Light-by-light forward scattering amplitudes in Lattice QCD

Antoine Gérardin

Institut für Kernphysik, Johannes Gutenberg-Universität Mainz

In collaboration with J. Green, O. Gryniuk, G. von Hippel, H. Meyer, V. Pascalutsa and H. Wittig

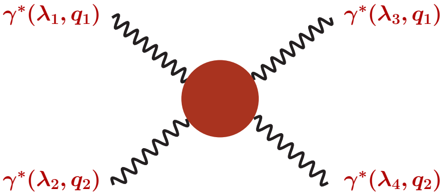
March 14, 2018

Muon g-2 Theory Initiative Hadronic Light-by-Light working group workshop

Motivations

- Model calculations and dispersive approach
  - ↪ The pion-pole contribution to the HLbL scattering in the $(g-2)_\mu$ is dominant
  - ↪ η and η' are not negligible : their contribution can be computed using a similar method (more challenging)
  - ↪ Other contributions : more difficult on the lattice (resonances : require a dedicated study)

- Light-by-light forward scattering amplitudes



$$\gamma^*(\lambda_1, q_1)\,\gamma^*(\lambda_2, q_2) \to \gamma^*(\lambda_3, q_1)\,\gamma^*(\lambda_4, q_2)$$

  - ↪ Can be computed on the lattice (involve spacelike photons)
  - ↪ Independent calculation : no QED kernel needed here
  - ↪ But the lattice object to be computed is the same : four-point correlation function

Strategy :

1) Compute all the forward LbL scattering amplitudes on the lattice [Green et. al '15]
2) Use a simple model to describe the lattice data (input : TFFs)
3) Extract information about TFFs by fitting the model parameters to lattice data

- ↪ The LbL forward scattering amplitudes contain more information than the scalar a_μ^{HLbL}
- ↪ Can obtain information about TFFs of resonances in an indirect way [de Rafael '94, Colangelo et. al '14]
- ↪ Complementary to the direct lattice QCD calculation [RBC-UKQCD, Mainz group]

Outline

1) Overall strategy

2) Lattice calculation and assumptions

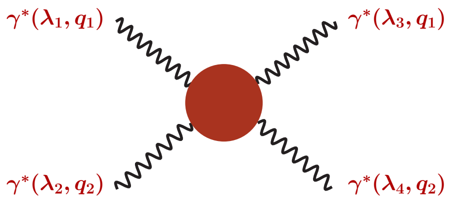
3) Description of the phenomenological model used to describe lattice amplitudes

4) Results

Overall Strategy

Light-by-light scattering amplitudes

- Forward scattering amplitudes $M_{\lambda_3\lambda_4\lambda_1\lambda_2}$: $\gamma^*(\lambda_1, q_1)\ \gamma^*(\lambda_2, q_2)\ \rightarrow\ \gamma^*(\lambda_3, q_1)\ \gamma^*(\lambda_4, q_2)$



- 81 helicity amplitudes ($\lambda_i = 0, \pm 1$)

$$\mathcal{M}_{\lambda'_1\lambda'_2\lambda_1\lambda_2} = \mathcal{M}_{\mu\nu\rho\sigma}\ \epsilon^{*\mu}(\lambda'_1)\ \epsilon^{*\nu}(\lambda'_2)\ \epsilon^{\rho}(\lambda_1)\ \epsilon^{\sigma}(\lambda_2)$$

- Photons virtualities : $Q_1^2 = -q_1^2 > 0$ and $Q_2^2 = -q_2^2 > 0$
- Cross symmetric variable : $\nu = q_1 \cdot q_2$

- Using parity and time invariance : only 8 independent amplitudes

$$(\mathcal{M}_{++,++} + \mathcal{M}_{+-,+-}),\ \mathcal{M}_{++,--},\ \mathcal{M}_{00,00},\ \mathcal{M}_{+0,+0},\ \mathcal{M}_{0+,0+},\ (\mathcal{M}_{++,00} + \mathcal{M}_{0+,-0}),$$
$$(\mathcal{M}_{++,++} - \mathcal{M}_{+-,+-}),\ (\mathcal{M}_{++,00} - \mathcal{M}_{0+,-0})$$

↪ Either even or odd with respect to ν

↪ The eight amplitudes have been computed on the lattice for different values of ν, Q_1^2, Q_2^2

- Relate the forward amplitudes to two-photon fusion cross sections using the optical theorem [Pascalutsa et. al '12]

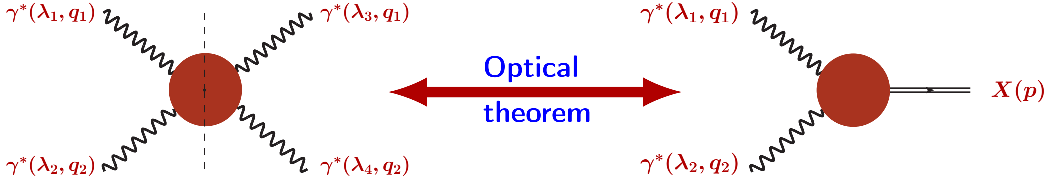
↪ Eight independent dispersion relations for $\mathcal{M}_{TT}$, $\mathcal{M}^t_{TT}$, $\mathcal{M}^a_{TT}$, $\mathcal{M}_{TL}$, $\mathcal{M}_{LT}$, $\mathcal{M}^a_{TL}$, $\mathcal{M}^t_{TL}$ and $\mathcal{M}_{LL}$

[Carlson et al.'71] [Brown et al. '71]

Dispersion relations

1) Optical theorem

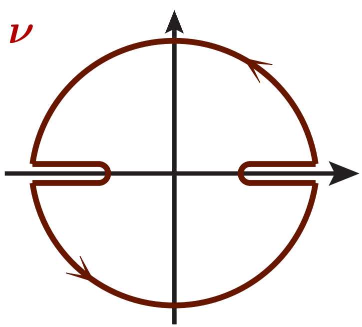
Imaginary part
$\mathcal{M}_{\lambda_3\lambda_4,\lambda_1\lambda_2}$



$\hookrightarrow$ X can be any C-even final state

$$W_{\lambda_3\lambda_4,\lambda_1\lambda_2} = \mathrm{Abs}\,\mathcal{M}_{\lambda_3\lambda_4,\lambda_1\lambda_2} = \frac{1}{2}\int \mathrm{d}\Gamma_X (2\pi)^4 \delta(q_1 + 1_2 - p_X)\mathcal{M}_{\lambda_1\lambda_2}(q_1, q_2, p_X)\,\mathcal{M}^*_{\lambda_3\lambda_4}(q_1, q_2, p_X)$$

2) Dispersion relations [Pascalutsa et. al '12]



Once-subtracted sum rules : cross-symmetric variable $\nu = q_1 \cdot q_2$

$$\mathcal{M}_{\mathrm{even}}(\nu) = \mathcal{M}_{\mathrm{even}}(0) + \frac{2\nu^2}{\pi}\int_{\nu_0}^{\infty} d\nu' \frac{1}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} W_{\mathrm{even}}(\nu')$$

$$\mathcal{M}_{\mathrm{odd}}(\nu) = \nu\mathcal{M}_{\mathrm{odd}}(\nu) + \frac{2\nu^3}{\pi}\int_{\nu_0}^{\infty} d\nu' \frac{1}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} W_{\mathrm{odd}}(\nu')$$

3) Higher mass singularities are suppressed with ν^2 :

$\hookrightarrow$ Only a few states X are necessary to saturate the sum rules and reproduce the lattice data

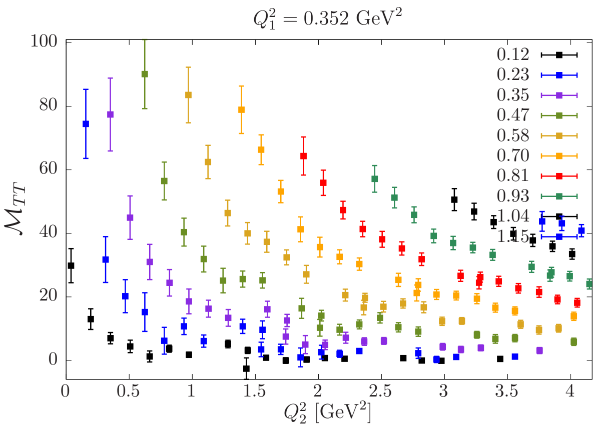
Description of the lattice data using phenomenology

→ For each of the 8 amplitudes, we have a dispersion relation ($\sigma_\alpha/\tau_\alpha(\nu')$: cross-section / interference term) :

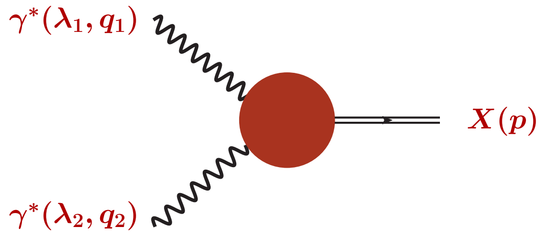
$$\overline{\mathcal{M}}_\alpha(\nu) = \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{X'}\,\sigma_\alpha/\tau_\alpha(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$

Lattice calculation

↪ 4-pt correlation function



$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \mathrm{X}(p_\mathrm{X})$ fusion cross sections



↪ X : any C-even states contribute

↪ Main contribution is expected from mesons :

Pseudoscalars (0^{-+}) Axial-vectors (1^{++})
Scalar (0^{++}) Tensors (2^{++})

↪ Described in terms of TFFs

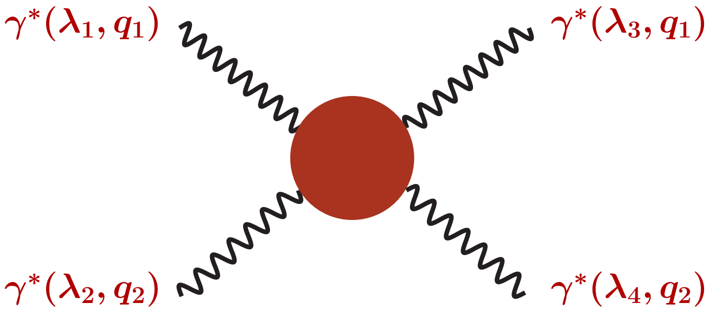
Contributions to the eight independent amplitudes

$$\overline{\mathcal{M}}(\nu) = \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{X'}\,\sigma/\tau(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$

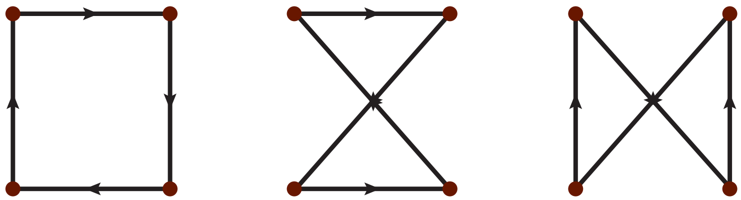
Amplitude	Pseudoscalar	Scalar	Axial	Tensor	Scalar QED
$\mathcal{M}_{TT}$	$\sigma_0/2$	$\sigma_0/2$	$\sigma_0/2$	$\frac{\sigma_0+\sigma_2}{2}$	σ_{TT}
$\mathcal{M}^t_{TT}$	$-\sigma_0$	σ_0	$-\sigma_0$	σ_0	τ_{TT}
$\mathcal{M}^a_{TT}$	$\sigma_0/2$	$\sigma_0/2$	$\sigma_0/2$	$\frac{\sigma_0-\sigma_2}{2}$	τ^a_{TT}
$\mathcal{M}_{TL}$	$\times$	$\times$	σ_{TL}	σ_{TL}	σ_{TL}
$\mathcal{M}_{LT}$	$\times$	$\times$	σ_{LT}	σ_{LT}	σ_{LT}
$\mathcal{M}^t_{TL}$	$\times$	τ_{TL}	τ_{TL}	τ_{TL}	τ_{TL}
$\mathcal{M}^a_{TL}$	$\times$	τ_{TL}	$-\tau_{TL}$	τ^a_{TL}	τ^a_{TL}
$\mathcal{M}_{LL}$	$\times$	σ_{LL}	$\times$	σ_{LL}	σ_{LL}

- σ : physical cross-section (positive contributions)
- τ : interference terms

Lattice calculation of the amplitudes



Lattice calculation



- Four-point correlation function

$$\Pi^{E}_{\mu\nu\rho\sigma}(Q_1,Q_2) = \sum_{X_1,X_2,X_3} \langle J^c_\mu(X_1)\, J^c_\nu(X_2)\, J^l_\rho(X_3)\, J^c_\sigma(0)\rangle \; e^{iQ_1(X_1-X_3)}\; e^{iQ_2 X_2} \; + \; \text{contact terms}$$

↪ $J_\mu(x) = \frac{2}{3}\overline{u}(x)\gamma_\mu u(x) - \frac{1}{3}\overline{d}(x)\gamma_\mu d(x)$ (3 conserved + 1 local vecor currents)

↪ Method of sequential propagators (two values of Q_1^2, all Q_2 such that $Q_2^2 < 4\ \mathrm{GeV}^2$)

↪ The eight amplitudes are obtained from

$$\mathcal{M}(q_1^2, q_2^2, \nu) = e^4\, T^{E}_{\mu\nu\mu'\nu'}(Q_1,Q_2)\, \Pi^{E}_{\mu\nu\mu'\nu'}(Q_1,Q_2)$$

- Five CLS ensembles ($N_f = 2$ dynamical quarks, $\mathcal{O}(a)$-improved Wilson-Clover)

↪ 4 ensembles at $a \approx 0.065$ fm and m_π down to 180 MeV

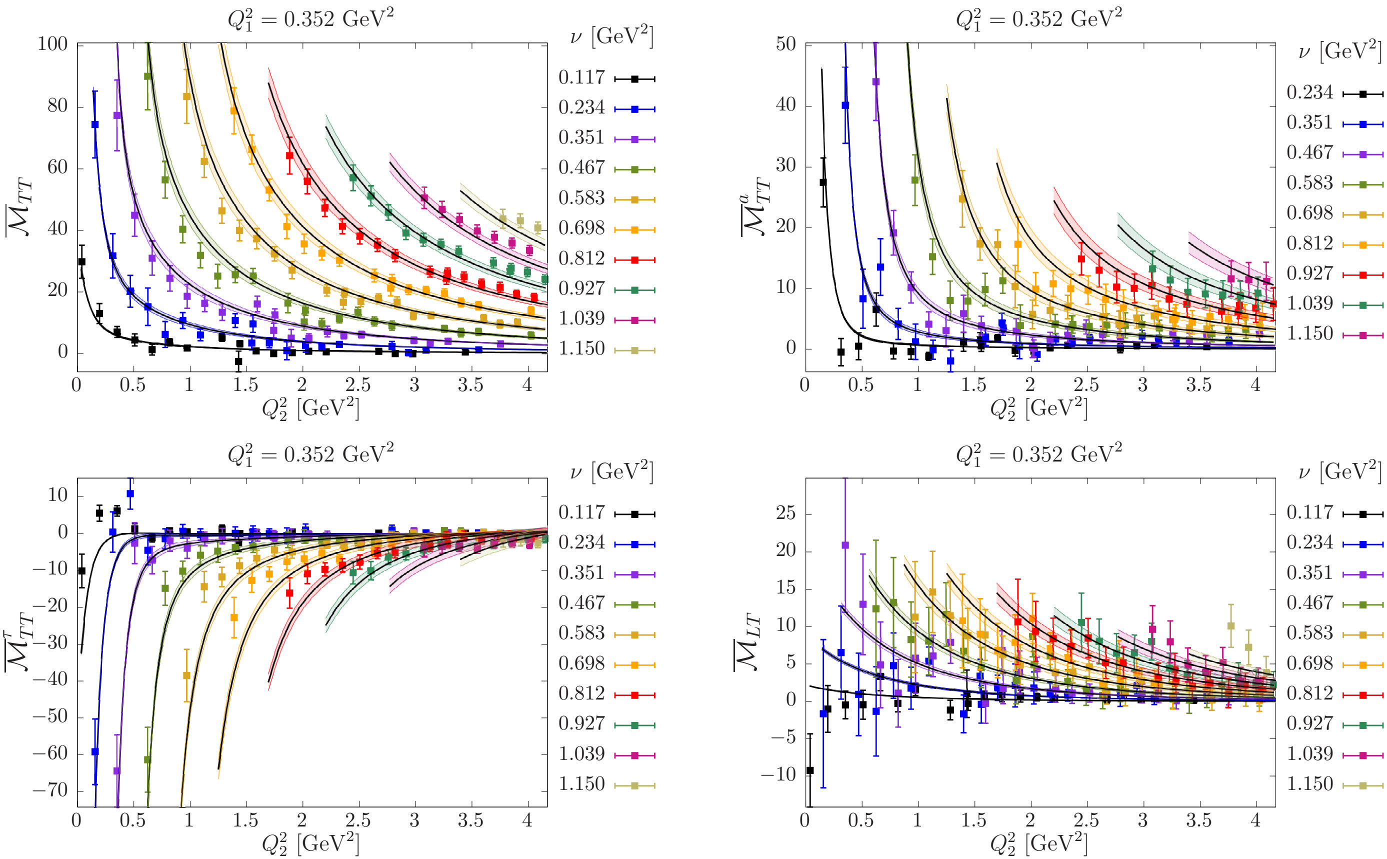
↪ 1 ensemble at $a \approx 0.048$ fm

- Only the fully-connected diagrams are included for all the ensembles

- Leading 2+2 disconnected diagrams computed for two ensembles

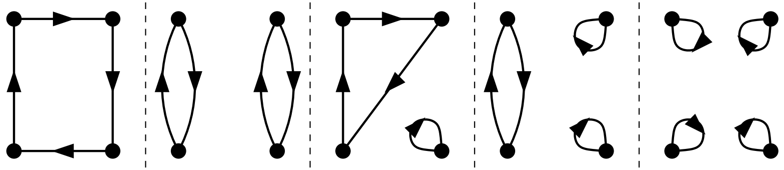
Lattice QCD amplitudes

- Results for a given ensemble at a given value of $Q_1^2 = 0.352\ \text{GeV}^2$
- Four of the eight lattice amplitudes



Connected and disconnected contributions : flavor structure

- There are five different topologies :



- Wick contractions lead to :

$$\Pi^{\mathrm{HLbL}} = \sum_{f} \mathcal{Q}_f^4 \ \Pi^4 + \sum_{f_1,f_2} \mathcal{Q}_{f_1}^2 \mathcal{Q}_{f_2}^2 \ \Pi^{2+2} + \sum_{f_1,f_2} \mathcal{Q}_{f_1}^3 \mathcal{Q}_{f_2} \ \Pi^{3+1} + \sum_{f_1,f_2,f_3} \mathcal{Q}_{f_1}^2 \mathcal{Q}_{f_2} \mathcal{Q}_{f_3} \ \Pi^{2+1+1} + \sum_{f_1,f_2,f_3,f_4} \mathcal{Q}_{f_1} \mathcal{Q}_{f_2} \mathcal{Q}_{f_3} \mathcal{Q}_{f_4} \ \Pi^{1+1+1+1}$$

- The contribution to Π^{HLbL} of an isovector (isoscalar) resonance M_1 (M_0) can be written as

$$\Pi^{\mathrm{HLbL}}(M_1) = (\mathcal{Q}_u^2 - \mathcal{Q}_d^2)^2 \ \Pi_{M_1}$$
$$\Pi^{\mathrm{HLbL}}(M_0) = (\mathcal{Q}_u^2 + \mathcal{Q}_d^2) \ \Pi_A + (\mathcal{Q}_u + \mathcal{Q}_d)^2 (\mathcal{Q}_u^2 + \mathcal{Q}_d^2) \ \Pi_B + (\mathcal{Q}_u + \mathcal{Q}_d)^4 \ \Pi_C$$

$\rightarrow$ consequence of the isospin decomposition of the electromagnetic current :

$$J_\mu^{\mathrm{e.m.}} = J_\mu^1 + J_\mu^0, \qquad J_\mu^1 = \frac{\mathcal{Q}_u - \mathcal{Q}_d}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \qquad J_\mu^0 = \frac{\mathcal{Q}_u + \mathcal{Q}_d}{2} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d).$$

$$\mathcal{F}_{M_1\gamma^*\gamma^*} = (\mathcal{Q}_u^2 - \mathcal{Q}_d^2) \ \mathcal{F} \ , \quad \mathcal{F}_{M_0\gamma^*\gamma^*} = (\mathcal{Q}_u^2 + \mathcal{Q}_d^2) \ \mathcal{F}_C + (\mathcal{Q}_u + \mathcal{Q}_d)^2 \ \mathcal{F}_D$$

Connected and disconnected contributions : flavor structure

- Identification of the polynomials in $\mathcal{Q}_u$ and $\mathcal{Q}_d$: leads to two sets of three equations

$$\begin{cases} \Pi_{M_1} = \Pi^{4}_{M_1} + \Pi^{2+2}_{M_1} + \Pi^{3+1}_{M_1} + \Pi^{2+1+1}_{M_1} + \Pi^{1+1+1+1}_{M_1} \\ -\Pi_{M_1} = 0 + \Pi^{2+2}_{M_1} + 0 + \Pi^{2+1+1}_{M_1} + 3\Pi^{1+1+1+1}_{M_1} \\ 0 = 0 + 0 + \Pi^{3+1}_{M_1} + 2\Pi^{2+1+1}_{M_1} + 4\Pi^{1+1+1+1}_{M_1} \end{cases}$$

$$\begin{cases} \Pi_A + \Pi_B + \Pi_C = \Pi^{4}_{M_0} + \Pi^{2+2}_{M_0} + \Pi^{3+1}_{M_0} + \Pi^{2+1+1}_{M_0} + \Pi^{1+1+1+1}_{M_0} \\ \Pi_A + \Pi_B + 3\Pi_C = 0 + \Pi^{2+2}_{M_0} + 0 + \Pi^{2+1+1}_{M_0} + 3\Pi^{1+1+1+1}_{M_0} \\ 3\Pi_B + 4\Pi_C = 0 + 0 + \Pi^{3+1}_{M_0} + 2\Pi^{2+1+1}_{M_0} + 4\Pi^{1+1+1+1}_{M_0} \end{cases}$$

Connected and disconnected contributions : flavor structure

- Identification of the polynomials in $\mathcal{Q}_u$ and $\mathcal{Q}_d$: leads to two sets of three equations

$$\begin{cases} \Pi_{M_1} = \Pi^4_{M_1} + \Pi^{2+2}_{M_1} + \Pi^{3+1}_{M_1} + \Pi^{2+1+1}_{M_1} + \Pi^{1+1+1+1}_{M_1} \\ -\Pi_{M_1} = 0 + \Pi^{2+2}_{M_1} + 0 + \Pi^{2+1+1}_{M_1} + 3\Pi^{1+1+1+1}_{M_1} \\ 0 = 0 + 0 + \Pi^{3+1}_{M_1} + 2\Pi^{2+1+1}_{M_1} + 4\Pi^{1+1+1+1}_{M_1} \end{cases}$$

$$\begin{cases} \Pi_A + \Pi_B + \Pi_C = \Pi^4_{M_0} + \Pi^{2+2}_{M_0} + \Pi^{3+1}_{M_0} + \Pi^{2+1+1}_{M_0} + \Pi^{1+1+1+1}_{M_0} \\ \Pi_A + \Pi_B + 3\Pi_C = 0 + \Pi^{2+2}_{M_0} + 0 + \Pi^{2+1+1}_{M_0} + 3\Pi^{1+1+1+1}_{M_0} \\ 3\Pi_B + 4\Pi_C = 0 + 0 + \Pi^{3+1}_{M_0} + 2\Pi^{2+1+1}_{M_0} + 4\Pi^{1+1+1+1}_{M_0} \end{cases}$$

- Assume that all disconnected contributions with at least one isolated quark loop are negligible (then $\Pi_A \approx \Pi_{M_0}$)

$$\begin{cases} \Pi_{M_1} + \Pi_{M_0} \approx (\Pi^4_{M_1} + \Pi^4_{M_1}) + (\Pi^{2+2}_{M_1} + \Pi^{2+2}_{M_0}) \\ -\Pi_{M_1} + \Pi_{M_0} \approx (\Pi^{2+2}_{M_1} + \Pi^{2+2}_{M_0}) \end{cases}$$

Connected and disconnected contributions : flavor structure

- Identification of the polynomials in $\mathcal{Q}_u$ and $\mathcal{Q}_d$: leads to two sets of three equations

$$\begin{cases} \Pi_{M_1} = \Pi^4_{M_1} + \Pi^{2+2}_{M_1} + \Pi^{3+1}_{M_1} + \Pi^{2+1+1}_{M_1} + \Pi^{1+1+1+1}_{M_1} \\ -\Pi_{M_1} = 0 + \Pi^{2+2}_{M_1} + 0 + \Pi^{2+1+1}_{M_1} + 3\Pi^{1+1+1+1}_{M_1} \\ 0 = 0 + 0 + \Pi^{3+1}_{M_1} + 2\Pi^{2+1+1}_{M_1} + 4\Pi^{1+1+1+1}_{M_1} \end{cases}$$

$$\begin{cases} \Pi_A + \Pi_B + \Pi_C = \Pi^4_{M_0} + \Pi^{2+2}_{M_0} + \Pi^{3+1}_{M_0} + \Pi^{2+1+1}_{M_0} + \Pi^{1+1+1+1}_{M_0} \\ \Pi_A + \Pi_B + 3\Pi_C = 0 + \Pi^{2+2}_{M_0} + 0 + \Pi^{2+1+1}_{M_0} + 3\Pi^{1+1+1+1}_{M_0} \\ 3\Pi_B + 4\Pi_C = 0 + 0 + \Pi^{3+1}_{M_0} + 2\Pi^{2+1+1}_{M_0} + 4\Pi^{1+1+1+1}_{M_0} \end{cases}$$

- Assume that all disconnected contributions with at least one isolated quark loop are negligible (then $\Pi_A \approx \Pi_{M_0}$)

$$\begin{cases} \Pi_{M_1} + \Pi_{M_0} \approx (\Pi^4_{M_1} + \Pi^4_{M_1}) + (\Pi^{2+2}_{M_1} + \Pi^{2+2}_{M_0}) \\ -\Pi_{M_1} + \Pi_{M_0} \approx (\Pi^{2+2}_{M_1} + \Pi^{2+2}_{M_0}) \end{cases}$$

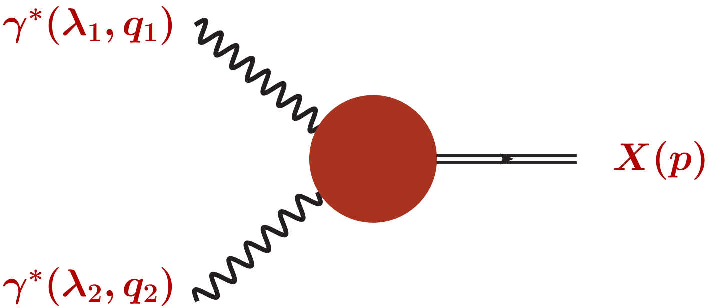
- Then the contribution to the fully connected and $2+2$ disconnected contributions read

$$(Q_u^4 + Q_d^4)\Pi^4_{M_1+M_0} \approx 2\ (Q_u^4 + Q_d^4)\Pi_{M_1} \approx 2\frac{Q_u^4 + Q_d^4}{(Q_u^2 - Q_d^2)^2}\ \Pi^{\mathrm{HLbL}}(M_1) \approx \frac{34}{9}\Pi^{\mathrm{HLbL}}(M_1)$$

$$(Q_u^2 + Q_d^2)^2\Pi^{2+2}_{M_1+M_0} \approx -\frac{(Q_u^2 + Q_d^2)^2}{(Q_u^2 - Q_d^2)^2}\ \Pi^{\mathrm{HLbL}}(M_1) + \Pi^{\mathrm{HLbL}}(M_0) \approx -\frac{25}{9}\Pi^{\mathrm{HLbL}}(M_1) + \Pi^{\mathrm{HLbL}}(M_0)$$

$\hookrightarrow$ This was already noticed by [Bijnens '16] using large-N_c arguments (See also talk from Thuesday)

Description of the model to describe lattice data



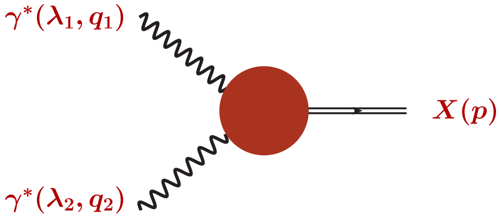
Consequences : particle content

- For the fully connected contribution :
  ↪ we consider only isovector resonances but with a factor $34/9$
- Higher mass resonances contributions are suppressed in the dispersion relations
  ↪ consider only the lightest state in each channel
- We work with $N_f = 2$ lattice simulations : no η !

	Isovector	Isoscalar	Isoscalar
0^{-+}	π	η'	η
0^{++}	$a_0(980)$	$f_0(980)$	$f_0(600)$
1^{++}	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$
2^{++}	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$

Two-photon fusion cross sections : modelisation

$$\overline{\mathcal{M}}(\nu) = \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{X'}\, \sigma/\tau(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$



- Example : contribution of the pseudoscalar to the amplitude M_{TT}

$$\sigma_{TT} = 8\pi^2 \delta(s - m_P^2) \frac{2\sqrt{X}}{m_P^2} \times \frac{\Gamma_{\gamma\gamma}}{m_P} \times \left[\frac{F_{\mathcal{P}\gamma^*\gamma^*}(Q_1^2, Q_2^2)}{F_{\mathcal{P}\gamma^*\gamma^*}(0,0)} \right]^2$$

- Similar results for other mesons (assume Breit-Wigner shape for resonances)

- We assume a constant mass shift in the spectrum (scalar, axial, tensor)

$$m_X = m_X^{\text{phys}} + (m_\rho^{\text{lat}} - m_\rho^{\text{phys}}) \quad , \quad X = A, S, T$$

- The two-photons decay width $\Gamma_{\gamma\gamma} = \frac{\pi\alpha^2}{4} m_S \left[F^T_{\mathcal{S}\gamma^*\gamma^*}(0,0) \right]^2$ is taken from experiment

All the non-perturbative information is encoded into the meson transition form factors

Assumptions on form factors

- Pseudoscalar meson

↪ experimental data are available only when at least one photon is on-shell

↪ $F_{\mathcal{P}\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ has been computed on the lattice [Gerardin, Nyffeler, Meyer '16]

↪ We use the lattice result (no free parameter is introduced)

- Scalar mesons

↪ can be produced by two transverse photons (T) or two longitudinal photons (L) : $F^T_{\mathcal{S}\gamma^*\gamma^*}$ and $F^L_{\mathcal{S}\gamma^*\gamma^*}$

↪ $F^T_{\mathcal{S}\gamma^*\gamma^*}$ has been measured experimentally in the region $Q^2 < 30~\mathrm{GeV}^2$ for the $f_0(980)$ meson [Masuda '15]

↪ results are compatible with a monopole form factor with $M_S = 0.800(50)~\mathrm{MeV}$

$$\frac{F^T_{\mathcal{S}\gamma^*\gamma^*}(Q_1^2, Q_2^2)}{F^T_{\mathcal{S}\gamma^*\gamma^*}(0,0)} = \frac{1}{(1+Q_1^2/M_S^2)(1+Q_2^2/M_S^2)}$$

↪ In the following, M_S is considered as a free parameter

Assumptions on form factors

- Axial mesons

↪ Two form factors $F^{(0)}_{\mathcal{A}\gamma^*\gamma^*}$ and $F^{(1)}_{\mathcal{A}\gamma^*\gamma^*}$ ($\Lambda = 0, 1$ corresponds to the two helicity states of the axial meson)

↪ Quark model inspired parametrisation [N. Cahn '87]

$$F^{(0)}_{\mathcal{A}\gamma^*\gamma^*}(Q_1^2, Q_2^2) = m_A^2 A(Q_1^2, Q_2^2)\,,$$
$$F^{(1)}_{\mathcal{A}\gamma^*\gamma^*}(Q_1^2, Q_2^2) = -\frac{\nu}{X}\left(\nu + Q_2^2\right)\, m_A^2 A(Q_1^2, Q_2^2)\,,$$
$$F^{(1)}_{\mathcal{A}\gamma^*\gamma^*}(Q_2^2, Q_1^2) = -\frac{\nu}{X}\left(\nu + Q_1^2\right)\, m_A^2 A(Q_1^2, Q_2^2)$$

in which $2\nu = m_A^2 + Q_1^2 + Q_2^2$ with m_A the meson mass,

$$\frac{A(Q_1^2, 0)}{A(0,0)} = \frac{1}{(1 + Q_1^2/M_A^2)^2}\,,$$

↪ Single virtuel case : L3 Collaboration in the region $Q^2 < 5\ \mathrm{GeV}^2$ [Achard '01 '07]

↪ $M_A = 1040(78)\ \mathrm{MeV}$ for the $f_1(1285)$ meson

↪ One free fit parameter : M_A

Assumptions on form factors

- Tensor mesons

↪ Amplitudes are described by four form factors $F^{(\Lambda)}_{\mathcal{T}\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ with $\Lambda = (0,T),\ (0,L),\ 1,\ 2$

↪ The single-virtual form factors for helicities $\Lambda = (0,T), 1, 2$ have also been measured experimentally in the region $Q^2 < 30\ \mathrm{GeV}^2$ by the Belle Collaboration [Masuda '15]

↪ data are compatible with a dipole form factor [Danilkin '16]

$$\frac{F^{(\Lambda)}_{\mathcal{T}\gamma^*\gamma^*}(Q_1^2, Q_2^2)}{F^{(\Lambda)}_{\mathcal{T}\gamma^*\gamma^*}(0,0)} = \frac{1}{(1 + Q_1^2/M^2_{T,(\Lambda)})^2(1 + Q_2^2/M^2_{T,(\Lambda)})^2}$$

↪ Four free fit parameters : $M_{T,(\Lambda)}$

- Scalar QED

$\gamma^*\gamma^* \to \pi^+\pi^-$ evaluated using scalar QED dressed with monopole form factors

↪ Monopole mass set to the (lattice) rho mass

- Conclusion

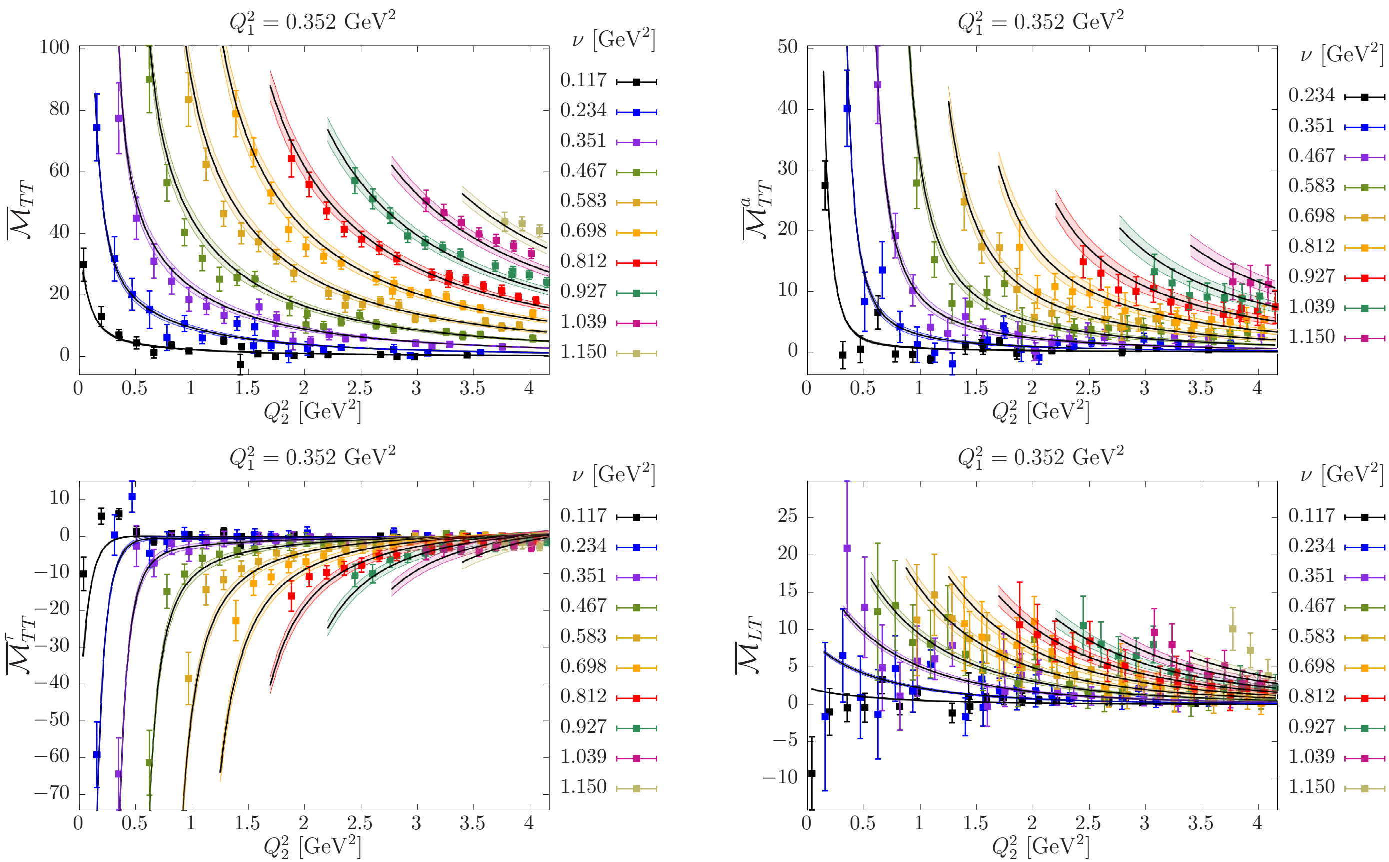
The model has 6 independent parameters (monopole and dipole masses of TFFs)

↪ global fit of the model to the eight lattice amplitudes (for all Q_1^2 and Q_2^2)

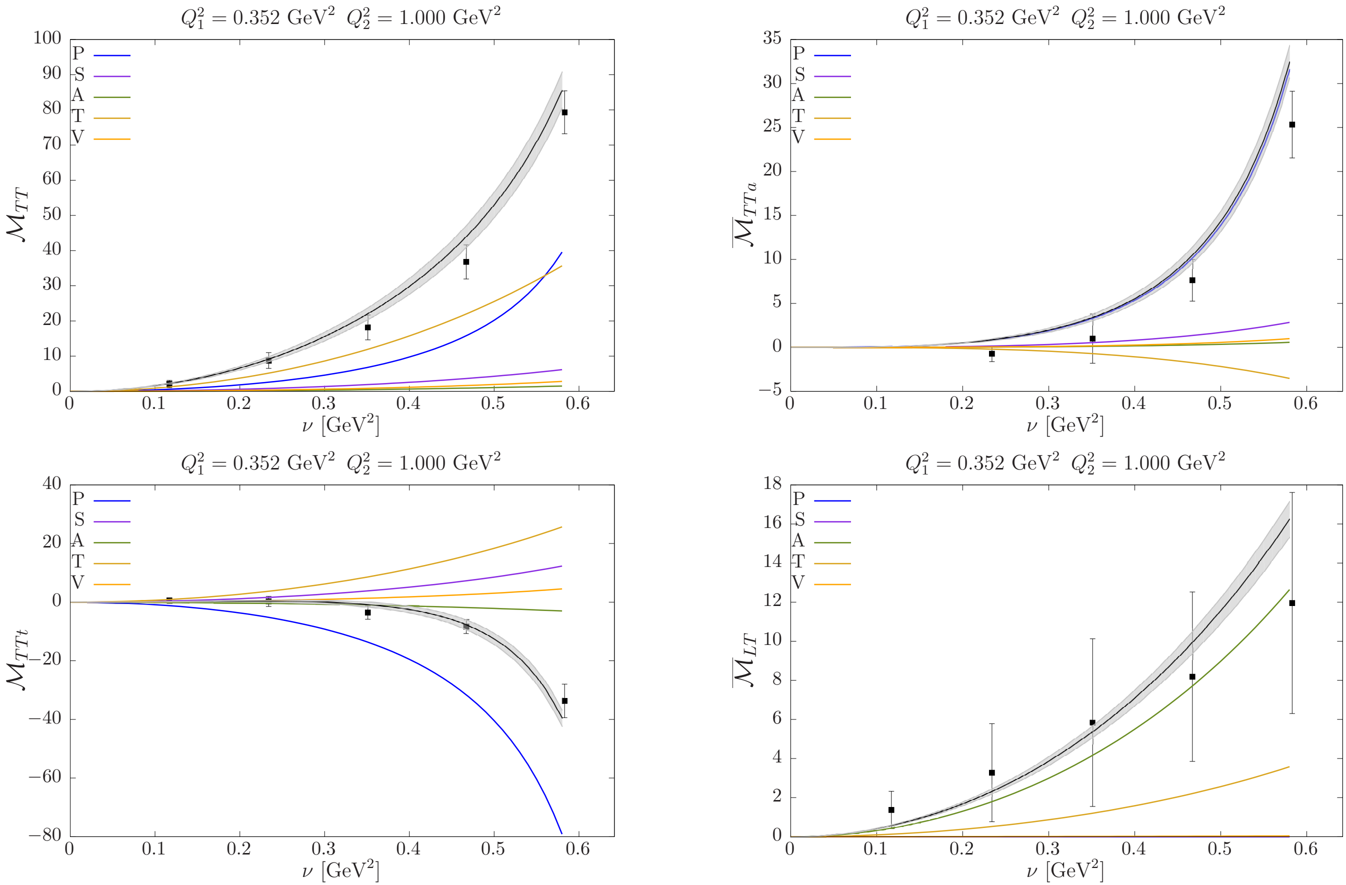
Results

Results on F7 - dependance on ν and Q_2^2

- Each plot correspond to a fixed Q_1^2
- Different colours correspond to different values of $\nu = Q_1^2 \cdot Q_2^2$



Preliminary results : F7 - contributions from different channels



Monopole and dipole masses

Monopole FF

$$F_X(Q_1^2,Q_2^2) = \frac{F_X(0,0)}{\left(1+Q_1^2/\Lambda_X^2\right)\left(1+Q_2^2/\Lambda_X^2\right)}$$

,

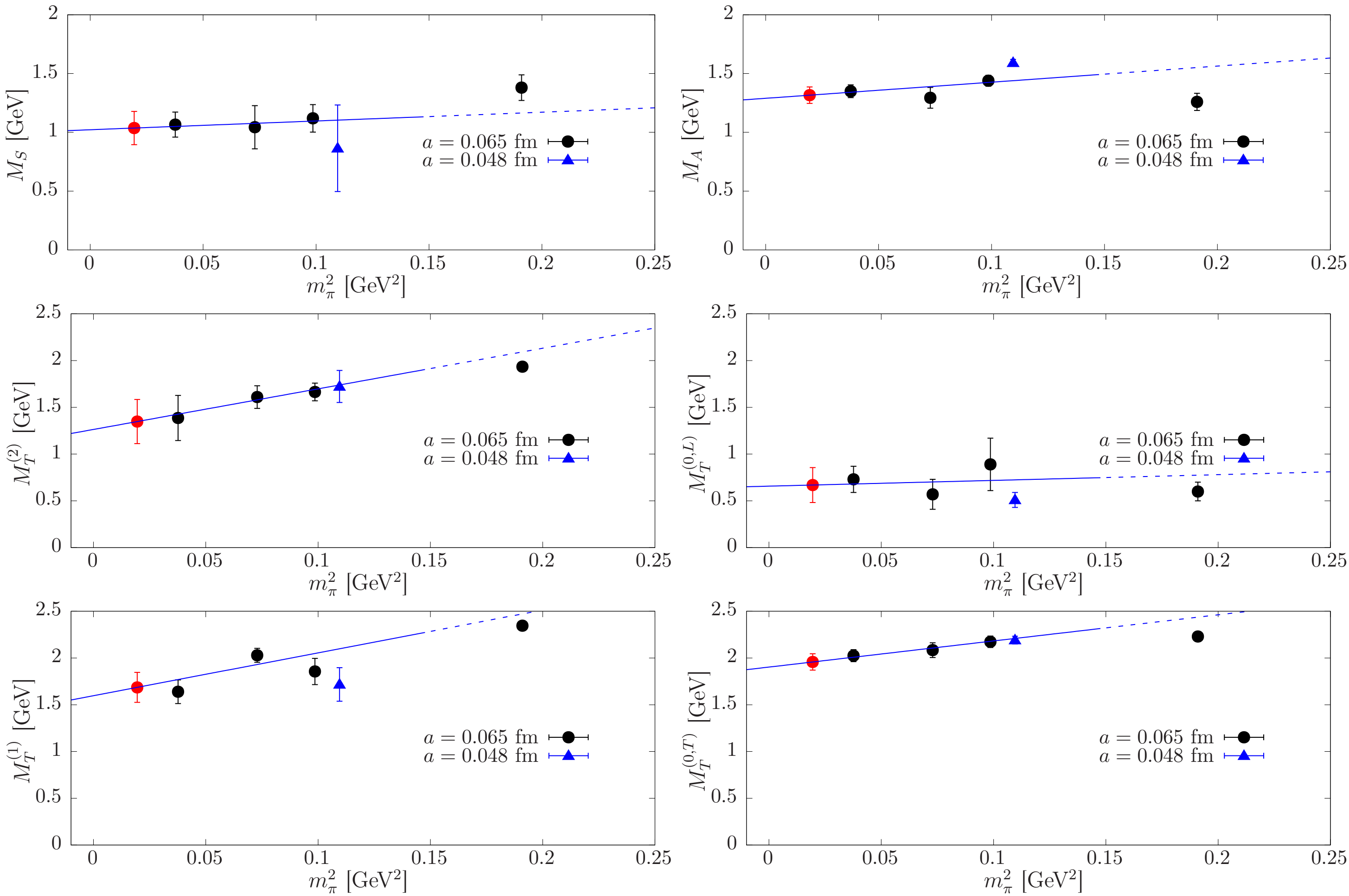
Dipole FF

$$F_X(Q_1^2,Q_2^2) = \frac{F_X(0,0)}{\left(1+Q_1^2/\Lambda_X^2\right)^2\left(1+Q_2^2/\Lambda_X^2\right)^2}$$

- Global fit of the eight amplitudes

	M_S [GeV]	M_A [GeV]	$M_T^{(2)}$ [GeV]	$M_T^{(0,T)}$ [GeV]	$M_T^{(1)}$ [GeV]	$M_T^{(0,L)}$ [GeV]	χ^2/d.o.f
E5	1.38(11)	1.26(10)	1.93(3)	2.24(5)	2.36(4)	0.60(10)	4.22
F6	1.12(14)	1.44(5)	1.66(9)	2.17(5)	1.85(14)	0.89(28)	1.15
F7	1.04(18)	1.29(8)	1.61(12)	2.08(7)	2.03(7)	0.57(16)	1.19
G8	1.07(10)	1.36(5)	1.37(24)	2.03(6)	1.63(13)	0.73(14)	1.13
N6	0.86(37)	1.59(3)	1.72(17)	2.19(4)	1.72(18)	0.51(8)	1.35

Monopole and dipole masses : chiral extrapolations (preliminary, stat error only)



Preliminary results : monopole and dipole masses

Monopole FF **Dipole FF**

$$F_X(Q_1^2, Q_2^2) = \frac{F_X(0,0)}{(1 + Q_1^2/\Lambda_X^2)\,(1 + Q_2^2/\Lambda_X^2)} \quad , \quad F_X(Q_1^2, Q_2^2) = \frac{F_X(0,0)}{(1 + Q_1^2/\Lambda_X^2)^2\,(1 + Q_2^2/\Lambda_X^2)^2}$$

- Global fit of the eight amplitudes

	M_S [GeV]	M_A [GeV]	$M_T^{(2)}$ [GeV]	$M_T^{(0,T)}$ [GeV]	$M_T^{(1)}$ [GeV]	$M_T^{(0,L)}$ [GeV]	χ^2/d.o.f
E5	1.38(11)	1.26(10)	1.93(3)	2.24(5)	2.36(4)	0.60(10)	4.22
F6	1.12(14)	1.44(5)	1.66(9)	2.17(5)	1.85(14)	0.89(28)	1.15
F7	1.04(18)	1.29(8)	1.61(12)	2.08(7)	2.03(7)	0.57(16)	1.19
G8	1.07(10)	1.36(5)	1.37(24)	2.03(6)	1.63(13)	0.73(14)	1.13
N6	0.86(37)	1.59(3)	1.72(17)	2.19(4)	1.72(18)	0.51(8)	1.35

- $M_S = 1.04(14)$ GeV : slightly above the experimental result from the Belle Collaboration ($M_S = 796(54)$ MeV for the *isoscalar* scalar meson [Masuda '15])
- $M_A = 1.32(7)$ GeV to be compared with the experimental value by the L3 Collaboration $M_A = 1040(80)$ MeV for the *isoscalar* meson $f_1(1285)$ [Achard '01 '07].
- $M_T^{(2)} = 1.35(24)$ GeV, $M_{0,T}^{(1)} = 1.69(16)$ GeV and $M_{(1)}^{(0,T)} \approx 1.96(9)$ GeV, above the experimental values for the $f_2(1270)$ mesons obtained by fitting the single-virtual form factor [Masuda '15, Danilkin '16].

Fully-connected contribution : test of our assumption

- Only isovector particles contribute to the fully connected diagram with a factor $34/9$

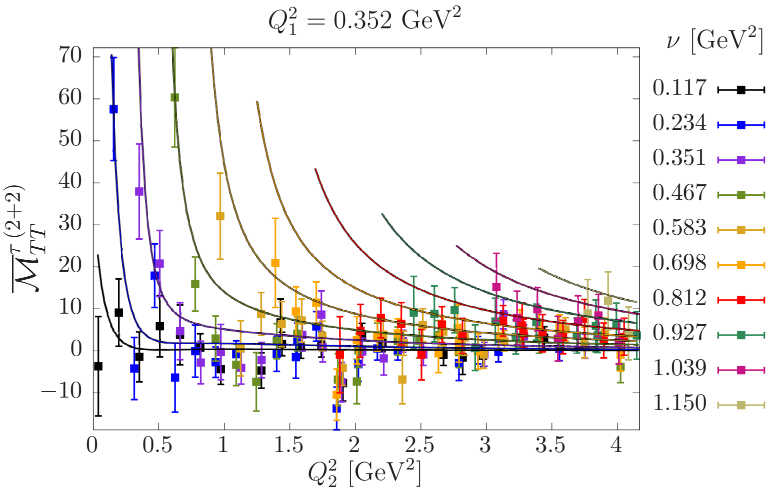
$$(Q_u^4 + Q_d^4)\Pi^4_{M_1+M_0} \approx \frac{34}{9}\Pi^{\mathrm{HLbL}}(M_1)$$
$$(Q_u^2 + Q_d^2)^2\Pi^{2+2}_{M_1+M_0} \approx -\frac{25}{9}\Pi^{\mathrm{HLbL}}(M_1) + \Pi^{\mathrm{HLbL}}(M_0)$$

- Large-N_c approximation :

  → Each J^{PC} sector : the non-singlet resonances cancel the contribution of the flavor-singlet resonances in Π^{2+2}

- Approximation badly broken in the pseudoscalar channel (pion is much lighter than the η' meson) :

$$\mathcal{M}^{(2+2)}_{..} \approx -\frac{25}{9}\mathcal{M}^{(\pi^0)}_{..} + \mathcal{M}^{(\eta')}_{..} \,.$$



We can test this assumption on F6 where we have computed disconnected diagrams :

↪ Reasonable statistical precision for $\mathcal{M}_{TT}$

↪ This is not a fit

↪ Relatively good agreement, but large stat errors

Implications for lattice QCD calculations

1) With $N_f = 2$ dynamical quark, one expects

$$\mathcal{M}_{..}^{(2+2)} \approx -\frac{25}{9}\mathcal{M}_{..}^{(\pi^0)} + \mathcal{M}_{..}^{(\eta')}$$

2) Similar calculation with $N_f = 3$ dynamical quark :

$$\mathcal{M}_{..}^{(2+2)} \approx -2(a_\mu^{\mathrm{HLbL},\pi^0} + a_\mu^{\mathrm{HLbL},\eta}) + a_\mu^{\mathrm{HLbL},\eta'}$$

- LMD+V model for the pion-pole contribution : $a_\mu^{\mathrm{HLbL},\pi^0} = 62.9 \times 10^{-11}$
- VMD model for the η and η' contribution : $a_\mu^{\mathrm{HLbL},\eta} = 14.5 \times 10^{-11}$

$a_\mu^{\mathrm{HLbL},\eta'} = 12.5 \times 10^{-11}$

$$a_\mu^{\mathrm{HLbL},(2+2)} \approx \begin{cases} -\frac{25}{9}a_\mu^{\mathrm{HLbL},\pi^0} + a_\mu^{\mathrm{HLbL},\eta'} = -(162 \pm 27) \cdot 10^{-11} & m_s = \infty, \\ -2(a_\mu^{\mathrm{HLbL},\pi^0} + a_\mu^{\mathrm{HLbL},\eta}) + a_\mu^{\mathrm{HLbL},\eta'} = -(142 \pm 19) \cdot 10^{-11} & m_s = m_{ud}. \end{cases}$$

Assuming the model estimate for the full HLbL is correct $a_\mu^{\mathrm{HLbL}} \approx (102 \pm 39) \cdot 10^{-11}$, one expect

$$a_{\mu,\mathrm{model}}^{\mathrm{HLbL},(4)} \approx \begin{cases} (264 \pm 51) \cdot 10^{-11} & m_s = \infty, \\ (244 \pm 46) \cdot 10^{-11} & m_s = m_{ud}. \end{cases}$$

Conclusion

- We have shown the possibility to compute the eight forward light-by-light amplitudes on the lattice
  $\hookrightarrow$ same 4-pt correlation function as in the full HLbL calculation
- They are well described by the cross sections $\gamma^*\gamma^* \to$ a few resonances, via dispersive sum rules.
- Allows us to put constraints on TFFs used to estimate the HLbL contribution to the $(g-2)_\mu$

  $\hookrightarrow$ The amplitudes contain a lot of informations

  $\hookrightarrow$ 8 independant amplitudes, several kinematics
- It was a first feasibility study ($N_f = 2$ dynamical quarks, no continuum limit)
- It would be interesting to consider the more realistic case with $N_f = 2+1$ dynamical quarks and light quarks ($m_\pi \leq 200\ \mathrm{MeV}$)

  $\hookrightarrow$ extract information about TFFs