac{(2J+1)\,\Gamma_{\gamma\gamma}\left(J^P\right)}{m} \left(1+\frac{Q^2}{m^2}\right)\left[\mathrm{T}^{(\Lambda)}\left(Q^2\right)\right]^2$$

Sum rules will relate 2γ width or TFFs:

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q^2)}\,[\sigma_2 - \sigma_0]$$

$Q^2 = 0$:

$$0 = -\sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3} - \sum_{\mathcal{S}} \ldots$$

$Q^2 \neq 0$:

$$0 = -\sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3}\,[T(Q^2)]^2 - \sum_{\mathcal{S}} \ldots$$

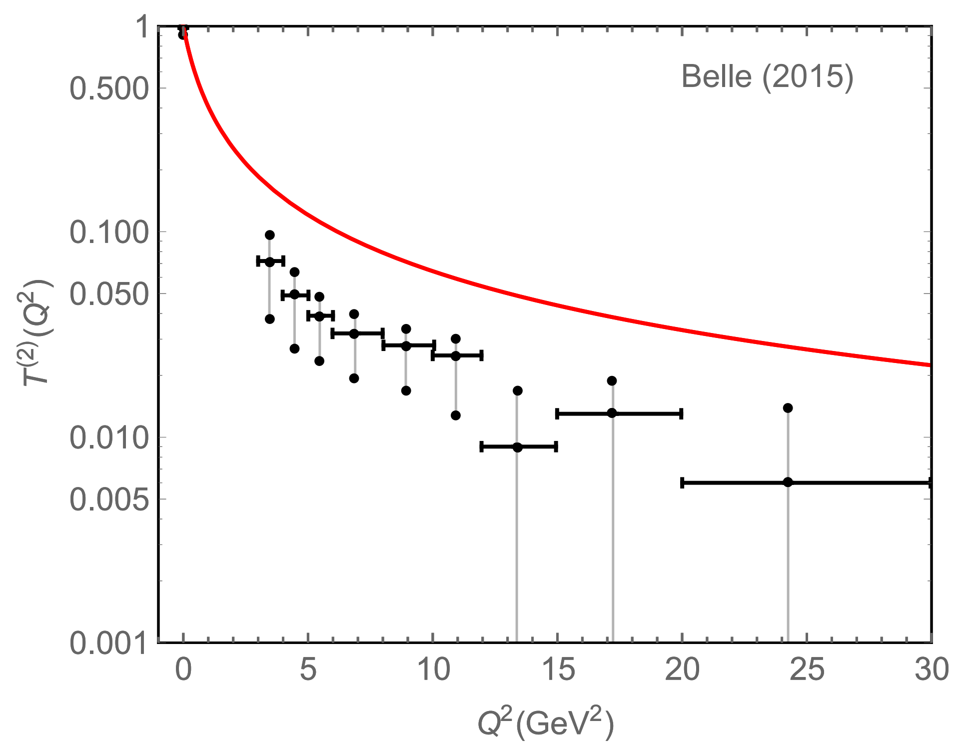
Sum rule I (isospin=0)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q^2)} \left[\sigma_2 - \sigma_0\right]$$

$$0 = -\sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3} \left[T_{\mathcal{P}}(Q^2)\right]^2 - \sum_{\mathcal{S},\mathcal{A}} \cdots + \sum_{\mathcal{T}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{T})}{m_{\mathcal{T}}^3} \left(\left[T_{\mathcal{T}}^{(\Lambda=2)}(Q^2)\right]^2 - \left[T_{\mathcal{T}}^{(\Lambda=0)}(Q^2)\right]^2 \right)$$

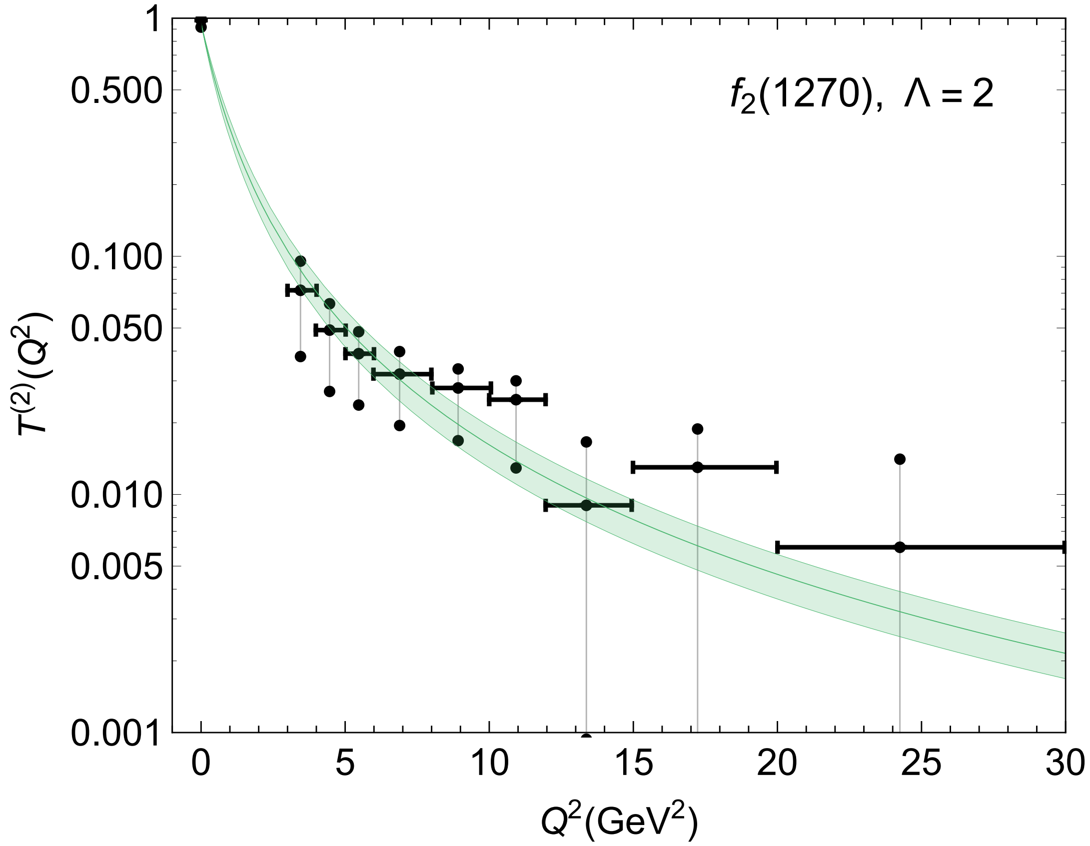
Dominant contributions

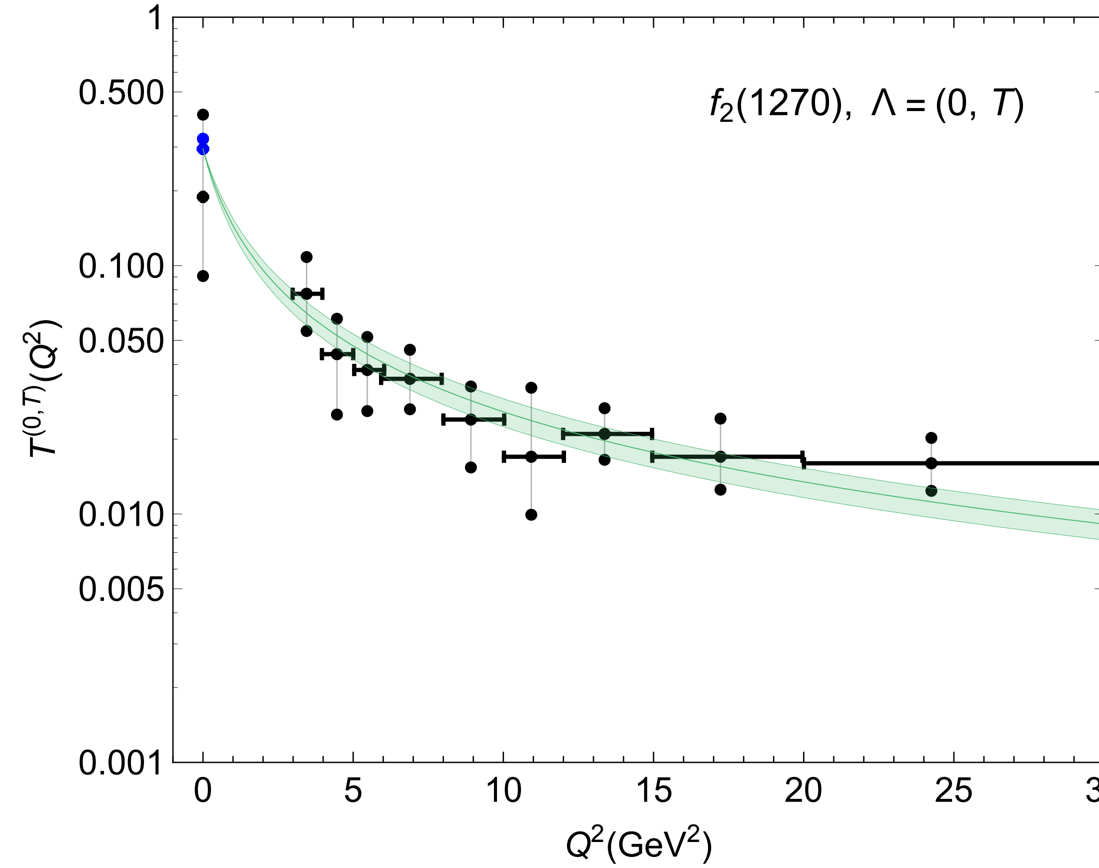
State	m (MeV)	$\Gamma_{\gamma\gamma}$(keV)		SR_1 $(Q^2 = 0)$ (nb)
η	547.862±0.017	0.516±0.020		-193±7
η'	957±0.06	4.35±0.25		-304±17
$f_2(1270)$	1275.5±0.8	2.93±0.40	(Λ=2)	434±60
			(Λ=0)	≈0
$f_2(1565)$	1562±13	0.70±0.14		56±11
.....				
sum				-7±64

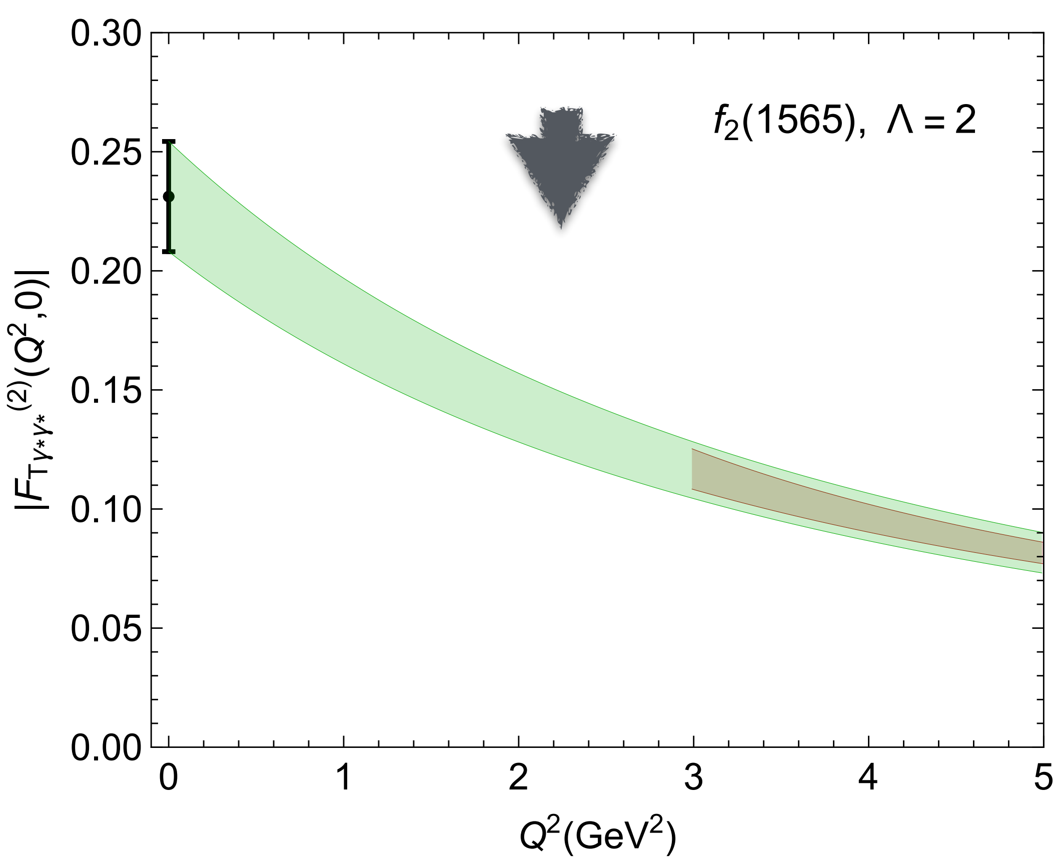


Pascalutsa, Pauk Vanderhaeghen (2012)

Belle (2015)







$$T^{(\Lambda)}(Q^2) \sim \frac{1}{\left(1 + Q^2/\lambda^2_{(\Lambda)}\right)^2}$$

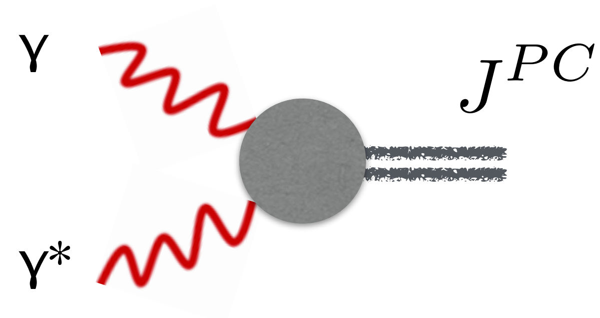
Prediction:

$$f_2(1565)$$
$$\lambda_{\Lambda=2} = 2719 \pm 53\,\mathrm{MeV}$$

I.D., Vanderhaeghen (2016)

future Belle data

Sum rules II and III



C=+1: $J^{PC}=0^{-+}, 0^{++}, 1^{++}, 2^{++}, \ldots$

$Q^2 \neq 0$

Landau-Yang theorem

$$0 = \int\limits_{s_0}^{\infty} ds \, \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\|} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^{a} \right]_{Q_2^2=0}$$

$$0 = \int\limits_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s, Q_1^2, Q_2^2)}{Q_1 Q_2} \right]_{Q_2^2=0}$$

Axial-vector mesons 1^{++} are **allowed** if one of the photons is virtual: interplay between $\mathcal{A}, \mathcal{T}$

Equivalent 2γ width: $\tilde{\Gamma}_{\gamma\gamma}(\mathcal{A}) \equiv \lim\limits_{Q_1^2 \to 0} \frac{m_{\mathcal{A}}^2}{Q_1^2} \frac{1}{2} \Gamma\left(\mathcal{A} \to \gamma_L^* \gamma_T\right)$

TFFs $\gamma^*\gamma \to f_1(1285), f_1(1420)$ were measured

L3 Collaboration (2002), (2007)

Sum rules II and III ($Q^2 \approx 0$)

	m [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int ds \left[\frac{1}{s^2}\sigma_\parallel + \frac{1}{s}\frac{\tau^a_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]	$\int_{s_0}^\infty ds \left[\frac{\tau_{\mathrm{TL}}(s,Q_1^2,Q_2^2)}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	-93 ± 21	$+153 \pm 35$
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	-50 ± 14	$+102 \pm 29$
$f_0(980)$	990 ± 20	0.31 ± 0.05	$+40 \pm 13$	$+19 \pm 10$
$f_2(1270)$	1275.5 ± 0.8	2.93 ± 0.40		
$\Lambda = 2$			$+122 \pm 17$	-
$\Lambda = (0, T)$			$+23 \pm 3$	-
$\Lambda = (0, L)$			??	??
$\Lambda = 1$			??	??
Sum				
$f_2(1565)$	1562 ± 13	0.70 ± 0.14		
$\Lambda = 2$			$+12 \pm 2$	-
Sum			≈ 0 (def.)	≈ 0 (def.)

Sum rules II and III ($Q^2 \approx 0$)

	m [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int ds \left[\frac{1}{s^2}\sigma_\parallel + \frac{1}{s}\frac{\tau^a_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]	$\int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s,Q_1^2,Q_2^2)}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb]
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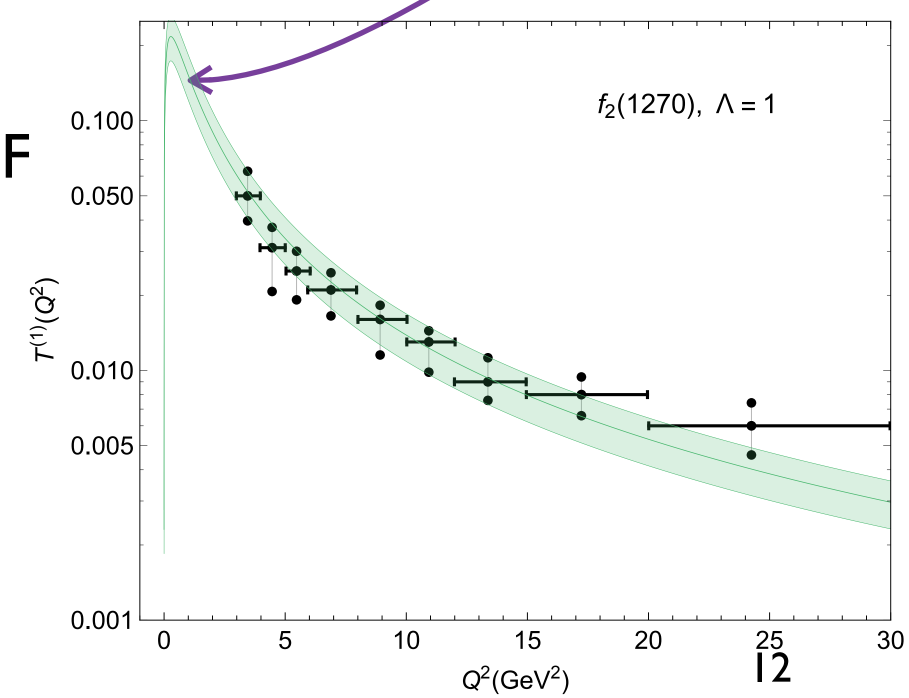
$\Lambda = 2$			$+122 \pm 17$	-
$\Lambda = (0, T)$			$+23 \pm 3$	-
$\Lambda = (0, L)$			-111 ± 15	-180 ± 43
$\Lambda = 1$			$+58 \pm 24$	-94 ± 40
Sum			$+92 \pm 26$	-274 ± 53
$f_2(1565)$	1562 ± 13	0.70 ± 0.14		
$\Lambda = 2$			$+12 \pm 2$	-
Sum			≈ 0 (def.)	≈ 0 (def.)

Constrain low Q^2 region of (Λ=1) TFF

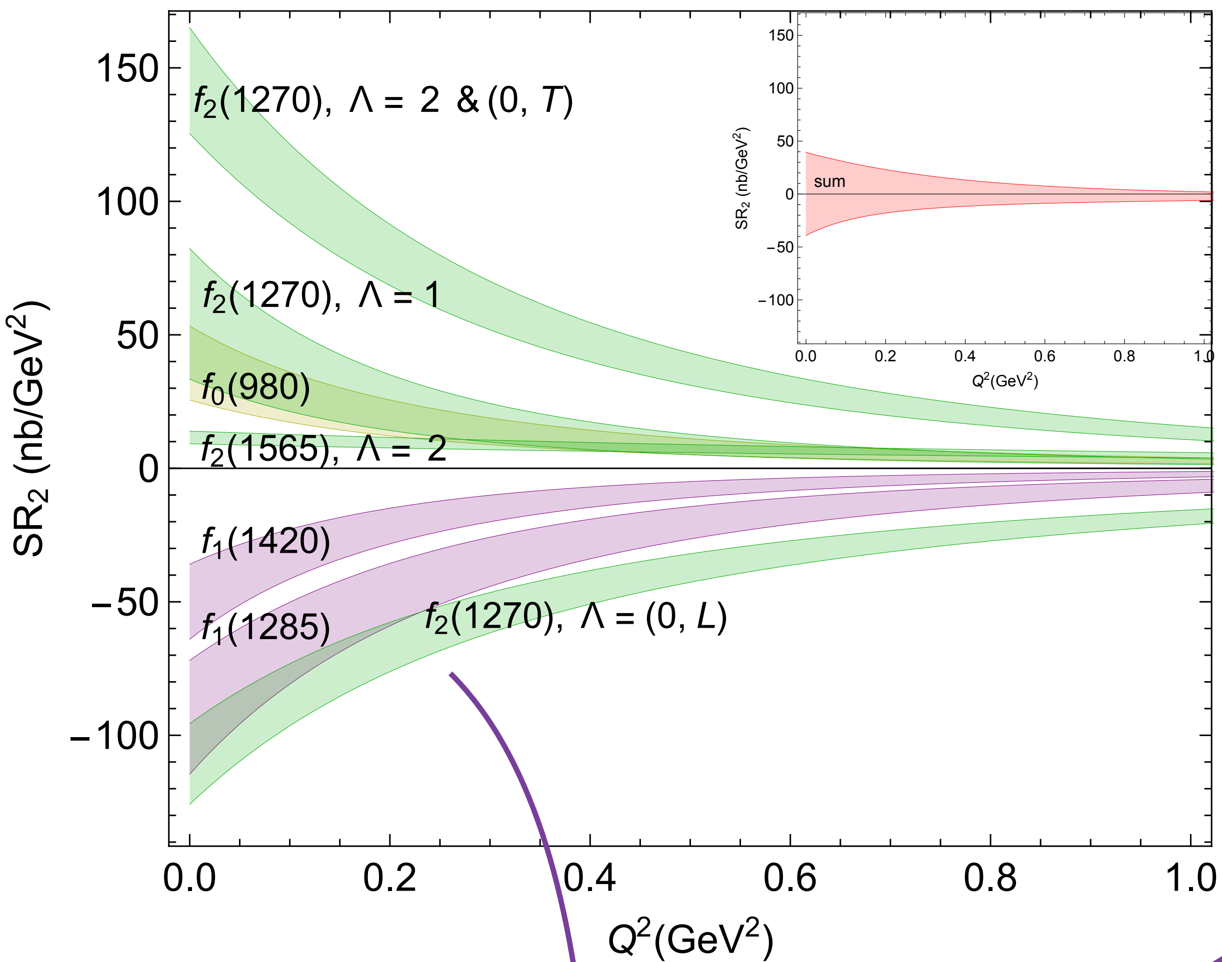
future BES III

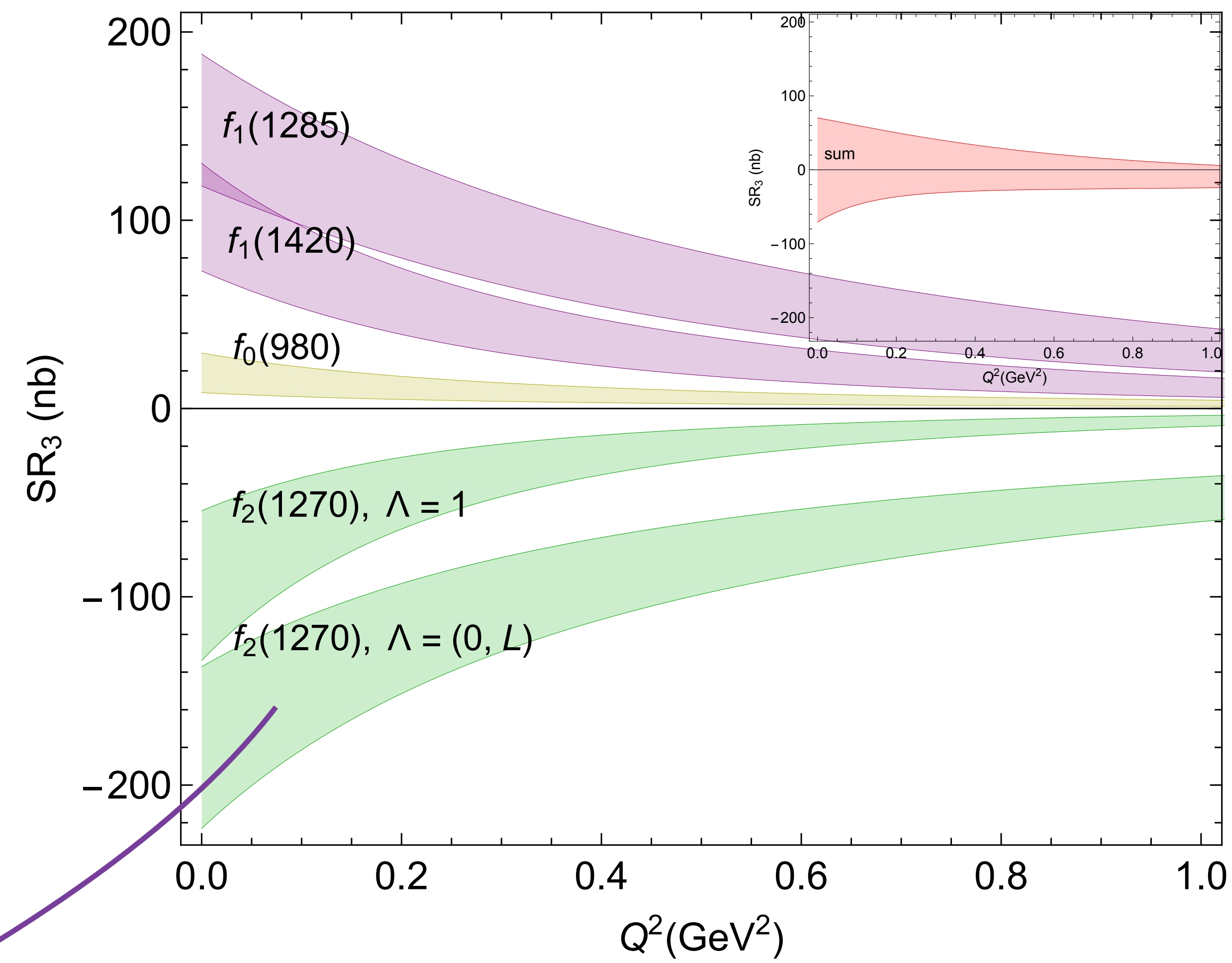


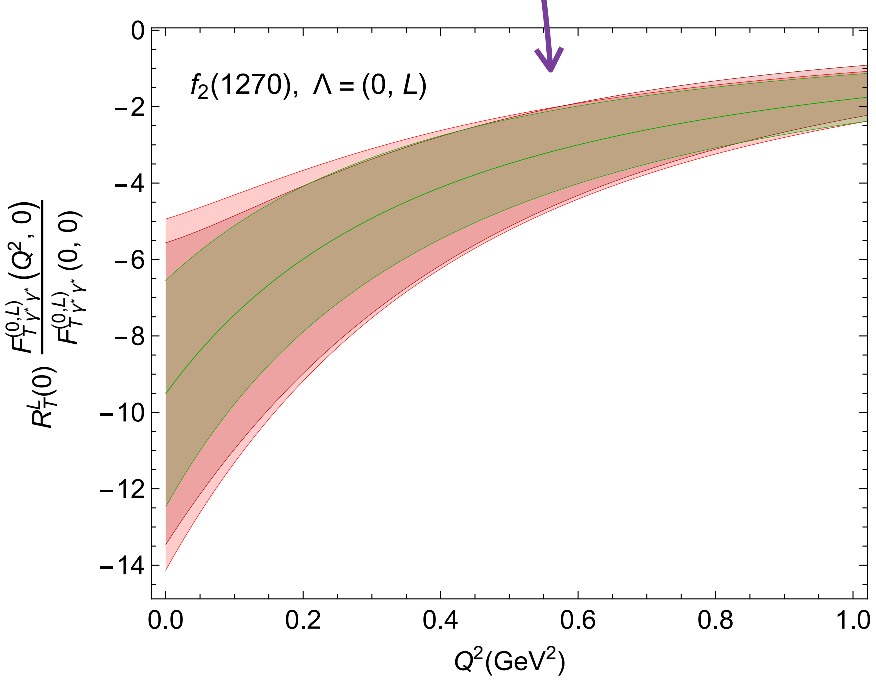
Constrain longitudinal coupling ratio for $Q^2 \to 0$

$$R^L_T(Q^2) \equiv \frac{T^{(0,L)}(Q^2)}{T^{(0,T)}(Q^2)}$$

Sum rules II and III (Q^2>0)







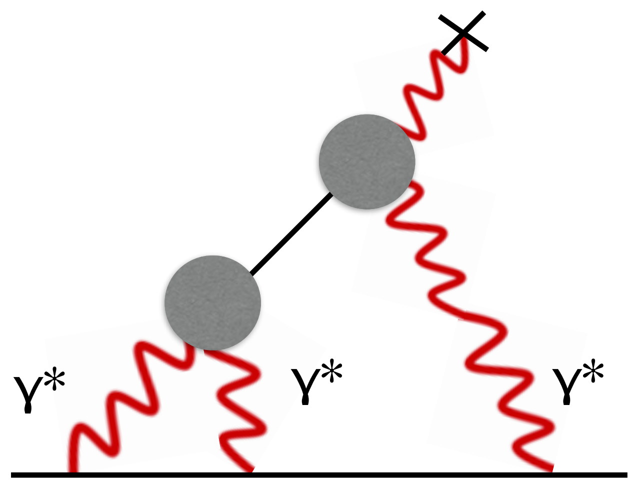
Prediction:

$$f_2(1270)$$

$$\lambda_{\Lambda=(0,L)} = 977 \pm 66\,\text{MeV}$$

Danilkin, Vanderhaeghen, (2016)

Meson contributions to (g-2)



Lepton tensor: well known

Hadron tensor: requires input from **TFFs**

$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)\, \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$
$$\frac{1}{q_1^2}\frac{1}{q_2^2}\frac{1}{(k-q_1-q_2)^2}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(p'-q_2)^2-m^2}$$

Results (excluding low energy region):

$$a_\mu[f_2(1270), f_2(1565)] = (0.1 \pm 0.01) \times 10^{-10}$$

New evaluation of axial vector contributions (satisfying Landau-Yang theorem)

$$a_\mu[f_1(1285), f_1(1420)] = (0.64 \pm 0.20) \times 10^{-10}$$
$$= (0.75 \pm 0.27) \times 10^{-10}$$

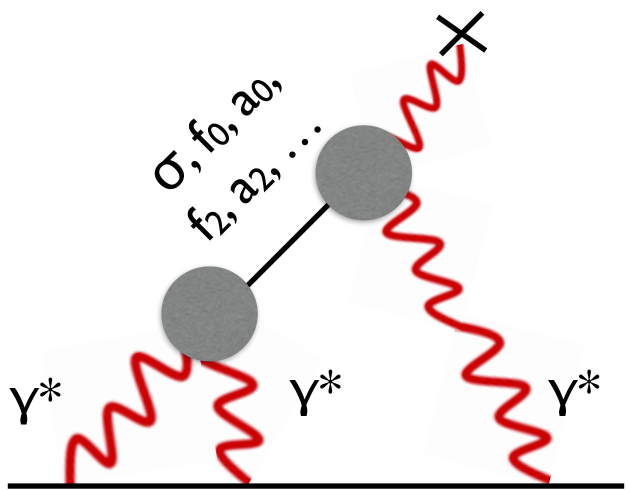
Pauk, Vdh (2013)
Jegerlehner (2015)

$$\delta a_\mu^{exp} = 1.6 \times 10^{-10}$$

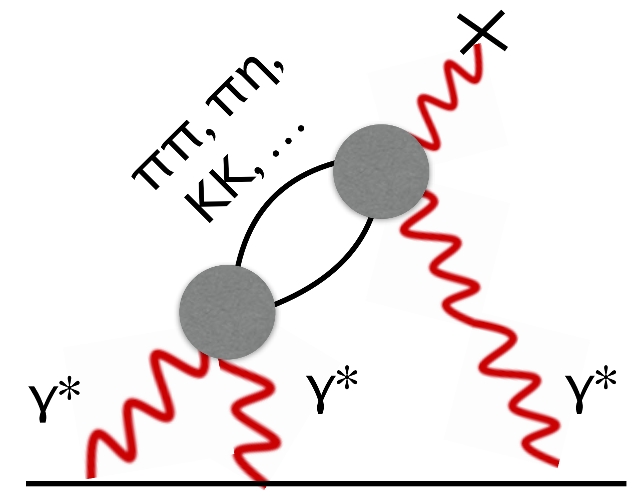
FNAL, J-PARC experiments

Improvements: Multi-meson production

Important contributions beyond **pseudo-scalar** poles



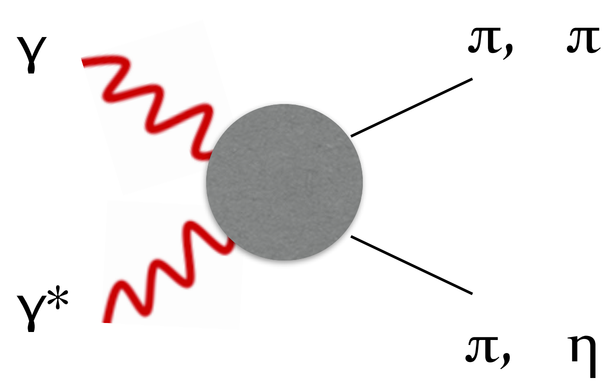
dispersive analysis for
ππ, πη, … loops



Pauk, Vanderhaeghen, (2014)

Colangelo, Hoferichter, Procura, Stoffer, (2017)

Important ingredient: γγ*→ππ, πη, …

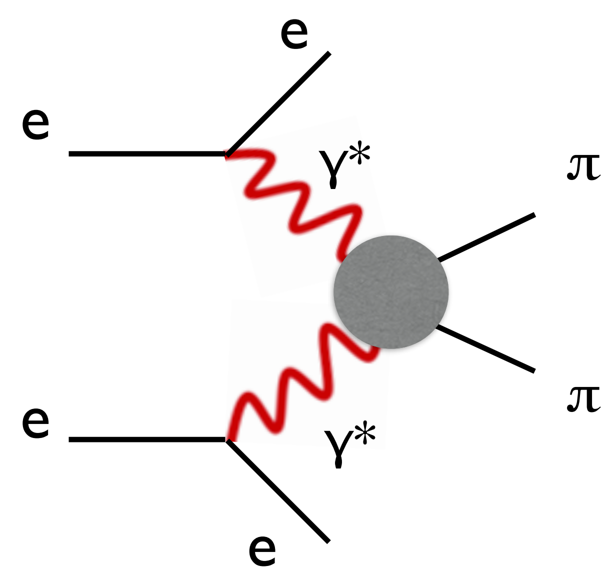


$$a_\mu^{\pi\text{box}} + a_{\mu,J=0}^{\pi\pi,\pi\text{-pole LHC}} = -2.4(0.1) \times 10^{-10}$$

γγ→ππ, KK, ηη, πη (Belle: 07,08, 09,10, ..)
γγ*→ππ, πη (BESIII in progress)

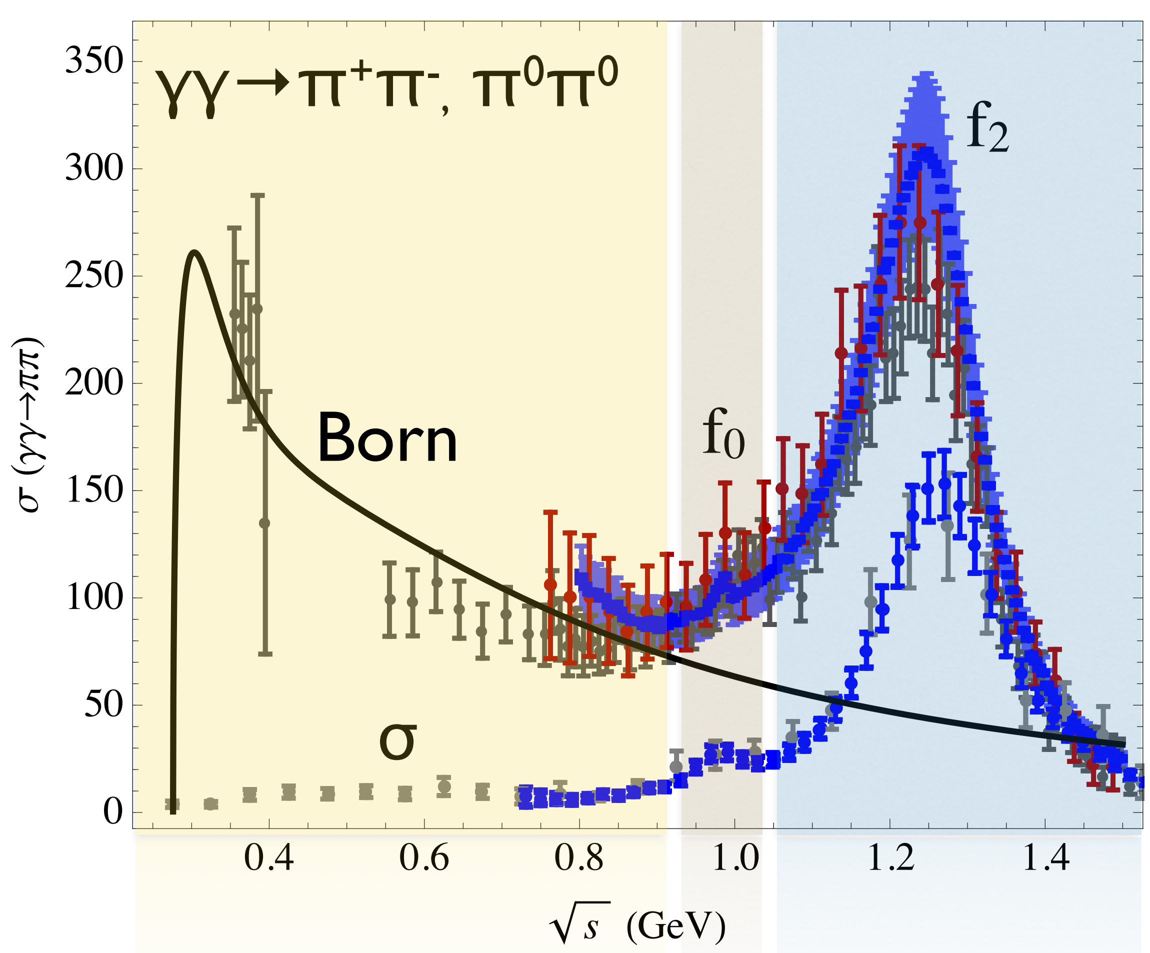
Experiment

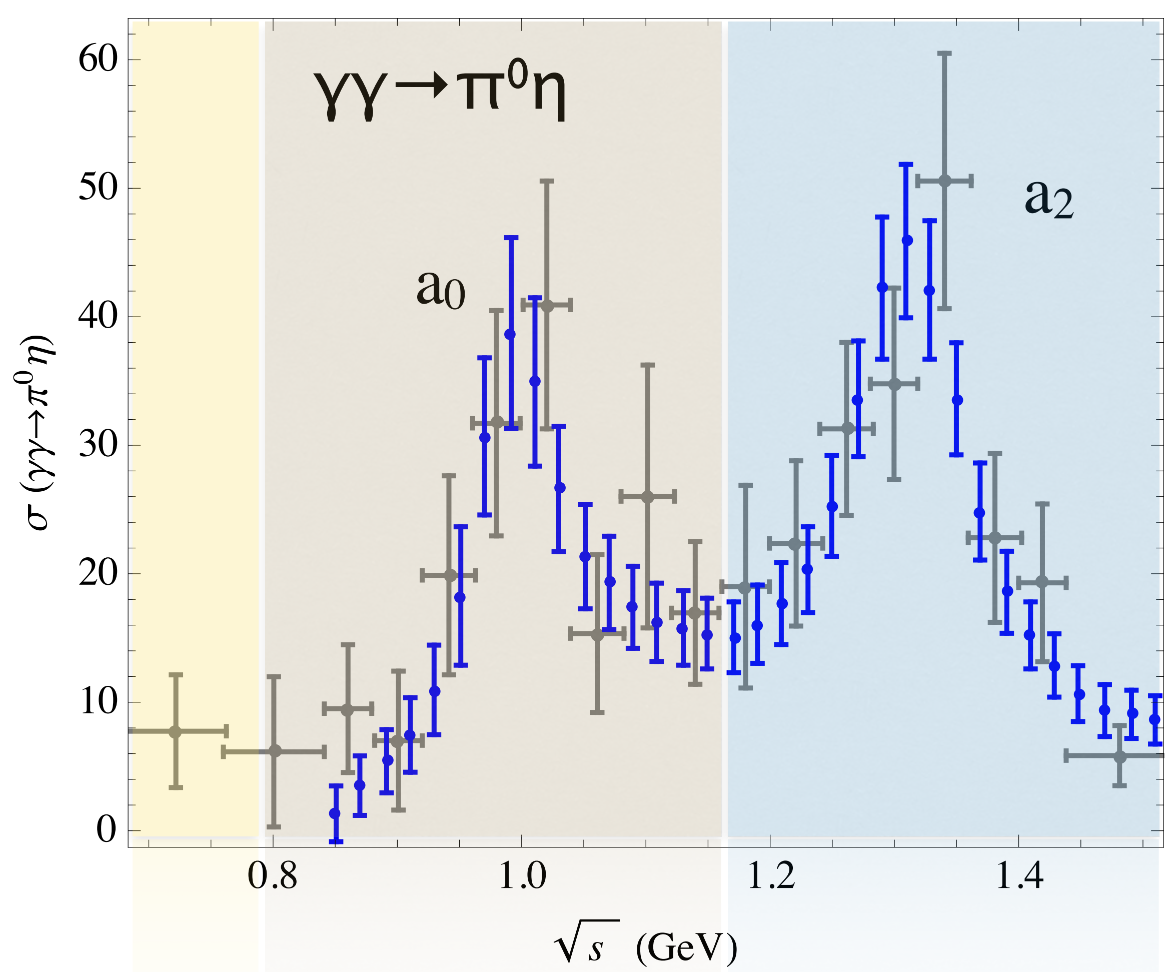
Observables in experiment $e^+e^- \to e^+e^-\pi\pi$



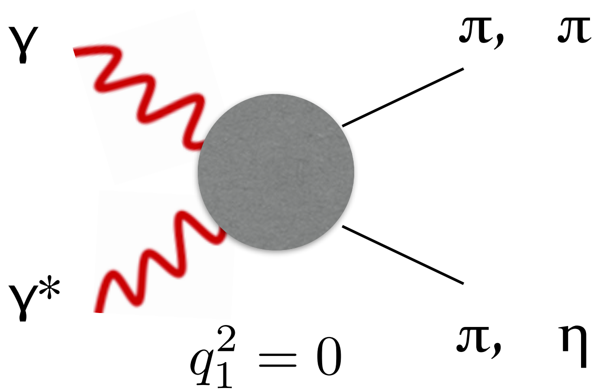
$$d\sigma = \frac{\alpha^2}{16\pi^4\,Q_1^2\,Q_2^2}\,\frac{2\sqrt{X}}{s(1-4m^2/s)^{1/2}}\cdot\frac{d^3\vec{p}_1^{\,\prime}}{E_1^{\prime}}\cdot\frac{d^3\vec{p}_2^{\,\prime}}{E_2^{\prime}}$$
$$\times\;\{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT}+\rho_1^{00}\,\rho_2^{00}\,\sigma_{LL}+2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL}+\ldots\}\,,$$

γγ→ππ, KK, ηη, πη (Belle: 07,08, 09,10, ..)
γγ*→ππ, πη (BESIII in progress)





Cross section



$$q_1^2 = 0$$
$$q_2^2 = -Q^2$$

C=+1: $J^{PC}=0^{++}, 2^{++}, \ldots.$

Helicity amplitudes

$$\langle \pi(p_1)\pi(p_2)|T|\gamma(q_1,\lambda_1)\gamma(q_2,\lambda_2)\rangle = (2\pi)^4\,\delta^{(4)}(p_1+p_2-q_1-q_2)\,H_{\lambda_1\lambda_2}$$

$$H_{\lambda_1\lambda_2} = H^{\mu\nu}\epsilon_\mu(\lambda_1)\,\epsilon_\nu(\lambda_2), \quad \lambda_1 = \pm 1,\ \lambda_2 = \pm 1, 0$$

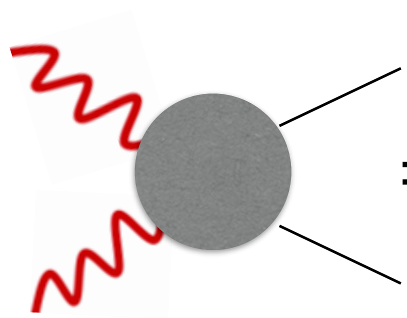
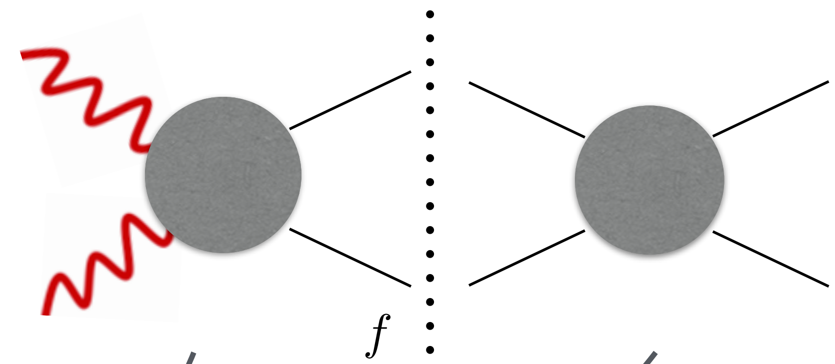
P symmetry: **6** → **3** independent amplitudes H_{++}, H_{+-}, H_{+0}

Cross sections

$$\sigma_{TT} = \pi\alpha^2 \frac{\rho(s)}{4\,(s+Q^2)} \int d\cos\theta\,\left(|H_{++}|^2 + |H_{+-}|^2\right)$$

$$\sigma_{TL} = \pi\alpha^2 \frac{\rho(s)}{2\,(s+Q^2)} \int d\cos\theta\,|H_{+0}|^2$$

Unitarity

2 Im  $= \sum_f \int d\Pi_2$ 

Partial wave expansion

$$H_{\lambda_1\lambda_2}(s,t) = \sum_{J=0}^{\overset{J_{max}=2}{\cancel{\infty}}} (2J+1)\, h_{J,\lambda_1\lambda_2}(s)\, d^J_{\lambda_1-\lambda_2,0}(\theta)$$

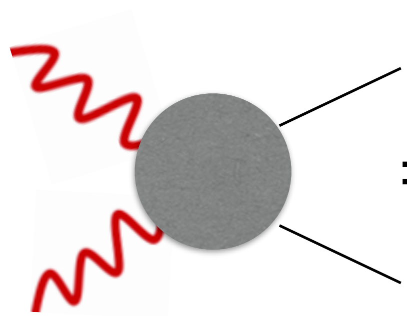
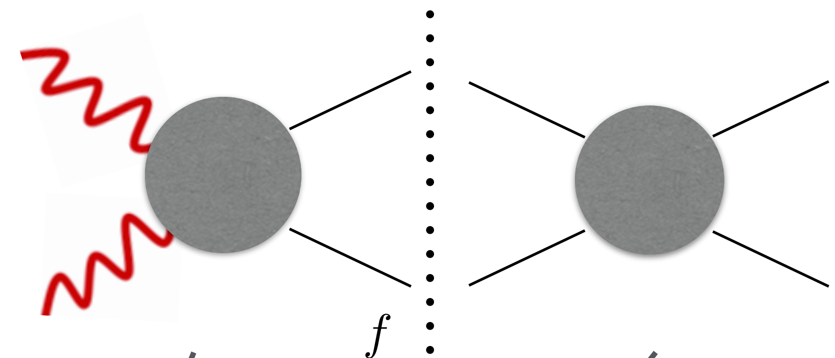
$$T(s,t) = \sum_{J=0}^{\overset{J_{max}=2}{\cancel{\infty}}} (2J+1)\, t_J(s)\, P_J(\theta)$$

These "diagonalise unitarity" and contain resonance information

Definite: J, λ_1, λ_2

$$\mathrm{Im}\, h_{\gamma\gamma^*\to\pi\pi}(s) = h_{\gamma\gamma^*\to\pi\pi}(s)\, \rho_{\pi\pi}(s)\, t^*_{\pi\pi\to\pi\pi}(s)$$

Unitarity

2 Im  $= \sum_f \int d\Pi_2$ 

Partial wave expansion

$$H_{\lambda_1\lambda_2}(s,t) = \sum_{J=0}^{\overset{J_{max}=2}{\cancel{\infty}}} (2J+1)\, h_{J,\lambda_1\lambda_2}(s)\, d^J_{\lambda_1-\lambda_2,0}(\theta)$$

$$T(s,t) = \sum_{J=0}^{\overset{J_{max}=2}{\cancel{\infty}}} (2J+1)\, t_J(s)\, P_J(\theta)$$

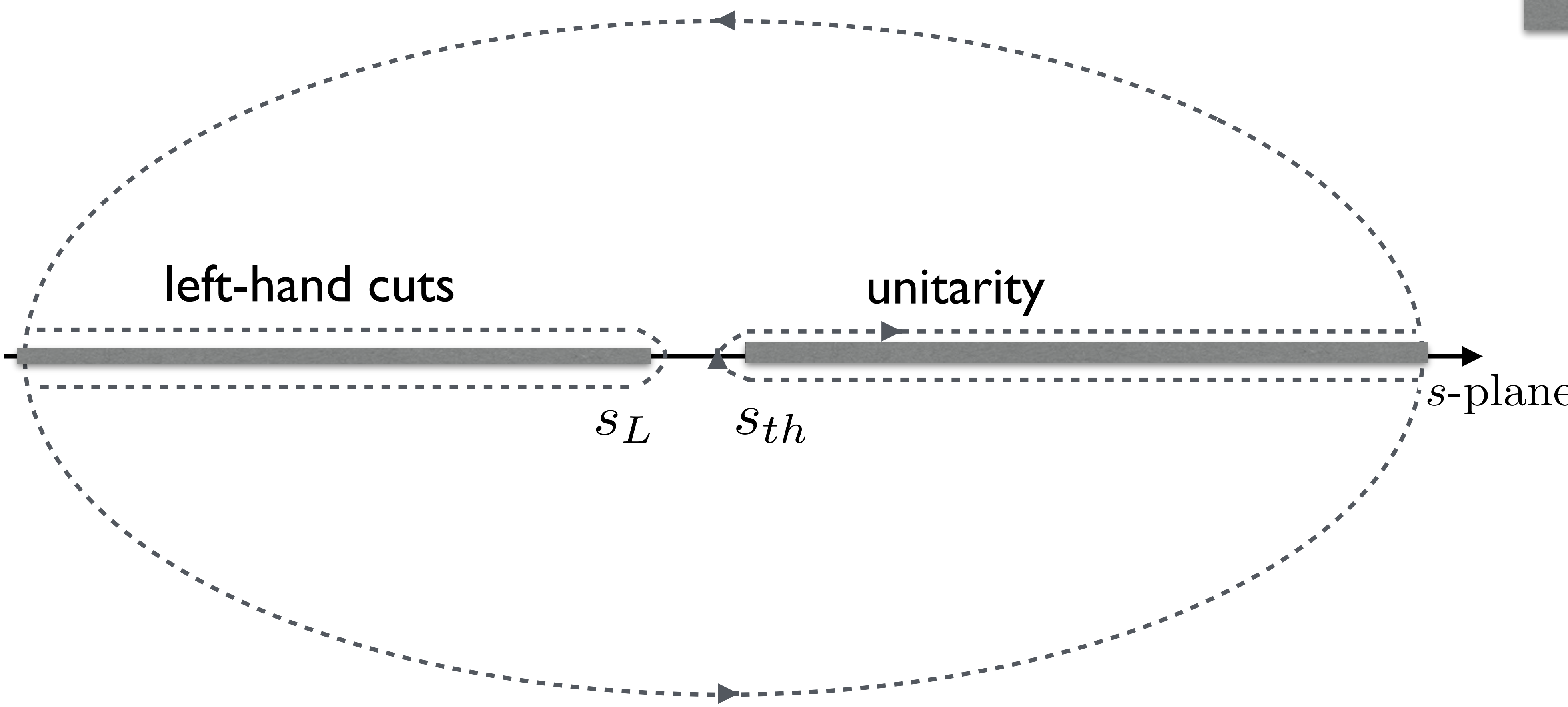
These “diagonalise unitarity” and contain resonance information

Definite: J, λ_1, λ_2

$$\mathrm{Im}\, h_{\gamma\gamma^*\to\pi\pi}(s) = \rho_{\pi\pi}\, h_{\gamma\gamma^*\to\pi\pi}\, t^*_{\pi\pi\to\pi\pi} + \rho_{KK}\, h_{\gamma\gamma^*\to KK}\, t^*_{KK\to\pi\pi} + \ldots$$

Dispersion relation

Definite: *J*, λ_1, λ_2

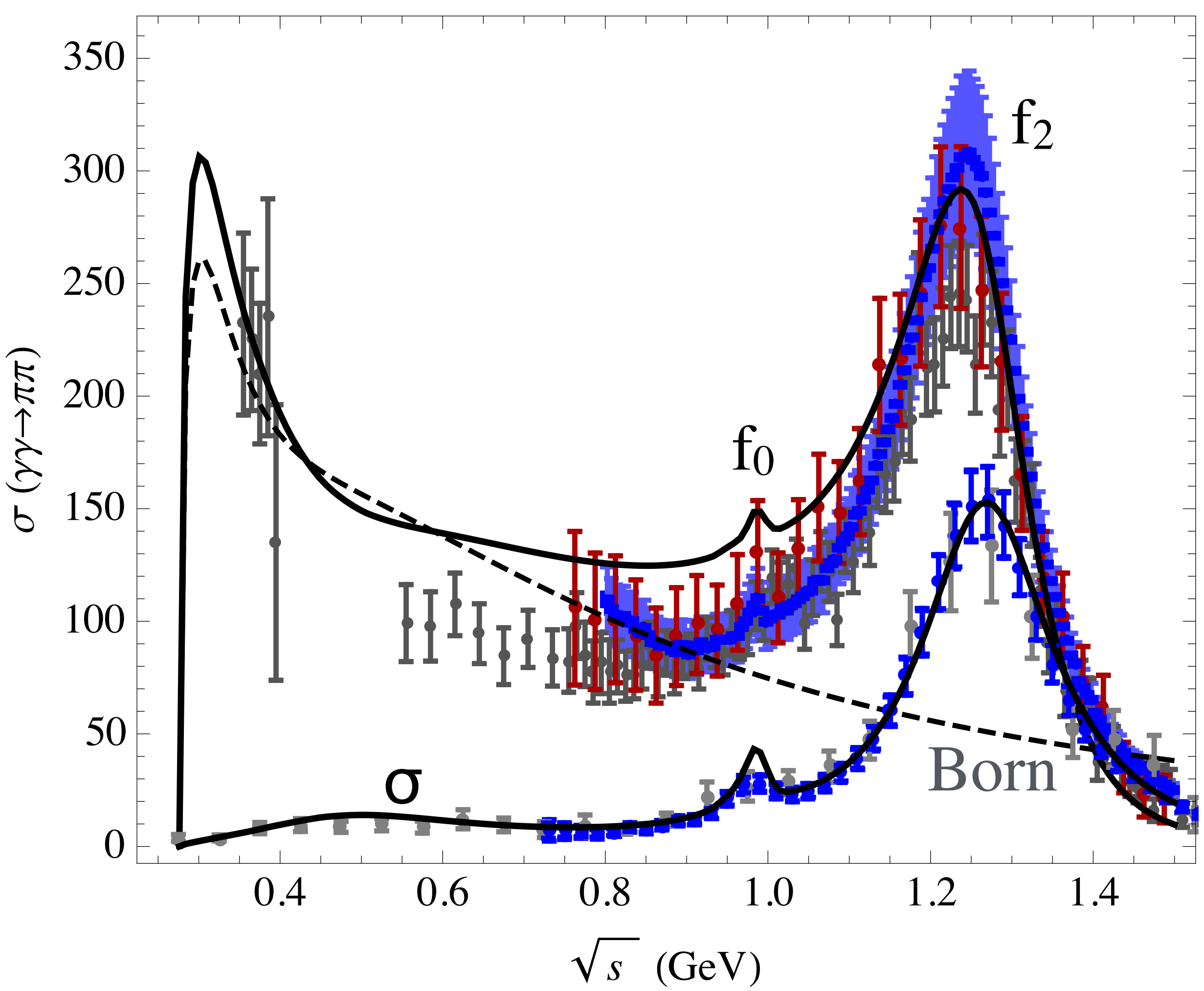


$$h(s) = \frac{1}{2\pi i}\int_C ds'\,\frac{h(s')}{s'-s} = \int_{-\infty}^{s_L}\frac{ds'}{\pi}\,\frac{\mathrm{Im}\,h(s')}{s'-s} + \int_{s_{th}}^{\infty}\frac{ds'}{\pi}\,\frac{\mathrm{Im}\,h(s')}{s'-s}$$

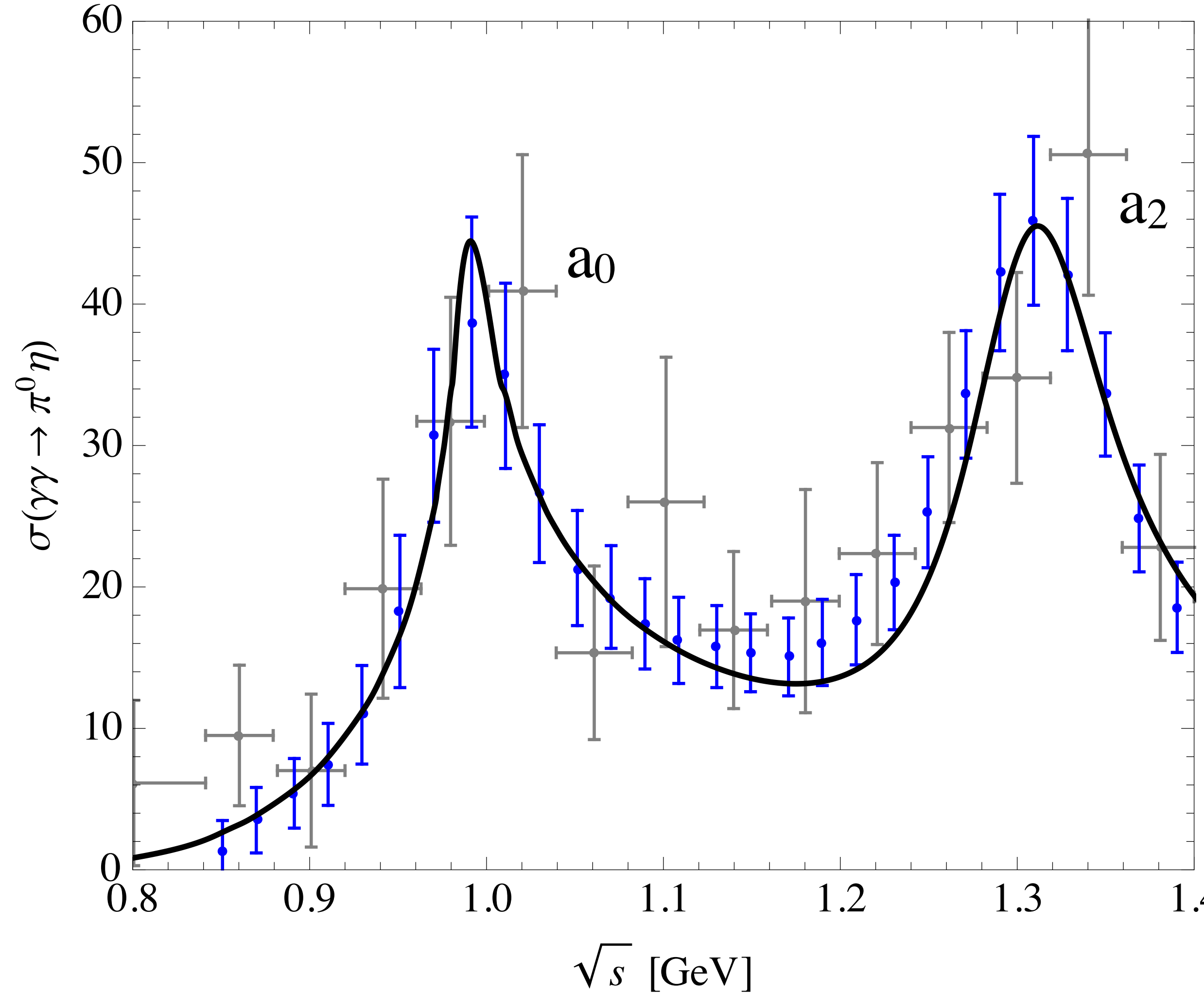
analyticity relates scattering amplitude at different energies

Results for $Q^2=0$

$\gamma\gamma \to \pi^+\pi^-$, $\pi^0\pi^0$



$\gamma\gamma \to \pi^0\eta$



- **Coupled-channel** dispersive treatment of $f_0(980)$ and $a_0(980)$ is **crucial**
- $f_2(1270)$ described dispersively through Omnes function
- $a_2(1320)$ described as a Breit Wigner resonance

I.D., Deineka, Vanderhaeghen (2017)

I.D., Vanderhaeghen (work in progress)

What has been done so far?

$Q^2 = 0$	Approach	Inelasticity	Number of fitted parameters to $\sigma_{\gamma\gamma \to MM}$	Range of applicability
[Hoferichter et. al. 2011]	Roy-Steiner	$\pi\pi$	0	$\sqrt{s} < 0.98$ GeV
[Morgan et. al. 1998]	Disp, Omnes	$\pi\pi$	0	$\sqrt{s} \lesssim 0.6$ GeV
[Dai et. al. 2014]	Amplitude anal.	$\pi\pi$, KK	>20	$\sqrt{s} < 1.5$ GeV
[Garcia-Martin et.al. 2010]	Disp, Omnes	$\pi\pi$, KK	6	$\sqrt{s} < 1.3$ GeV
[Current work]	Disp, Omnes	$\pi\pi$, KK $\pi\eta$, KK	0	$\sqrt{s} < 1.4$ GeV
$Q^2 \neq 0$				
[Moussallam 2013]	Disp, Omnes	$\pi\pi$, J=0	0	$\sqrt{s} \lesssim 0.8$ GeV
[Colangelo et.al. 2017]	Roy-Steiner	$\pi\pi$, J=0	0	$\sqrt{s} \lesssim 0.8$ GeV
[Current work]	Disp, Omnes	$\pi\pi$, KK, J=0,2 $\pi\eta$, KK	0	$\sqrt{s} < 1.4$ GeV

Only dispersive analyses are shown

Formalism

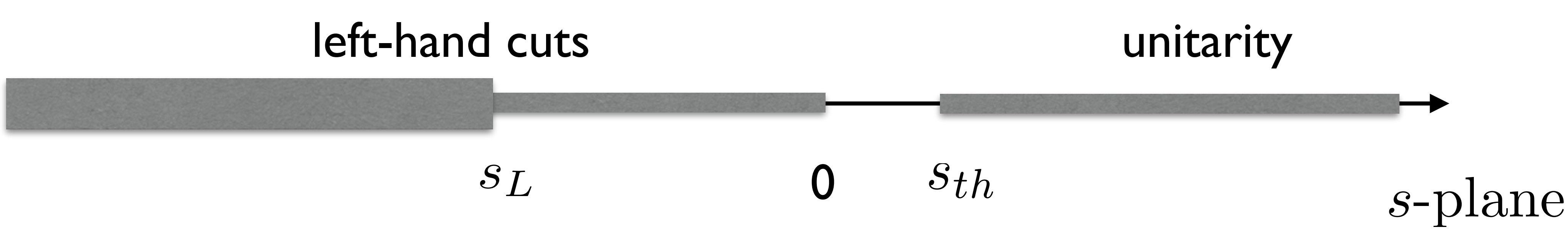
Morgan et. al. (1998)
Garcia-Martin et. al. (2010)
Moussallam (2013)

Write a dispersive representation for

$$\Omega^{-1}(s)(h(s) - h^{Born}(s))$$

Helicity - 0, s-wave

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} h_{++}^{Born}(s) \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s) \left[\begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} b \\ d \end{pmatrix} s + \frac{s^2}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'^2} \frac{\Omega(s')^{-1}}{s' - s} \begin{pmatrix} \mathrm{Im}\, \bar{h}_{++}(s') \\ \mathrm{Im}\, \bar{k}_{++}(s') \end{pmatrix} \right.$$
$$\left. - \frac{s^2}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'^2} \frac{\mathrm{Im}\, \Omega(s')^{-1}}{s' - s} \begin{pmatrix} h_{++}^{Born}(s') \\ k_{++}^{Born}(s') \end{pmatrix} \right]$$



Formalism

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Write a dispersive representation for

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$$- \frac{s^2}{\pi}\int_{s_{th}}^{\infty} \frac{ds'}{s'^2}\,\frac{\mathrm{Im}\,\Omega(s')^{-1}}{s'-s}\begin{pmatrix} h_{++}^{Born}(s') \\ k_{++}^{Born}(s') \end{pmatrix}\Bigg]$$

Coupled-channel Omnes function

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi\to\pi\pi} & \Omega_{\pi\pi\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\pi} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

Unitarity $\quad s \geq s_{th}$

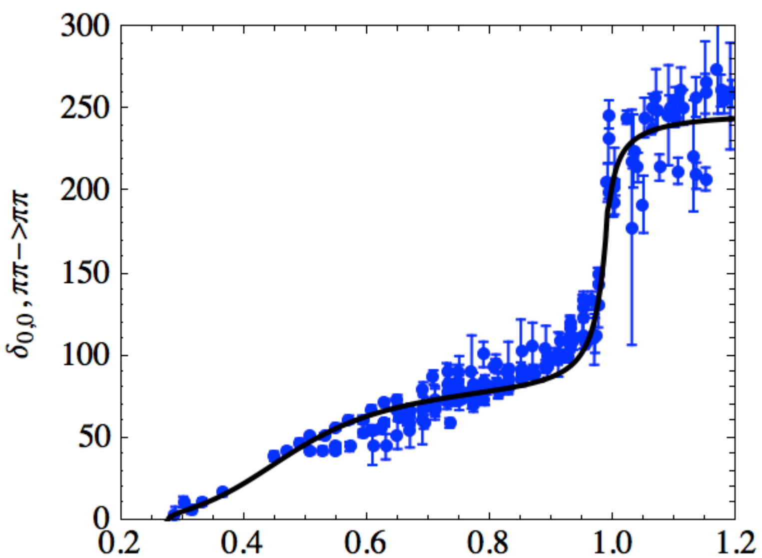
$$\mathrm{Im}\,h(s) = h(s)\,\rho(s)\,t^*(s)$$
$$\mathrm{Im}\,\Omega(s) = \Omega(s)\,\rho(s)\,t^*(s)$$

Omnes function I=0, {ππ, KK}

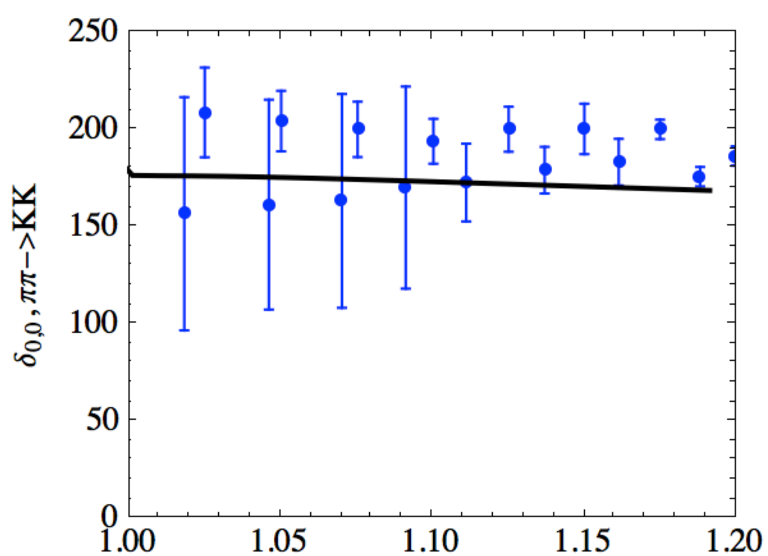
Coupled channel Omnes

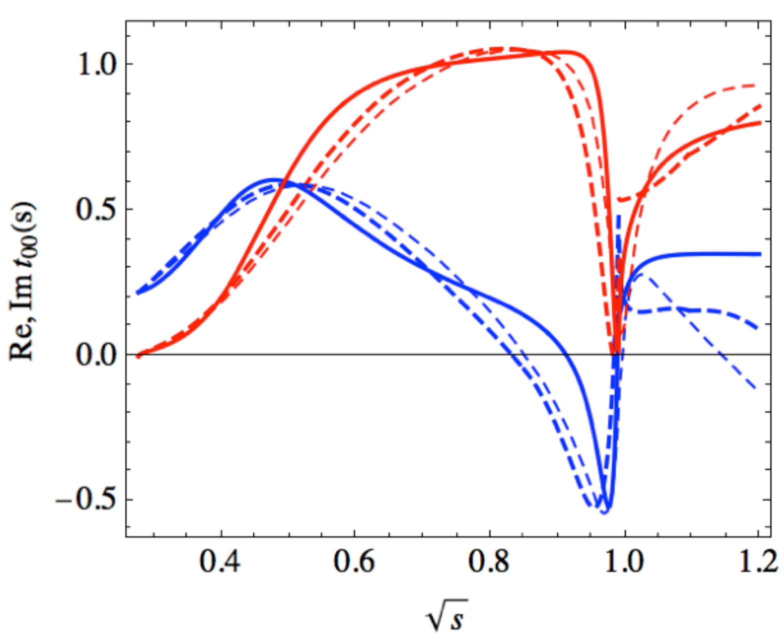
$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi\to\pi\pi} & \Omega_{\pi\pi\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\pi} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

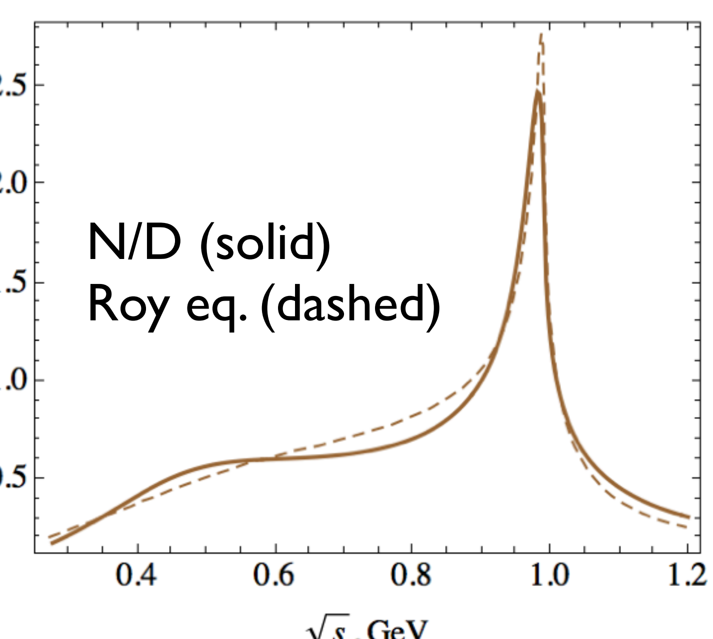
ππ->ππ



ππ->KK







Bounded p.w. amplitudes and Omnes at large energies

$$T(s) = \Omega(s)\, N(s)$$

$$N(s) = U(s) + \frac{s}{\pi}\int_R \frac{ds'}{s'}\,\frac{\rho(s')N(s')(U(s)-U(s'))}{s'-s}$$

$$\Omega^{-1}(s) = 1 - \frac{s}{\pi}\int_R \frac{ds'}{s'}\,\frac{\rho(s')N(s')}{s'-s}$$

$$U(s) = \sum_k C_k\, \xi(s)^k$$

Chew, Mandelstam
Lutz, I.D, Gasparyan

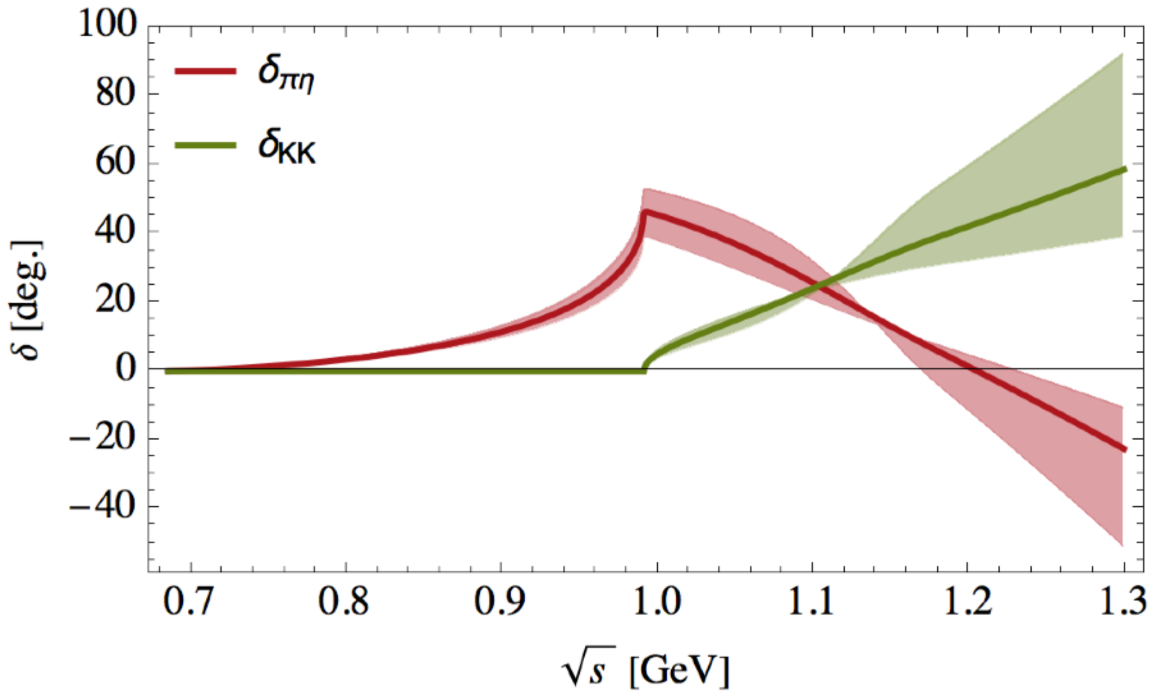
C_k **fitted** to Exp. **data** and Roy Eq. solutions

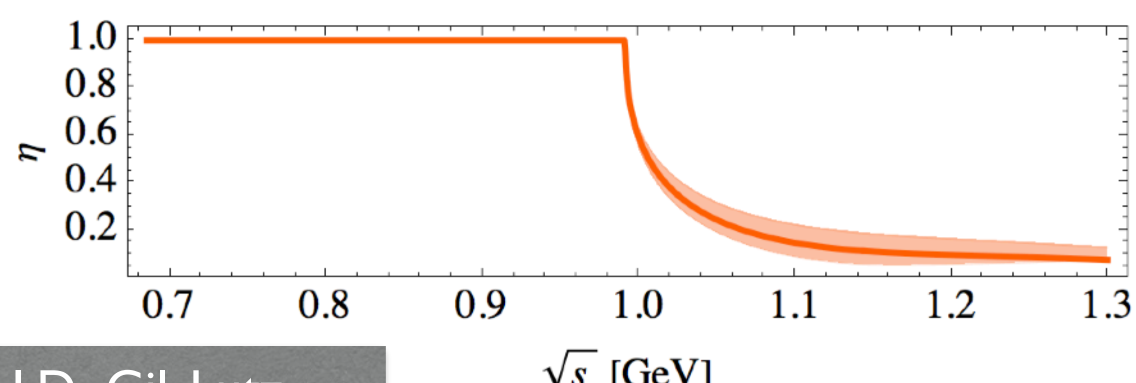
Omnes function I=1, {πη, KK}

Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\eta\to\pi\eta} & \Omega_{\pi\eta\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\eta} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

πη->πη





I.D., Gil, Lutz (2011), (2013)

Bounded p.w. amplitudes and Omnes at large energies

$$T(s) = \Omega(s)\, N(s)$$

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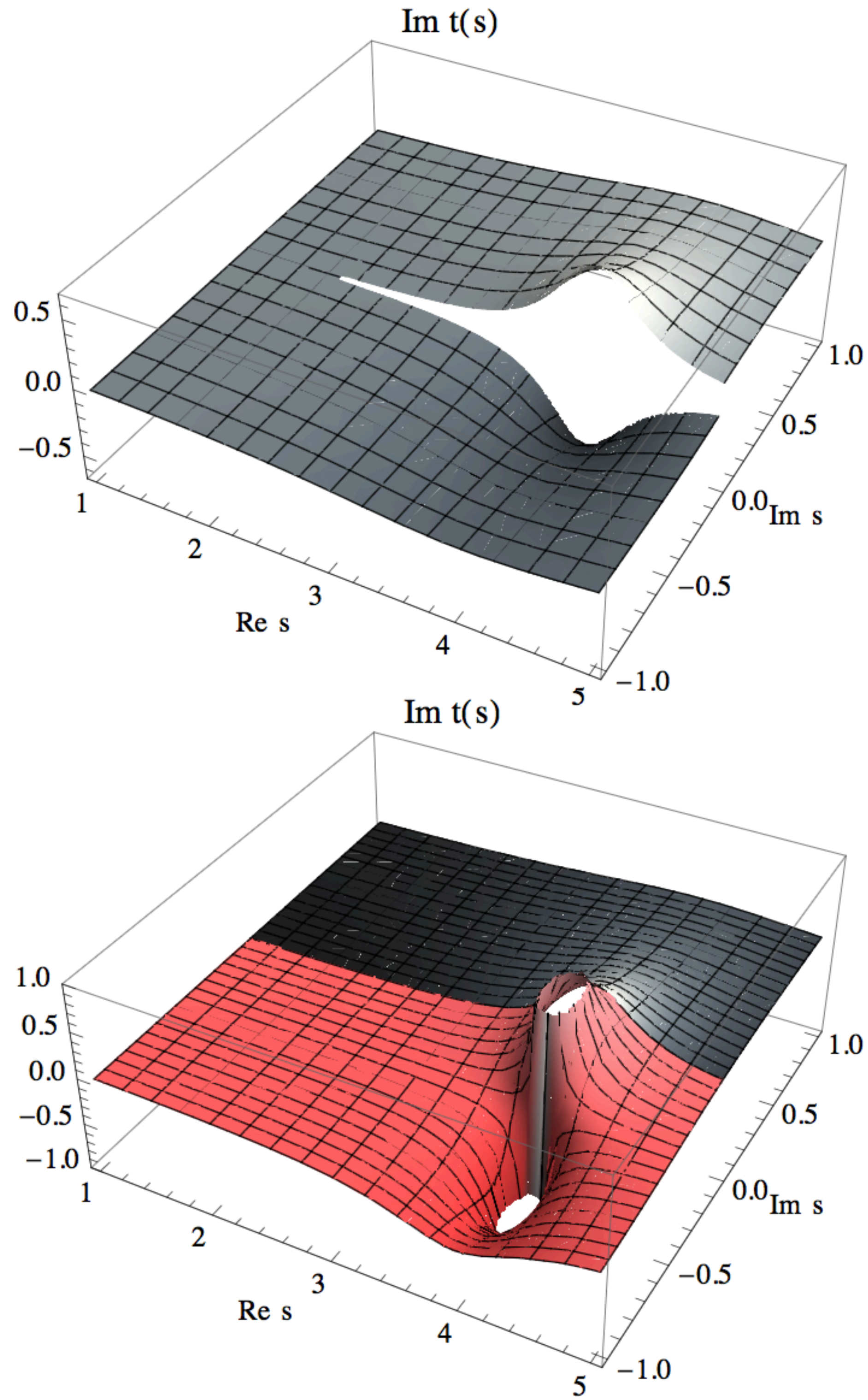
$$U(s) = \sum_k C_k\,\xi(s)^k$$

C_k **matched** to SU(3) **ChPT** at threshold

Poles in the complex plane

Single channel case (II Riemann sheet)



Unitarity:

$$t^I(s+i\epsilon)-t^I(s-i\epsilon)=2i\rho(s)t^I(s+i\epsilon)t^I(s-i\epsilon)$$

$$t^I(s+i\epsilon)=\frac{t^I(s-i\epsilon)}{1-2i\rho(s)t^I(s-i\epsilon)}$$

$$t^{II}(s-i\epsilon)\overset{\epsilon\to 0}{=}t^I(s+i\epsilon)$$

$$t^{II}(s)=\frac{t^I(s)}{1-2i\rho(s)t^I(s)}$$

sigma: $\sqrt{s_\sigma^{\mathrm{II}}}=0.436(5)\pm\frac{i}{2}0.357(40)\ \mathrm{GeV}$

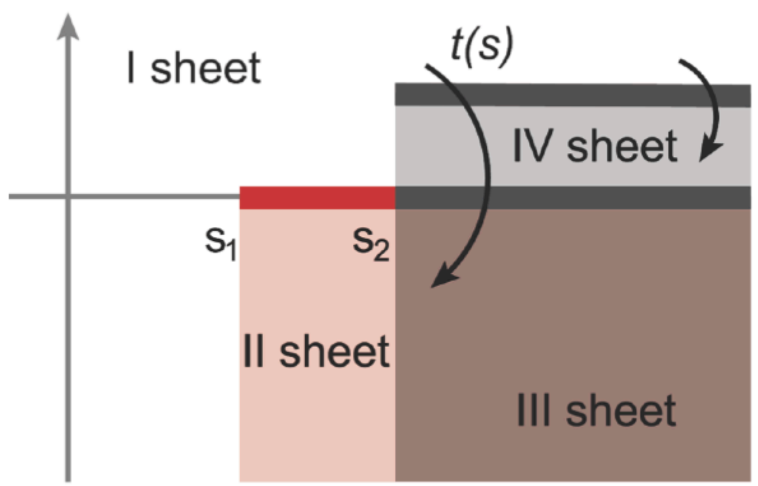
$f_0(980)$: $\sqrt{s_{f_0}^{\mathrm{II}}}=0.990(5)\pm\frac{i}{2}0.033(20)\ \mathrm{GeV}$

$a_0(980)$: no poles found on for the II Riemann sheet!

Poles in the complex plane

Coupled-channel case

$$t^I_{11}(s+i\epsilon)-t^I_{11}(s-i\epsilon)=2i\rho_1(s)t^I_{11}(s+i\epsilon)t^I_{11}(s-i\epsilon)+2i\rho_2(s)t^I_{12}(s+i\epsilon)t^I_{12}(s-i\epsilon)$$
$$t^I_{12}(s+i\epsilon)-t^I_{12}(s-i\epsilon)=2i\rho_1(s)t^I_{11}(s+i\epsilon)t^I_{12}(s-i\epsilon)+2i\rho_2(s)t^I_{12}(s+i\epsilon)t^I_{22}(s-i\epsilon)$$
$$t^I_{22}(s+i\epsilon)-t^I_{22}(s-i\epsilon)=2i\rho_1(s)t^I_{12}(s+i\epsilon)t^I_{12}(s-i\epsilon)+2i\rho_2(s)t^I_{22}(s+i\epsilon)t^I_{22}(s-i\epsilon)$$



Extensions to II, III, IV Riemann sheets

$$t^{II}_{11}(s)=\frac{t^I_{11}(s)}{1-2\,i\,\rho_1(s)\,t^I_{11}(s)}$$
$$t^{III}_{11}(s)=t^{II}_{11}(s)+\frac{2\,i\,\rho_2(s)\,t^{II}_{12}(s)^2}{1-2\,i\,\rho_2(s)\,t^{II}_{22}(s)}$$
$$t^{IV}_{11}(s)=t^{I}_{11}(s)+\frac{2\,i\,\rho_2(s)\,t^{I}_{12}(s)^2}{1-2\,i\,\rho_2(s)\,t^{I}_{22}(s)}$$

$$\sqrt{s^{\rm IV}_{a_0}}=(1.12^{-0.07}_{+0.02})\pm\frac{i}{2}(0.28^{+0.08}_{-0.13})\ \text{GeV}$$
$$\left|\frac{c_{K\bar K}}{c_{\pi\eta}}\right|=0.98^{-0.07}_{+0.20}$$

$$\rho_i(s)=2k_i(s)/\sqrt{s}$$

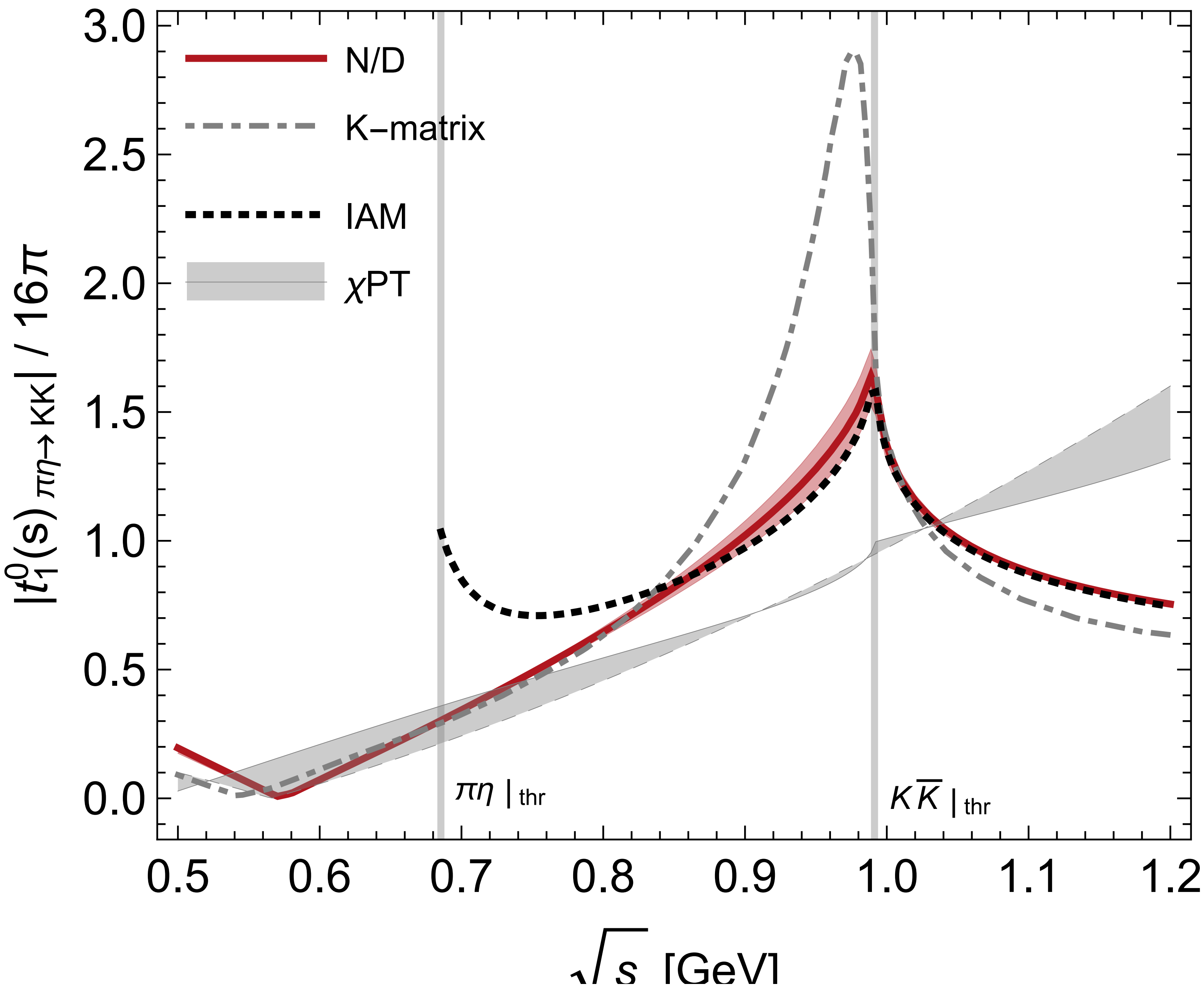
Sheet	Im k_1	Im k_2
I	+	+
II	−	+
III	−	−
IV	+	−

II sheet: $1-2i\rho_1(s)t^I_{11}(s)=0$

III sheet: $1-2i\rho_2(s)t^{II}_{22}(s)=0$

IV sheet: $1-2i\rho_2(s)t^{I}_{22}(s)=0$

Scattering amplitude πη→KK



K-matrix:
Albaladejo et. al. (2017)

Inverse Amplitude Method (IAM)
Gomez Nicola et.al. (2002)

Chiral Perturbation Theory
Gasser et. al. (1985)

- First **lattice** analysis for m_π=391 MeV [Jozef Dudek et. al. (2016)]
- **Chiral extrapolation** of the lattice results [Zhi-Hui Guo et. al. (2017)]

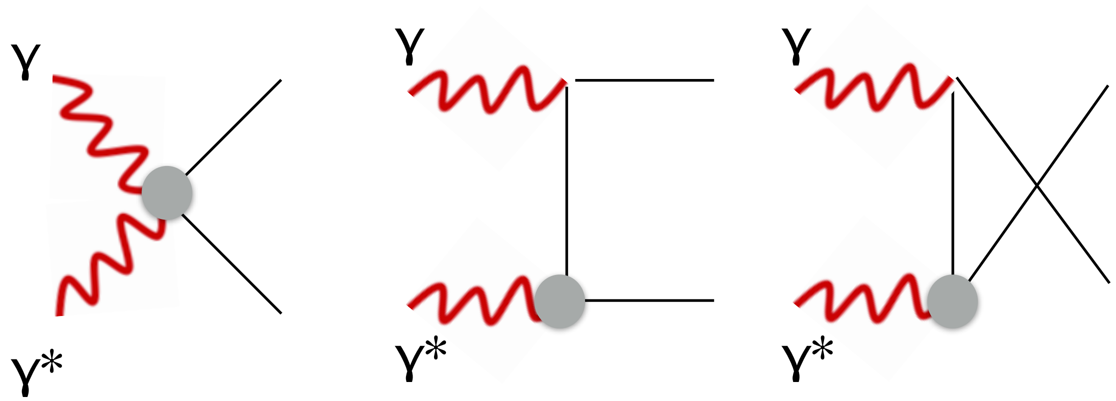
Left-hand cuts

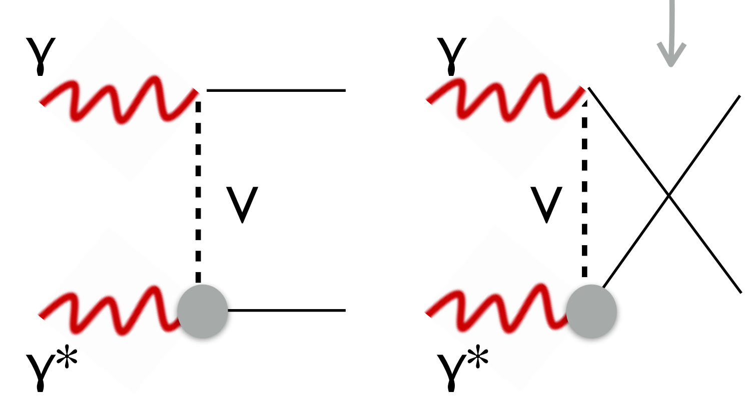
Morgan et. al. (1998)
Moussallam (2013)

Dispersive integral for J=0

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} h_{++}^{Born}(s) \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s) \left[\begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} b \\ d \end{pmatrix} s + \frac{s^2}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'^2} \frac{\Omega(s')^{-1}}{s'-s} \begin{pmatrix} \mathrm{Im}\,\bar{h}_{++}(s') \\ \mathrm{Im}\,\bar{k}_{++}(s') \end{pmatrix} - \frac{s^2}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'^2} \frac{\mathrm{Im}\,\Omega(s')^{-1}}{s'-s} \begin{pmatrix} h_{++}^{Born}(s') \\ k_{++}^{Born}(s') \end{pmatrix} \right]$$

Scalar QED (pion pole contribution)





Fearing Scherer (1998)
Colangelo et.al. (2015)

Vertex ππγ*

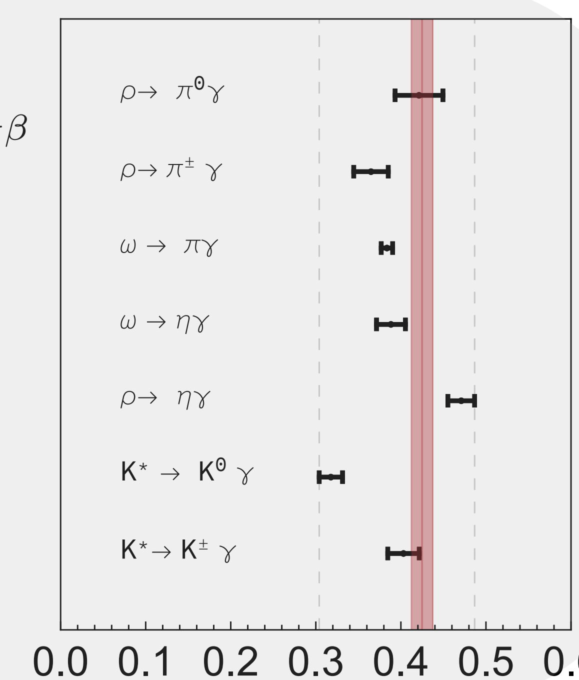
$$\langle \pi^+ | j_\mu(0) | \pi^+(p') \rangle = (p+p')_\mu\, F_\pi(Q^2)$$

$$F_\pi(Q^2) = \frac{1}{1+Q^2/M_\rho^2}$$

$$\mathcal{L} = e\, C_V\, \epsilon_{\mu\nu\alpha\beta}\, F^{\mu\nu}\, \partial^\alpha \phi\, V^\beta$$

$$F_{\pi\omega}(Q^2) = \frac{1}{1+Q^2/M_\rho^2}$$

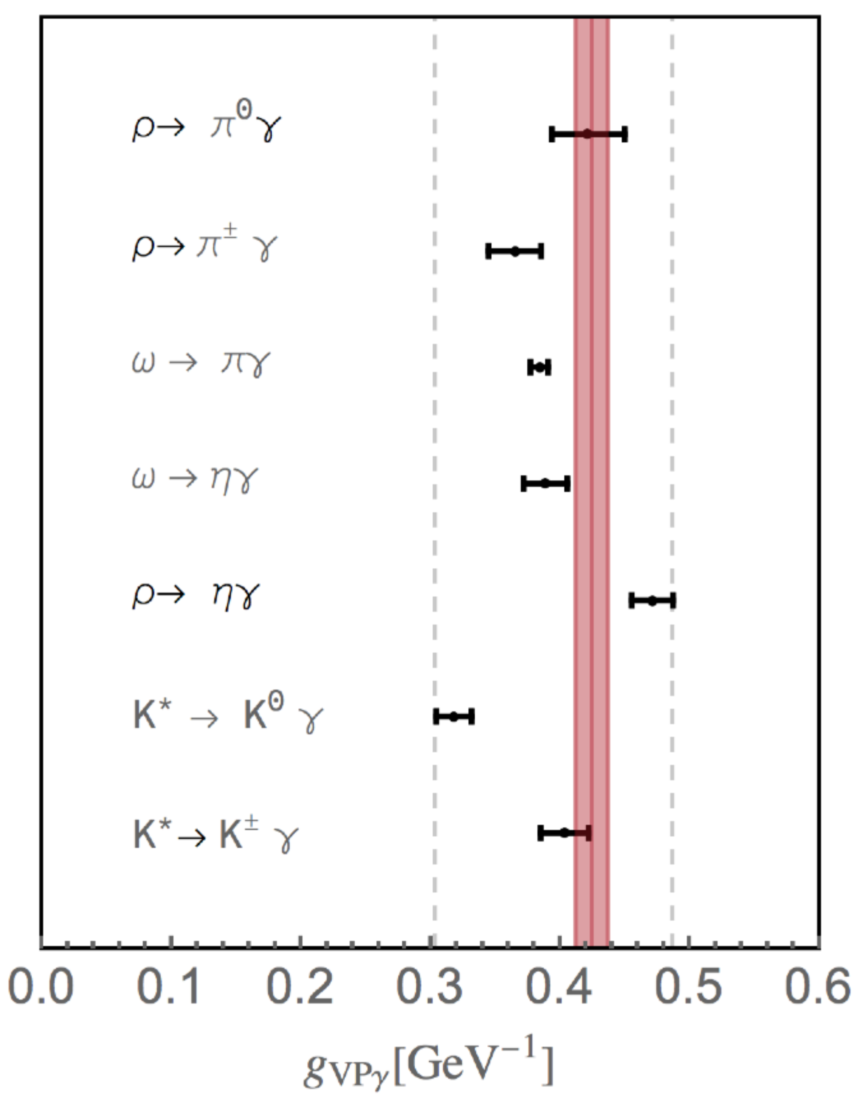
$$F_{\pi\rho}(Q^2) = \frac{1}{1+Q^2/M_\omega^2}$$



Left-hand cuts and $\eta \to \pi^0\gamma\gamma$

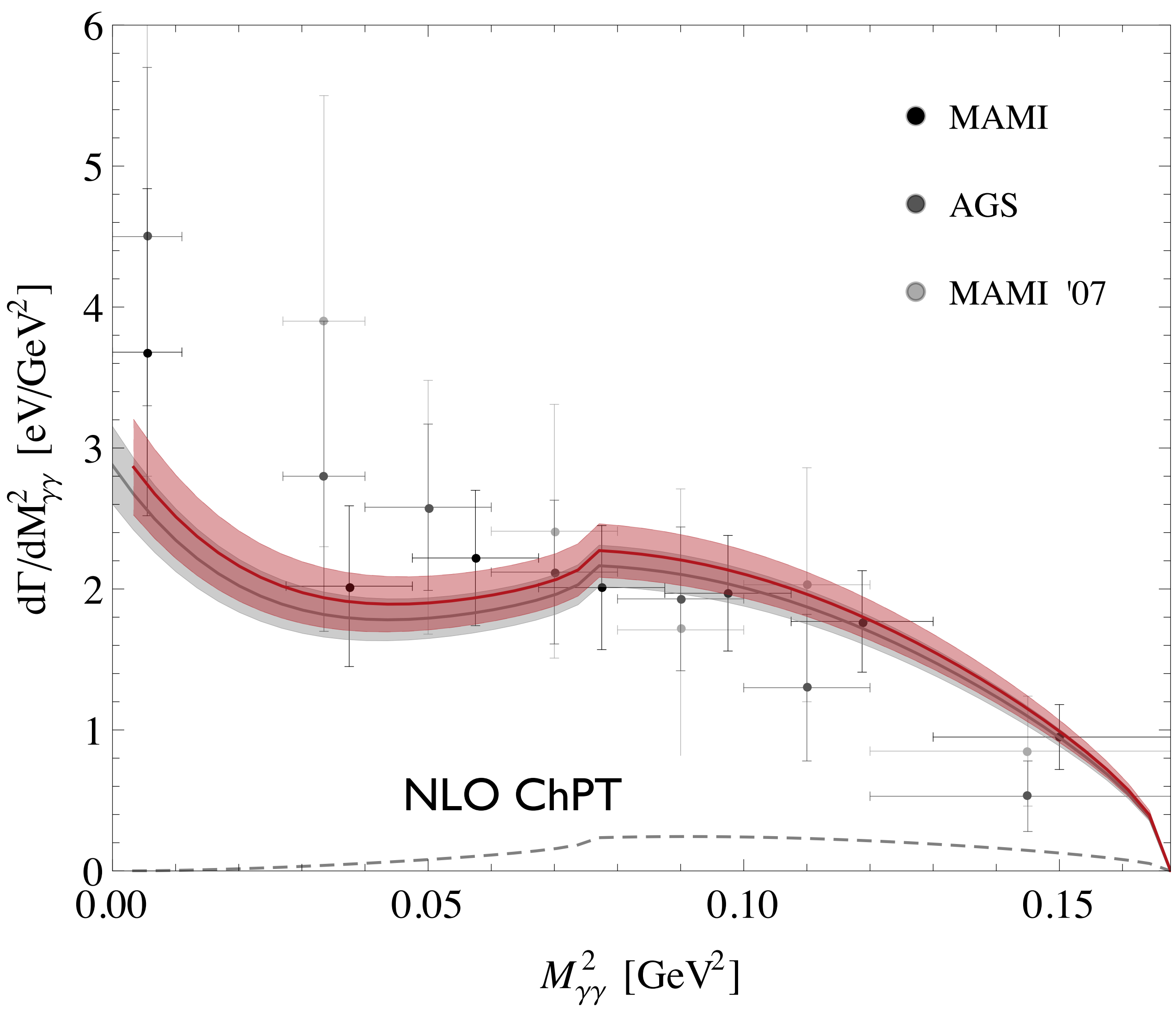
Approximate left-hand cuts by vector meson exchanges

$$\mathcal{L} = e\,C_V\,\epsilon_{\mu\nu\alpha\beta}\,F^{\mu\nu}\,\partial^\alpha\phi\,V^\beta$$



$\eta \to \pi^0\gamma\gamma$ is linked to $\gamma\gamma \to \pi^0\eta$ by crossing symmetry

$$\frac{d^2\Gamma}{ds\,dt} = \frac{1}{(2\pi)^3}\frac{1}{32\,m_\eta^3}\sum_{\lambda_1,\lambda_2}|H_{\lambda_1\lambda_2}|^2$$



Subtraction constants

Morgan et. al. (1998)
Moussallam (2013)

Dispersive integral for J=0

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} h_{++}^{Born}(s) \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s)\left[\begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} b \\ d \end{pmatrix} s + \frac{s^2}{\pi}\int_{-\infty}^{s_L} \frac{ds'}{s'^2}\frac{\Omega(s')^{-1}}{s'-s}\begin{pmatrix} \mathrm{Im}\,\bar{h}_{++}(s') \\ \mathrm{Im}\,\bar{k}_{++}(s') \end{pmatrix} - \frac{s^2}{\pi}\int_{s_{th}}^{\infty} \frac{ds'}{s'^2}\frac{\mathrm{Im}\,\Omega(s')^{-1}}{s'-s}\begin{pmatrix} h_{++}^{Born}(s') \\ k_{++}^{Born}(s') \end{pmatrix}\right]$$

Soft photon limit (q_1=0)

$$H_{\lambda_1\lambda_2} \to H_{\lambda_1\lambda_2}^{Born}$$
$$s = -Q^2,\ t = u = m_\pi^2$$

Low, Gell-Mann, Goldberger (1954)

For space like photons: generalized polarizabilities

$$\pm\frac{2\alpha}{m_\pi}\frac{H_{+\pm}^n}{s+Q^2} = (\alpha_1 \mp \beta_1)_{\pi^0} + \ldots$$
$$\pm\frac{2\alpha}{m_\pi}\frac{(H_{+\pm}^c - H_{+\pm}^{Born})}{s+Q^2} = (\alpha_1 \mp \beta_1)_{\pi^+} +$$

COMPASS data on $(\alpha_1-\beta_1)_{\pi^+}$
(future Hall D (JLab) experiment)

$f_2(1270)$ and $a_2(1320)$ contributions

Watson theorem (for elastic unitarity) J=2: $\phi(\gamma\gamma \to \pi\pi) = \phi(\pi\pi \to \pi\pi) = \delta(\pi\pi \to \pi\pi)$

$$\Omega(s) = \exp\left(\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\phi_{\gamma\gamma\to\pi\pi}(s)}{s'-s} \right)$$

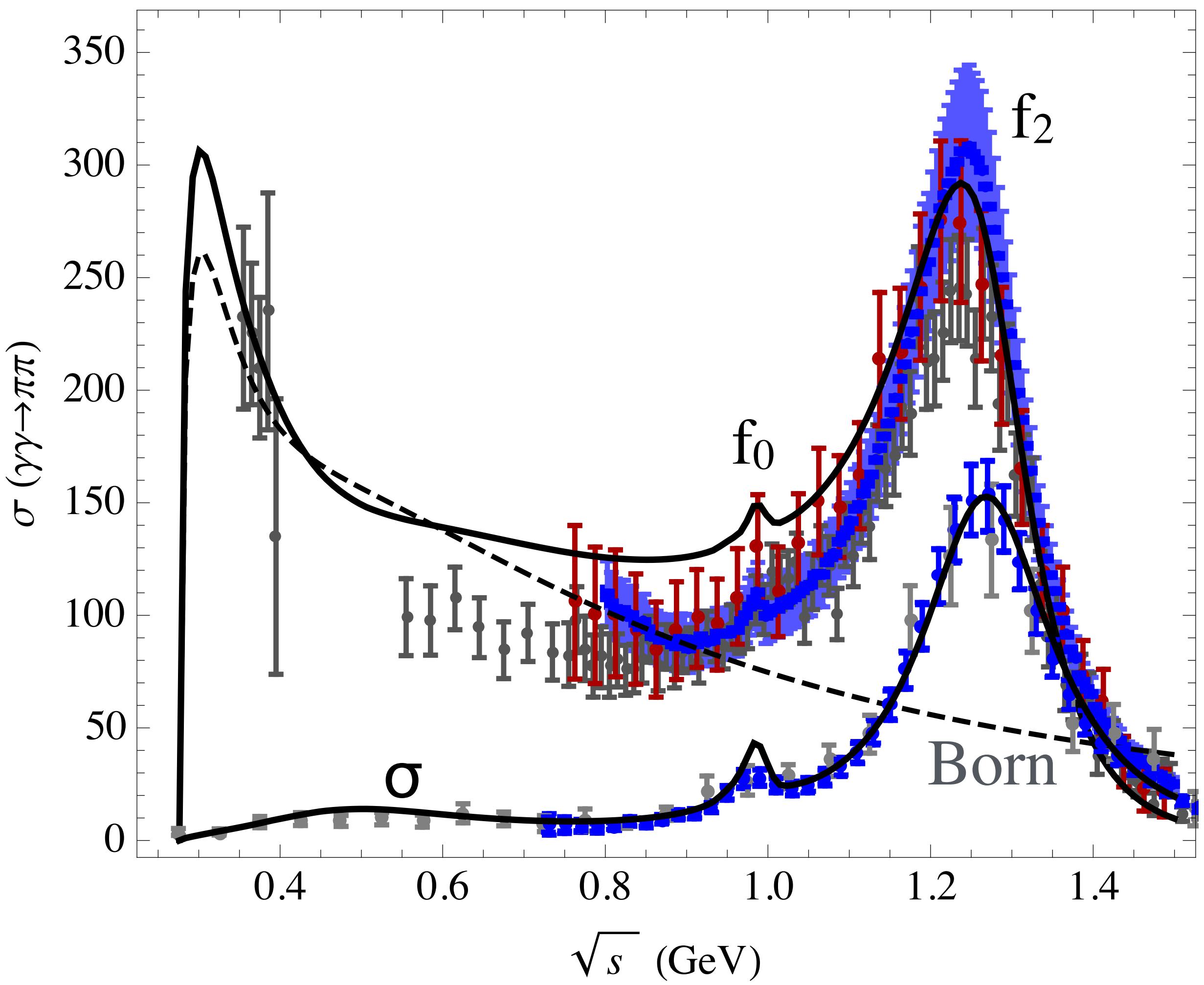
Roy analysis (2011)
R. Garcia-Martin
at.al.

Breit Wigner + Background

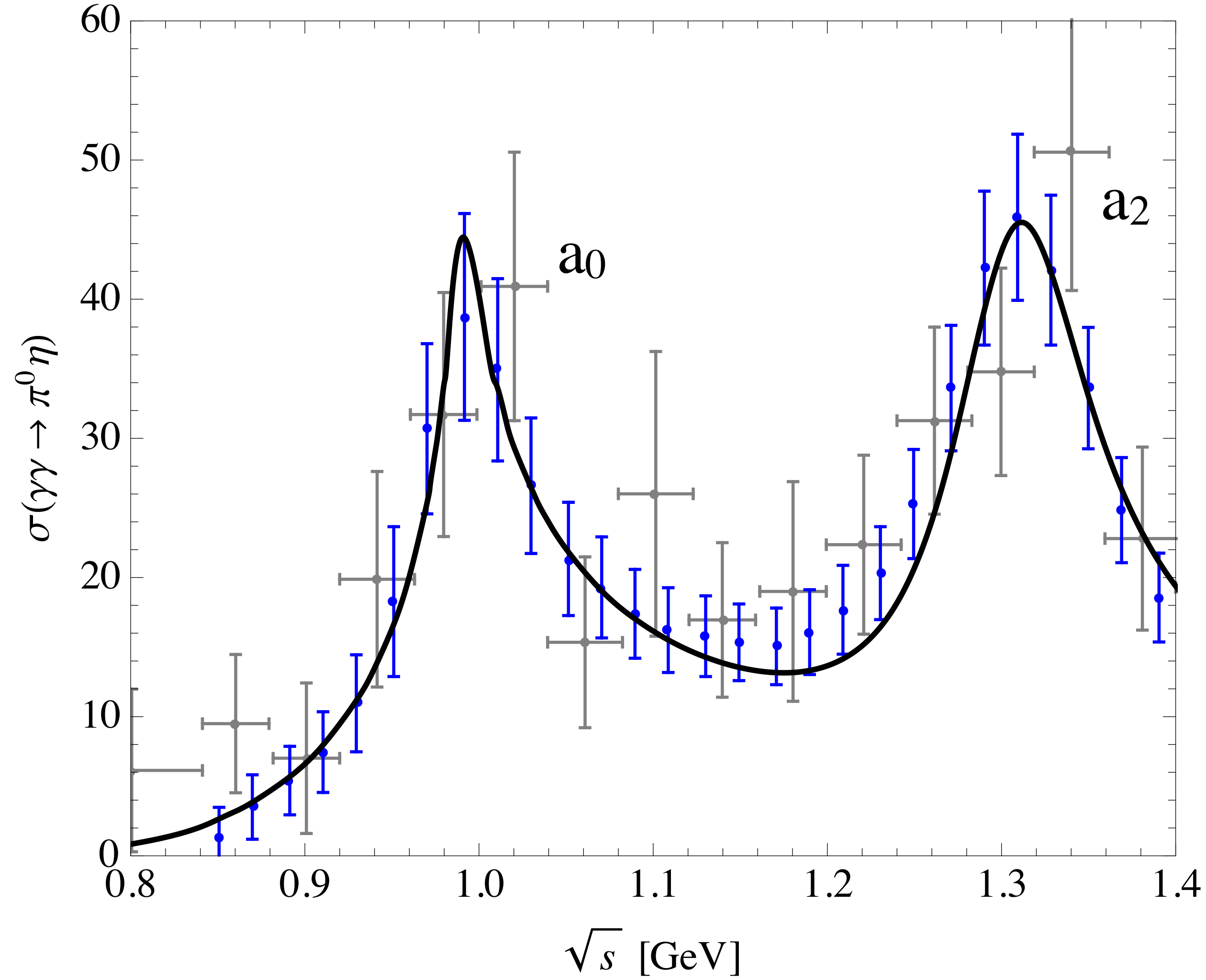
$$h_{J=2}^{f_2} = \frac{C_{f_2\to\gamma\gamma} C_{f_2\to\pi\pi}}{10\sqrt{6}} \frac{s(s+Q^2)\beta(s)}{s - M^2 + i\, M\, \Gamma(s)}\, T_{f_2}^{(\Lambda=2)}(Q^2)$$

Results for Q^2=0

$\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$



$\gamma\gamma \to \pi^0\eta$



- **Coupled-channel** dispersive treatment of $f_0(980)$ and $a_0(980)$ is **crucial**
- $f_2(1270)$ described dispersively through Omnes function
- $a_2(1320)$ described as a Breit Wigner resonance

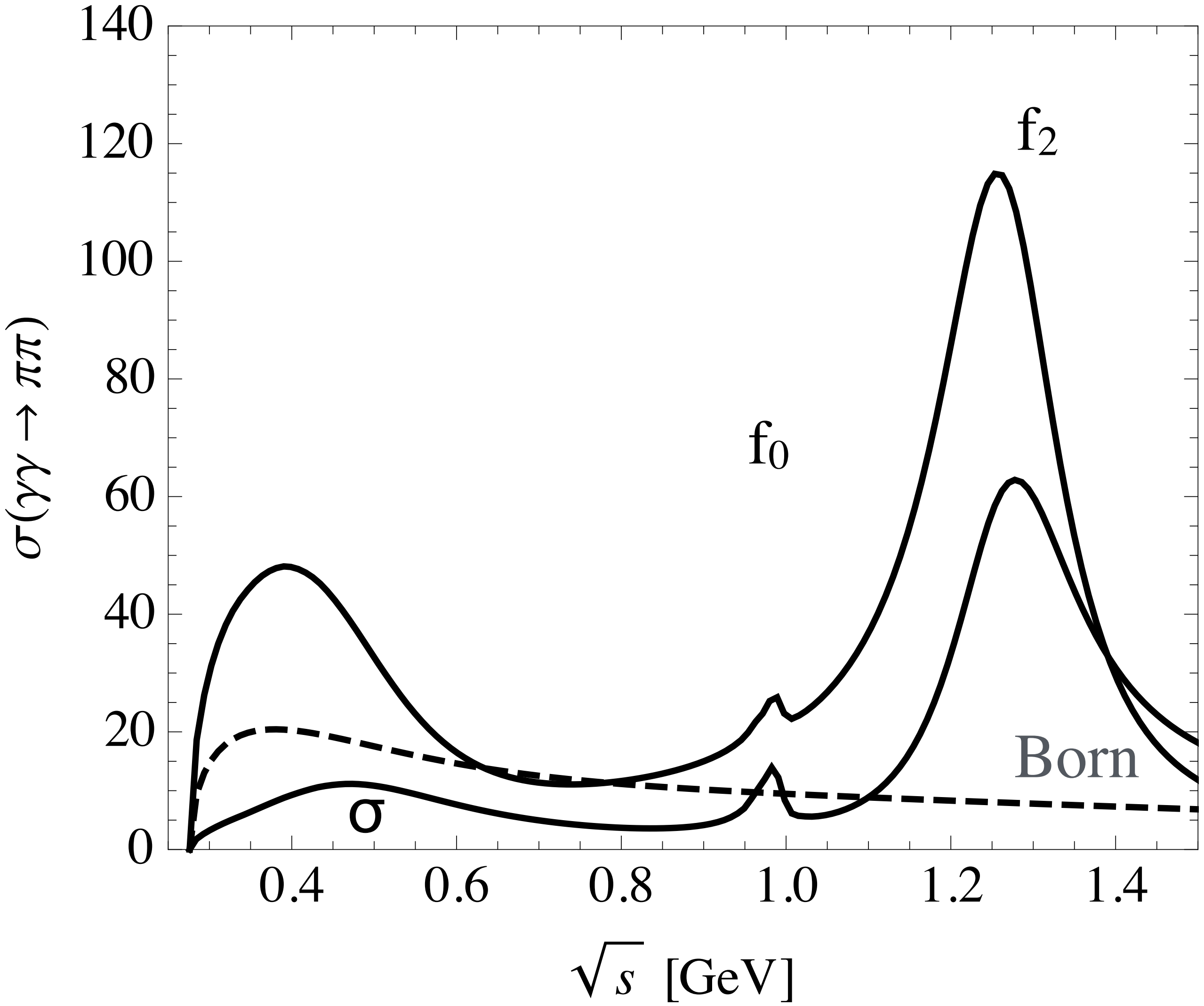
I.D., Deineka, Vanderhaeghen (2017)

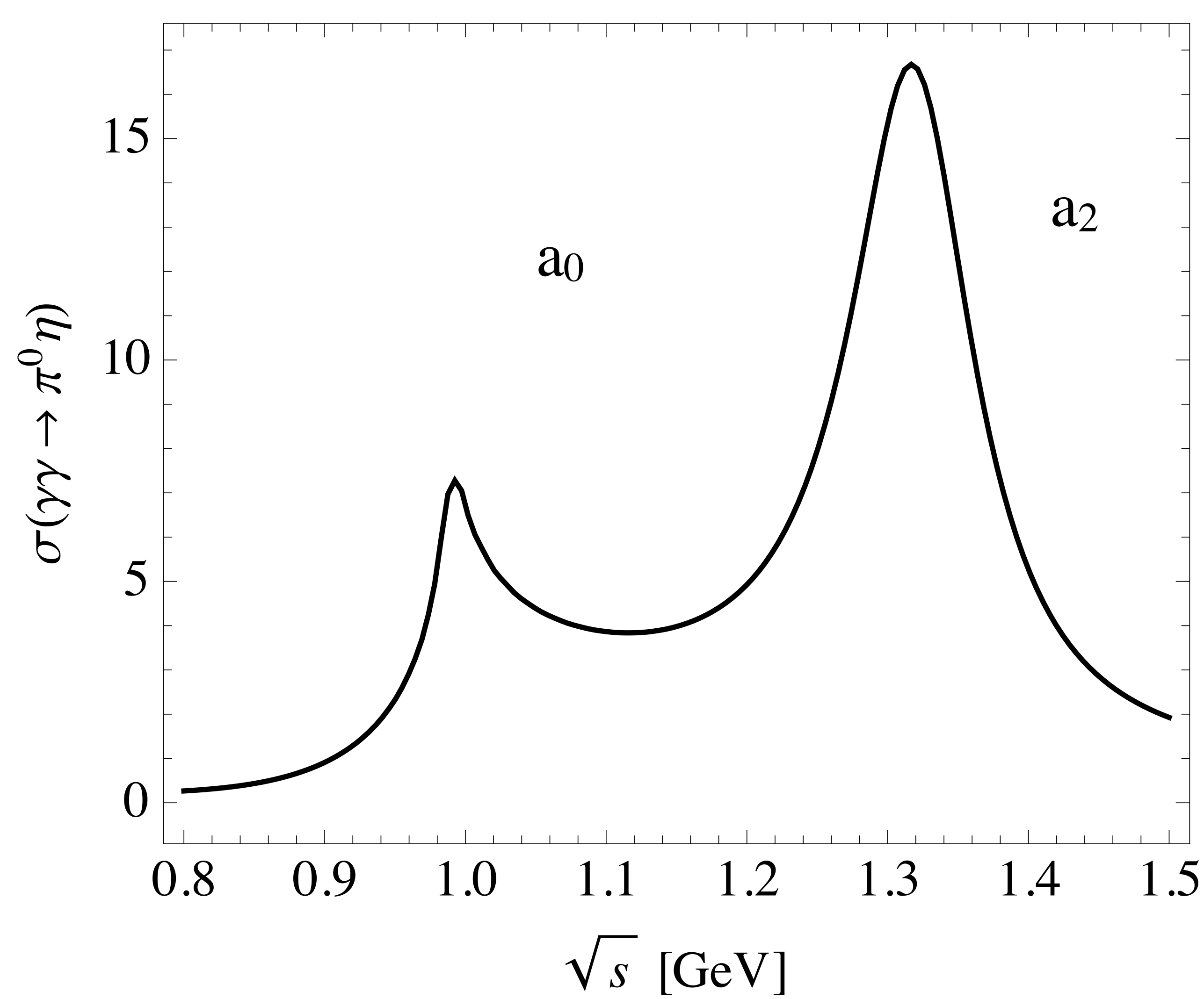
I.D., Vanderhaeghen (work in progress)

Results for Q^2=0.5 (prediction)

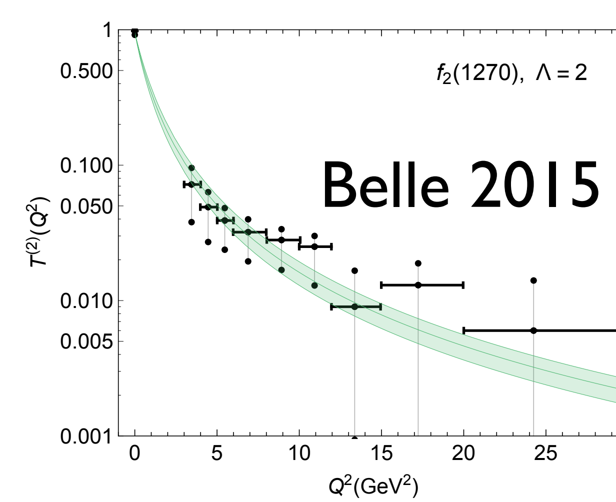
$\gamma\gamma \to \pi^+\pi^-$, $\pi^0\pi^0$

$\gamma\gamma \to \pi^0\eta$ (work in progress)





- **Coupled-channel** dispersive treatment for $f_0(980)$ and $a_0(980)$
- $f_2(1270)$, $a_2(1320)$ described as a Breit Wigner resonance and TFF is taken from Belle 2015 data



Summary and Outlook

- In light of the **new Belle data (2015)** for $f_2(1270)$ TFFs and using LbL sum rules we **predicted** (Λ=2) TFF for $f_2(1565)$

- **Update for meson contributions to (g-2) LbL**
  Tensor mesons contributions found to be small compared to anticipated exp. uncertainty $1.6 \cdot 10^{-10}$

  Axial vector mesons contributions (satisfying Landau-Yang theorem constraint) evaluated by 2 groups and found to be between $(0.64 - 0.75 \pm 0.27) 10^{-10}$

- **Next steps?**
  Need to take into account $f_0(500)$ and non resonant contributions in a dispersive approach

- Main ingredients: $\boldsymbol{\gamma\gamma^* \to \pi\pi, \pi\eta}$. Can be used in **different** (g-2) dispersive approaches.

  It is important to **validate** dispersive treatment of $\boldsymbol{\gamma\gamma^* \to \pi\pi, \pi\eta}$,… with upcoming BES III data

Extra slides

Omnes function {πη, KK}

Coupled-channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\eta\to\pi\eta} & \Omega_{\pi\eta\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\eta} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

Bounded p.w. amplitudes and Omnes at large energies

$$T(s) = \Omega(s)\, N(s)$$

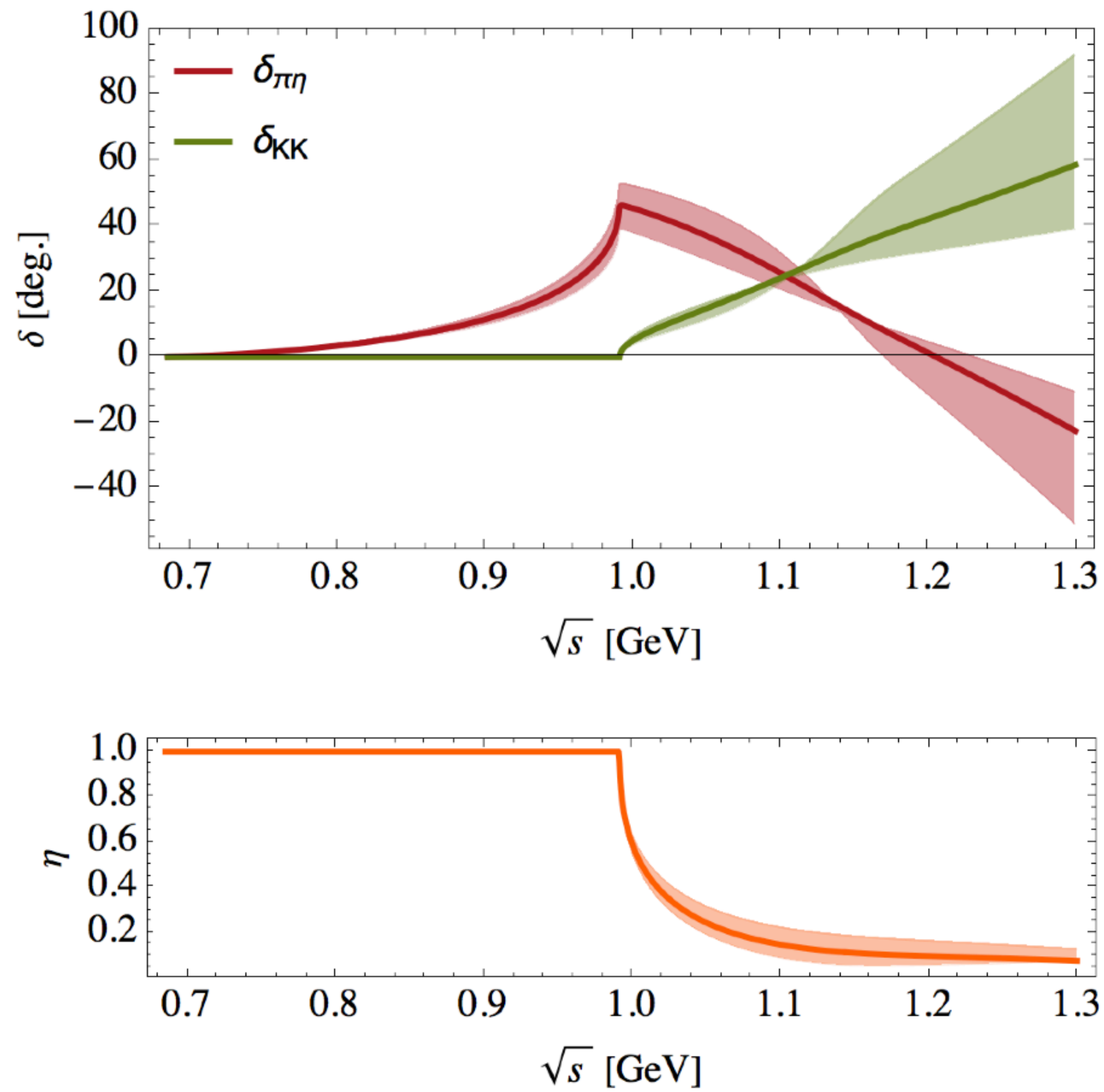
$$N(s) = U(s) + \frac{s - s_{th}}{\pi} \int_R \frac{ds'}{s' - s_{th}} \frac{\rho(s') N(s') (U(s) - U(s'))}{s' - s}$$

$$\Omega^{-1}(s) = 1 - \frac{s - s_{th}}{\pi} \int_R \frac{ds'}{s' - s_{th}} \frac{\rho(s') N(s')}{s' - s}$$

$$U(s) = \sum_k C_k\, \xi(s)^k$$

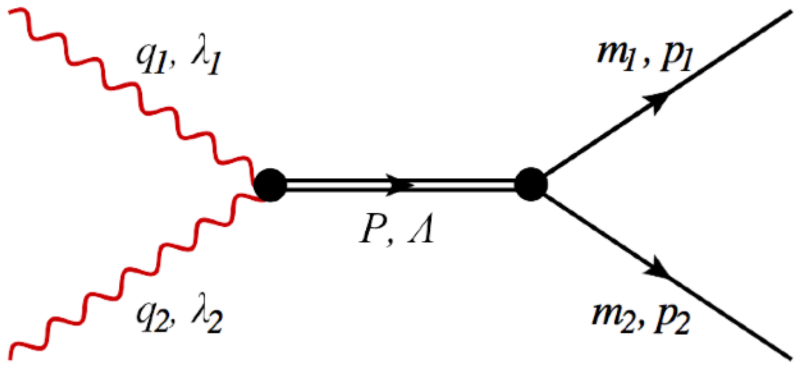
Chew, Mandelstam
Lutz, Gasparyan, I.D., Gill

C_k **matched** to SU(3) **ChPT** at threshold



a_2(1320) contribution

a_2(1320) resonance implemented as explicit d.o.f



$$\mathcal{L}_{T\to PP} = e^2\, C_{T\to PP}\, T_{\mu\nu}\, F^{\mu\lambda} F^{\nu}_{\lambda}$$
$$\mathcal{L}_{T\to\gamma\gamma} = C_{T\to\gamma\gamma}\, T^{\mu\nu}\, \partial_\mu P \partial_\nu P$$

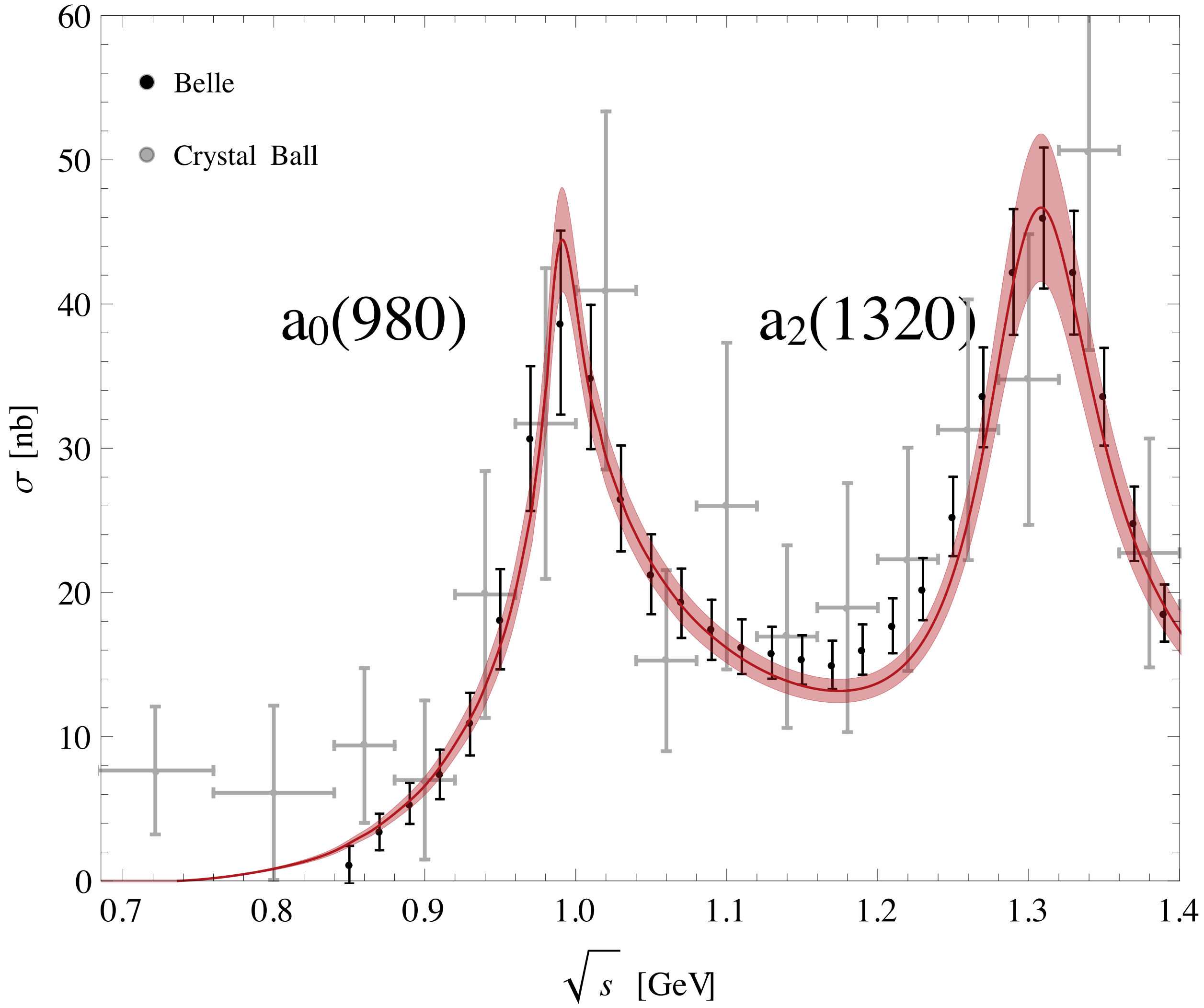
Helicity - 2, d-wave

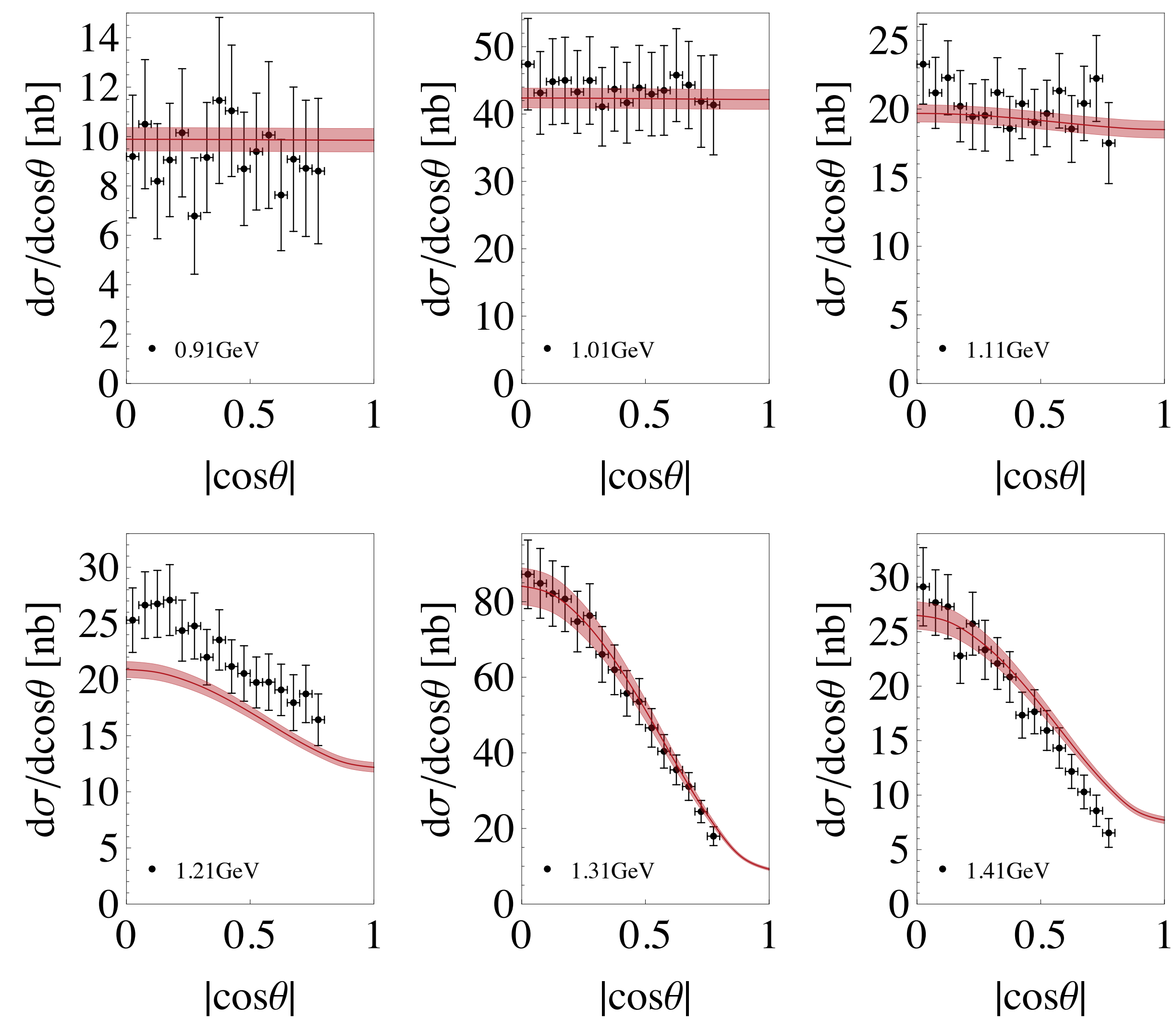
$$h_{+-}(s) = \frac{e^2\, C_{a_2\to\pi\eta}\, C_{a_2\to\gamma\gamma}\, s^2\, \beta^2_{\pi\eta}(s)}{10\sqrt{6}\,(s - M^2_{a_2} + i\, M_{a_2}\, \Gamma_{a_2}(s))}$$

Couplings

$$\Gamma_{a_2\to\pi\eta} = \frac{\beta^5_{\pi\eta}(M^2_{a_2})}{1920\,\pi}\, C^2_{a_2\to\pi\eta}\, M^3_{a_2} \overset{\mathrm{exp}}{=} 15.5(1.5)\,\mathrm{MeV}$$
$$\Gamma_{a_2\to\gamma\gamma} = \frac{\pi\alpha^2}{5}\, C^2_{a_2\to\gamma\gamma}\, M^3_{a_2} \overset{\mathrm{exp}}{=} 1.0(1)\,\mathrm{keV}$$

Results: $\gamma\gamma \to \pi\eta$





- Scrutinise the uncertainties of our **hadronic input** using $g_{VP\gamma}$ and fitting $\gamma\gamma \to \pi\eta$ data
- Total and differential cross sections are in good agreement with the data

Two-photon coupling

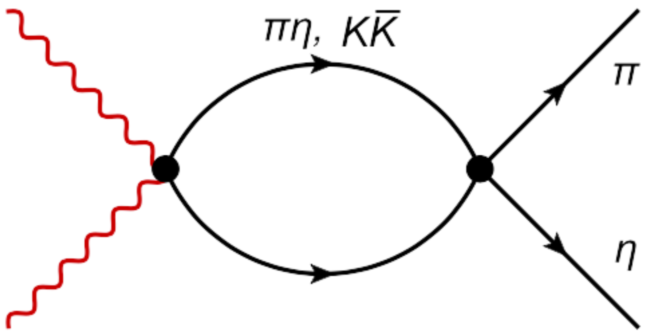
For the pole on the IV Riemann sheet unitarity implies

$$h^{IV}_{\gamma\gamma\to\pi\eta}(s) - h^{I}_{\gamma\gamma\to\pi\eta}(s) = 2\,i\,\rho_{K\bar{K}}(s)\,h^{I}_{\gamma\gamma\to K\bar{K}}(s)\,t^{IV}_{K\bar{K}\to\pi\eta}(s)$$

In the vicinity of the pole one can write

$$h^{IV}_{\gamma\gamma\to\pi\eta}(s) \simeq \frac{C_{\gamma\gamma}\,C_{\pi\eta}}{s^{\mathrm{IV}}_{a_0} - s}\,, \quad t^{IV}_{K\bar{K}\to\pi\eta}(s) \simeq \frac{C_{K\bar{K}}C_{\pi\eta}}{s^{\mathrm{IV}}_{a_0} - s}$$

$$\left(\frac{C_{\gamma\gamma}}{C_{K\bar{K}}}\right)^2 = -(2\rho_{K\bar{K}}(s^{IV}_{a_0}))^2(t^{I}_{\gamma\gamma\to K\bar{K}}(s^{IV}_{a_0}))^2$$



The two photon decay width

$$\Gamma_{\gamma\gamma} = \frac{|C_{\gamma\gamma}|^2}{16\pi m_0} = 0.27(4)\,\mathrm{keV}$$

Experimental value

$$\Gamma_{a_0\to\gamma\gamma}\mathcal{B}(\pi^0\eta) = 0.21^{+0.08}_{-0.04}\ \mathrm{keV}$$