μ g-2 Theory Initiative
Hadronic Light by Light WG Workshop

Uconn Phys. Dept., Storrs, CT, USA, March 12-14, 2018

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P pole contribution with RχL including flavor breaking

Adolfo Guevara, Pablo Roig & Juan José Sanz-Cillero

To appear soon

Motivation

- (No needed here for a_μ & a_μ^{HLbL})

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- Despite the remarkable advances in both the **dispersive computation** of a_μ^{HLbL} and its **lattice QCD evaluation**, **phenomenological Lagrangian calculations** are still useful.

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, B. Kubis, JHEP (2014) 091; PLB 738 (2014) 6; JHEP 1509 (2015) 074; PRL 118 (2017) no. 23 232001; JHEP 1704 (2017) 161.

V. Pascalutsa, V. Pauk & M. Vanderhaeghen PRD 85 (2012) 116001; V. Pauk & M. Vanderhaeghen, PRD 90 (2014) no.11, 113012.; J. Green, O. Gryniuk, G. von Hippel, H. B. Meyer and V. Pascalutsa, PRL 115 (2015) no.22, 222003; I. Danilkin & M. Vanderhaeghen, PRD 95 (2017) no. 1, 014019; Hagelstein & Pascalutsa PRL 120 (2018) no.7, 072002.

T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, C. Jung and C. Lehner, PRL 118 (2017) no. 2, 022005; PRD 96 (2017) no. 2, 022005.

A. Nyffeler, PRD 94 (2016) no.5, 053006; A. Gérardin, H. B. Meyer and A. Nyffeler, Phys. Rev. D94 (2016) no.7, 074507.

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From yesterday's J. Bijnens' talk: 'Do as well as you can'

Constrain as much as possible from experiment & theory, use common sense & improve with time.

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- Despite the remarkable advances in both the **dispersive computation** of a_μ^{HLbL} and its **lattice QCD evaluation**, phenomenological Lagrangian calculations are still useful.
- In this spirit, some recent articles have addressed the question:

K. Kampf and J. Novotný, Phys. Rev. D**84** (2011) 014036.

P. Roig, A. Guevara and G. López Castro, Phys. Rev. D**89** (2014) 073016.

H. Czyż, P. Kisza and S. Tracz, Phys. Rev. D **97** (2018) 016006.

(*Classic references results are quoted later on*)

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- Despite the remarkable advances in both the **dispersive computation** of a_μ^{HLbL} and its **lattice QCD evaluation**, **phenomenological Lagrangian calculations** are still useful.

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- In this work we use **chiral invariant Lagrangians** to describe the interactions betwen P^0 ($P=\pi,\eta,\eta'$) and the lightest V and P resonances **including order m_P^2 corrections**.

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- Despite the remarkable advances in both the **dispersive computation** of a_μ^{HLbL} and its **lattice QCD evaluation**, **phenomenological Lagrangian calculations** are still useful.
- In this spirit, some recent articles have addressed the question.
- In this work we use **chiral invariant Lagrangians** to describe the interactions betwen P^0 ($P=\pi,\eta,\eta'$) and the lightest V and P resonances **including order m_P^2 corrections**.
- The combination of **data, chiral symmetry relations and short-distance constraints** allows us to determine with **improved precision** $a_\mu^{P,HLbL}$.

Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{non-R}} + \sum_{R} \left(\mathcal{L}_R^{\mathrm{Kin}} + \mathcal{L}_R\right) + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \ldots$$

Do not contribute at tree level

Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm non-R} + \sum_R \left(\mathcal{L}_R^{\rm Kin} + \mathcal{L}_R\right) + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \ldots$$

In the antisymmetric tensor representation

G. Ecker, J. Gasser, A. Pich, E. De Rafael, Nucl. Phys. B**321** (1989) 311; G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. De Rafael, Phys. Lett. B**223** (1989) 425.

Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm non-R} + \sum_R \left(\mathcal{L}_R^{\rm Kin} + \mathcal{L}_R\right) + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \ldots$$

- **Operators without resonance fields:**

$$\mathcal{L}^{\rm even}_{\rm non-R} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle \,,$$

S. Weinberg, Physica A **96** (1979) 327. J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142; Nucl. Phys. B **250** (1985) 465.

$$\mathcal{L}^{\rm odd}_{\rm non-R} = \mathcal{L}_{WZW} + \sum_{j=7,8,22} C_j^W \mathcal{O}_j^W \,,$$

J. Bijnens, L. Girlanda and P. Talavera, Eur. Phys. J. C**23** (2002) 539.

J. Wess and B. Zumino, Phys. Lett. **37B** (1971) 95.

E. Witten, Nucl. Phys. B **223** (1983) 422.

$\mathcal{O}_7^W$	$i\epsilon_{\mu\nu\alpha\beta}\langle \chi_- f_+^{\mu\nu} f_+^{\alpha\beta}\rangle$
$\mathcal{O}_8^W$	$i\epsilon_{\mu\nu\alpha\beta}\langle \chi_- \rangle \langle f_+^{\mu\nu} f_+^{\alpha\beta}\rangle$
$\mathcal{O}_{22}^W$	$i\epsilon_{\mu\nu\alpha\beta}\langle u^\mu \{\nabla_\rho f_+^{\rho\nu}, f_+^{\alpha\beta}\}\rangle$

Double trace but could be enhanced ($\mathcal{O}_8^W$)

~~U(3)~~ explicitly ($\mathcal{O}_7^W$, $\mathcal{O}_8^W$)

P pole contribution with RχL including flavor breaking

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Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{non}-\text{R}} + \sum_R \left(\mathcal{L}_R^{\text{Kin}} + \mathcal{L}_R\right) + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \ldots$$

- **Operators with one vector resonance field:** (Ecker et. al. '89)

$$\mathcal{L}_R^{\text{Kin}} = -\frac{1}{2}\langle \nabla_\lambda V^{\lambda\nu} \nabla^\rho V_{\rho\nu} \rangle + \frac{1}{4} M_V^2 \langle V_{\mu\nu} V^{\mu\nu} \rangle ,$$

(G_V operator is not quoted)

$$\mathcal{L}_V^{\text{even}} = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{\lambda_V}{\sqrt{2}} \langle V_{\mu\nu} \{ f_+^{\mu\nu}, \chi_+ \} \rangle ,$$

~~U(3)~~ explicitly

$\lambda_6^V = \lambda_V / \sqrt{2}$ in V. Cirigliano, G. Ecker, M. Eidemuller, R. Kaiser, A. Pich and J. Portolés, Nucl. Phys. B **753** (2006) 139.

$$\mathcal{L}_V^{\text{odd}} = \sum_{j=1,2,3,5,6} \frac{c_j}{M_V} \mathcal{O}_{VJP}^j$$

P. D. Ruiz-Femenía, A. Pich and J. Portolés, JHEP **0307** (2003) 003.

Lagrangian

$$\mathcal{L} \;=\; \mathcal{L}_{\mathrm{non-R}} + \sum_{R} \left(\mathcal{L}_R^{\mathrm{Kin}} + \mathcal{L}_R\right) + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \ldots$$

- **Operators with one vector resonance field:**

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$\mathcal{O}^1_{VJP}$	$\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\,\nabla_\alpha u^\sigma\rangle$
$\mathcal{O}^2_{VJP}$	$\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha}, f_+^{\rho\sigma}\}\,\nabla_\alpha u^\nu\rangle$
$\mathcal{O}^3_{VJP}$	$i\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\sigma}\}\,\chi_-\rangle$ → ~~U(3)~~ explicitly
$\mathcal{O}^4_{VJP}$	$i\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}\,[f_-^{\rho\sigma}, \chi_+]\rangle$
$\mathcal{O}^5_{VJP}$	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha}\}\,u^\sigma\rangle$
$\mathcal{O}^6_{VJP}$	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma}\}\,u^\nu\rangle$
$\mathcal{O}^7_{VJP}$	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha}\}\,u_\alpha\rangle$

Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{non-R}} + \sum_{R} \left(\mathcal{L}_R^{\text{Kin}} + \mathcal{L}_R \right) + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \dots$$

- **Operators with two vector resonance fields:**

V. Cirigliano, G. Ecker, H. Neufeld and A. Pich, JHEP **0306** (2003) 012.
Z. H. Guo and J. J. Sanz-Cillero, Phys. Rev. D **79** (2009) 096006.

$$\mathcal{L}_{VV}^{\text{even}} = -\frac{1}{2} e_m^V \langle V_{\mu\nu} V^{\mu\nu} \chi_+ \rangle ,$$

$$\mathcal{L}_{VV}^{\text{odd}} = \sum_{j=1,2,3} d_j \mathcal{O}_{VVP}^j .$$

~~U(3)~~ explicitly

$\mathcal{O}_{VVP}^1$	$\varepsilon_{\mu\nu\rho\sigma} \langle \{ V^{\mu\nu}, V^{\rho\alpha} \} \nabla_\alpha u^\sigma \rangle$
$\mathcal{O}_{VVP}^2$	$i\varepsilon_{\mu\nu\rho\sigma} \langle \{ V^{\mu\nu}, V^{\rho\sigma} \} \chi_- \rangle$
$\mathcal{O}_{VVP}^3$	$\varepsilon_{\mu\nu\rho\sigma} \langle \{ \nabla_\alpha V^{\mu\nu}, V^{\rho\alpha} \} u^\sigma \rangle$
$\mathcal{O}_{VVP}^4$	$\varepsilon_{\mu\nu\rho\sigma} \langle \{ \nabla^\sigma V^{\mu\nu}, V^{\rho\alpha} \} u_\alpha \rangle$

P. D. Ruiz-Femenía, A. Pich and J. Portolés, JHEP **0307** (2003) 003.

For vertices involving more than one pseudoGoldstone, the general basis of VJ & VV operators can be found in Kampf & Novotny, PRD 84 (2011) 014036.

Lagrangian

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P-resonance contributions to the $\pi^0/\eta/\eta'$ TFFs are suppressed as m_p^2/M_P^2. However, the lightest P nonet need to be included to recover the OPE asymptotics of the VVP Green function: Kampf & Novotny, PRD 84 (2011) 014036.

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$$\mathcal{L} = \mathcal{L}_{\text{non}-\text{R}} + \sum_R \left(\mathcal{L}_R^{\text{Kin}} + \mathcal{L}_R\right) + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \dots$$

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$$\Delta\mathcal{L}_{P'}^{\text{even}} = \frac{1}{2}\langle \nabla_\mu P' \nabla^\mu P' \rangle + i d_m \langle P' \chi_- \rangle,$$

→ ~~U(3)~~ explicitly

$$\Delta\mathcal{L}_{P'}^{\text{odd}} = \varepsilon_{\mu\nu\alpha\beta}\langle \kappa_5^P \{f_+^{\mu\nu}, f_+^{\alpha\beta}\} P + \kappa_3^{VP}\{V^{\mu\nu}, f_+^{\alpha\beta}\}P + \kappa^{VVP} V^{\mu\nu} V^{\alpha\beta} P \rangle,$$

P’ effect is equivalent to considering a 2nd V multiplet: Roig & Sanz Cillero PLB 733 (2014) 158-163

η–η' mixing

$$(\Phi_{11},\Phi_{22},\Phi_{33}) = \left(\frac{C_\pi\pi^0 + C_q\eta + C_q'\eta'}{\sqrt{2}}, \frac{-C_\pi\pi^0 + C_q\eta + C_q'\eta'}{\sqrt{2}}, -C_s\eta + C_s'\eta'\right)$$

$$\phi^3 = (\Phi_{11} - \Phi_{22})/\sqrt{2} = C_\pi \pi^0$$

$$C_\pi = F/F_\pi$$ in the large Nc limit

($C_\pi^2 = Z_\pi$)

$$C_q := \frac{\boxed{F}}{\sqrt{3}\cos(\theta_8-\theta_0)}\left(\frac{\cos\theta_0}{f_8} - \frac{\sqrt{2}\sin\theta_8}{f_0}\right)$$

$$C_q' := \frac{\boxed{F}}{\sqrt{3}\cos(\theta_8-\theta_0)}\left(\frac{\sqrt{2}\cos\theta_8}{f_0} + \frac{\sin\theta_0}{f_8}\right)$$

$$C_s := \frac{\boxed{F}}{\sqrt{3}\cos(\theta_8-\theta_0)}\left(\frac{\sqrt{2}\cos\theta_0}{f_8} + \frac{\sin\theta_8}{f_0}\right)$$

$$C_s' := \frac{\boxed{F}}{\sqrt{3}\cos(\theta_8-\theta_0)}\left(\frac{\cos\theta_8}{f_0} - \frac{\sqrt{2}\sin\theta_0}{f_8}\right)$$

Kaiser & Leutwyler '00; Feldmann, Kroll & Stech '98, '99

't Hooft '74; '74; Witten '79

→ Computed up to NNLO in $1/N_C$ in Guo, Guo, Oller & Sanz-Cillero, JHEP 1506 (2015) 175

P pole contribution with RχL including flavor breaking

U(3) breaking in the φ and V nonets

Ideal mixing for V → $(V_{11}^{\mu\nu}, V_{22}^{\mu\nu}, V_{33}^{\mu\nu}) = ((\rho^{0\,\mu\nu} + \omega^{\mu\nu})/\sqrt{2}, (-\rho^{0\,\mu\nu} + \omega^{\mu\nu})/\sqrt{2}, \phi)$

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$$M_\rho^2 = M_V^2 - 4e_m^V m_\pi^2\,, \qquad M_{K^*}^2 = M_V^2 - 4e_m^V m_K^2\,,$$

$$M_\omega^2 = M_V^2 - 4e_m^V m_\pi^2\,, \qquad M_\phi^2 = M_V^2 - 4e_m^V \Delta_{2K\pi}^2\,.$$

$\Delta_{2K\pi}^2 = 2m_K^2 - m_\pi^2$

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$M_V = 764.3$ MeV, $e_m^V = -0.228$

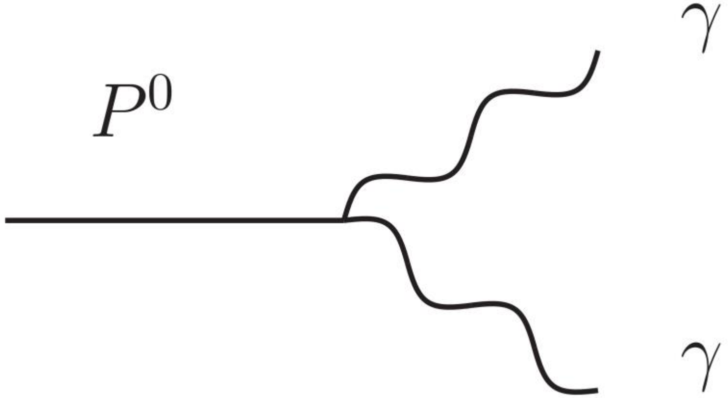
$$M_\rho^2 = M_V^2 - 4e_m^V m_\pi^2\,, \qquad M_{K^*}^2 = M_V^2 - 4e_m^V m_K^2\,,$$

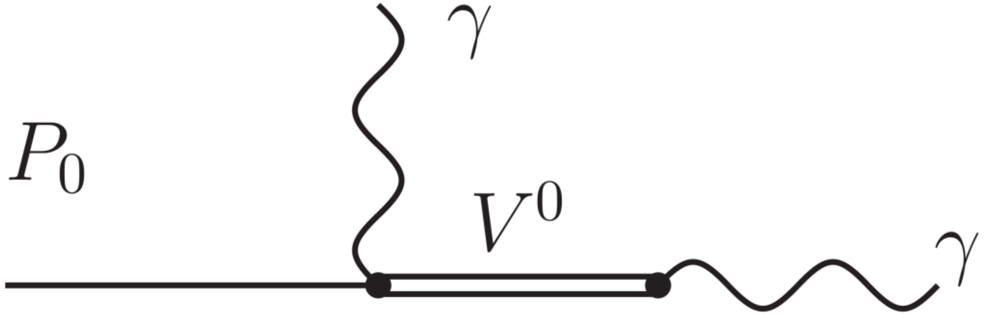
$$M_\omega^2 = M_V^2 - 4e_m^V m_\pi^2\,, \qquad M_\phi^2 = M_V^2 - 4e_m^V \Delta_{2K\pi}^2\,.$$

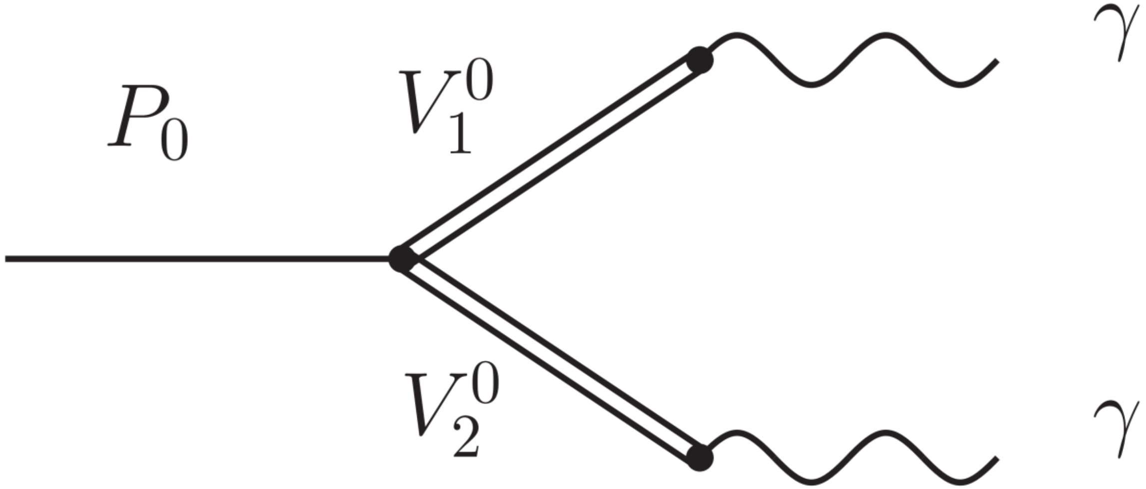
$$\Delta_{2K\pi}^2 = 2m_K^2 - m_\pi^2$$

$$\begin{aligned} \rho-\gamma: &\quad F_V \longrightarrow F_V + 8m_\pi^2\lambda_V\,, \\ \omega-\gamma: &\quad F_V \longrightarrow F_V + 8m_\pi^2\lambda_V\,, \\ \phi-\gamma: &\quad F_V \longrightarrow F_V + 8\Delta_{2K\pi}^2\lambda_V\,. \end{aligned}$$

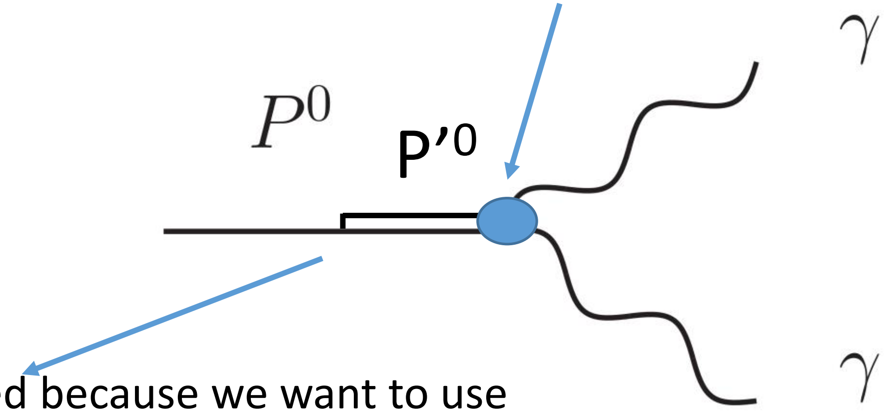
TFFs in RχT







It includes P'V(V) interactions



Needed because we want to use
the consistent set of SD
constraints applying to VVP Green
function & related FFs
Roig & Sanz Cillero
PLB 733 (2014) 158-163

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TFFs in RχT

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{2}{3F_\pi}\Bigg\{-\frac{N_C}{8\pi^2}+32m_\pi^2C_7^W+\Bigg[-8q_1^2C_{22}^W + \frac{2(F_V+8m_\pi^2\lambda_V)^2\left(d_3(q_1^2+q_2^2)+d_{123}m_\pi^2\right)}{D_\rho(q_1^2)D_\omega(q_2^2)} - \frac{\sqrt{2}(F_V+8m_\pi^2\lambda_V)}{M_V}\left(m_\pi^2c_{1235}-q_1^2c_{1256}+q_2^2c_{125}\right)\left(\frac{1}{D_\rho(q_1^2)}+\frac{1}{D_\omega(q_1^2)}\right) + (q_1\leftrightarrow q_2)\Bigg]\Bigg\},$$

$$D_R(s) = M_R^2 - s$$

$\mathcal{F}_{\eta^{(\prime)}\gamma^*\gamma^*}(q_1^2,q_2^2)$ in additional material

P pole contribution with RχL including flavor breaking

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TFFs in RχT

Convenient definitions:

$$\begin{cases} c_{1235} = c_1 + c_2 + 8c_3 - c_5\,, \\ c_{1256} = c_1 - c_2 - c_5 + 2c_6\,, \\ c_{125} = c_1 - c_2 + c_5\,, \\ d_{123} = d_1 + 8d_2 - d_3\,. \end{cases}$$

P-dependent coefficient for the local interaction

Kampf & Novotny, PRD 84 (2011) 014036

$$\mathcal{F}_{P\to P'^{*}\to\gamma^\star\gamma^\star} = -\frac{2}{3F}\frac{16 d_m C_{P7}}{M_P^2}\left[4\kappa_5^P - \sqrt{2}\kappa_3^{PV}F_V\left(\frac{1}{M_V^2-q_1^2}+\frac{1}{M_V^2-q_2^2}\right) + \frac{\kappa^{PVV}F_V^2}{(M_V^2-q_1^2)(M_V^2-q_2^2)}\right].$$

TFFs in RχT

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Starred couplings include P-resonance effects

$$C_7^W \longrightarrow C_7^{W\star} = C_7^W - \frac{2d_m\kappa_5^P}{M_P^2},$$

$$c_3 \longrightarrow c_3^\star = c_3 - \frac{d_m M_V \kappa_3^{PV}}{M_P^2},$$

$$d_2 \longrightarrow d_2^\star = d_2 - \frac{d_m \kappa^{PVV}}{2M_P^2}.$$

$$c_{1235} \longrightarrow c_{1235}^\star = c_1 + c_2 + 8c_3^\star - c_5,$$

$$d_{123} \longrightarrow d_{123}^\star = d_1 + 8d_2^\star - d_3.$$

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SD constraints $Q^2 = -q^2$

$$\mathcal{F}_{P\gamma^*\gamma^*}(q^2,q^2) \stackrel{Q^2\to\infty}{\longrightarrow} 0\,, \qquad \mathcal{F}_{P\gamma^*\gamma^*}(q^2,0) \stackrel{Q^2\to\infty}{\longrightarrow} 0\,,$$

Order by order in the m_P^2 expansion

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Order by order in the m_P^2 expansion

- π^0**-TFF**, $\mathcal{O}(m_P^0)$:

$$C_{22}^W = 0,$$

$$c_{125} = 0,$$

$$c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V},$$

$$d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2},$$

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- π^0**-TFF,** $\mathcal{O}(m_P^2)$ **:**

$$\lambda_V = -\frac{32\pi^2 F_V}{N_C} C_7^{W\star}\,,$$
$$c_{1235}^\star = \frac{N_C M_V e_m^V}{8\sqrt{2}\pi^2 F_V}.$$

SD constraints $Q^2 = -q^2$

$$\mathcal{F}_{P\gamma^*\gamma^*}(q^2,q^2) \stackrel{Q^2\to\infty}{\longrightarrow} 0\,, \qquad \mathcal{F}_{P\gamma^*\gamma^*}(q^2,0) \stackrel{Q^2\to\infty}{\longrightarrow} 0\,,$$

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- π^0**-TFF**, $\mathcal{O}(m_P^2)$:

$$\lambda_V = -\frac{32\pi^2 F_V}{N_C} C_7^{W\star},$$
$$c^\star_{1235} = \frac{N_C M_V e_m^V}{8\sqrt{2}\pi^2 F_V}.$$

- **Additional** η**-TFF constraints,** $\mathcal{O}(m_P^2)$:

$$C_8^W = 0,$$
$$c_3^\star = \frac{c^\star_{1235}}{8} = \frac{N_C M_V e_m^V}{64\sqrt{2}\pi^2 F_V}.$$

- **No additional** η'**-TFF constraints:**

SD constraints $Q^2 = -q^2$

$$\mathcal{F}_{P\gamma^*\gamma^*}(q^2,q^2) \overset{Q^2\to\infty}{\longrightarrow} 0\,, \qquad \mathcal{F}_{P\gamma^*\gamma^*}(q^2,0) \overset{Q^2\to\infty}{\longrightarrow} 0\,,$$

Order by order in the m_P^2 expansion

- VVP **Green's function,** $\mathcal{O}(m_P^0)$ **:**

Kampf & Novotny, PRD 84 (2011) 014036
Roig & Sanz Cillero PLB733 (2014) 158-163

$$c_{125} = c_{1235} = 0\,, \qquad c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V}\,,$$

$$d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2} + \frac{F^2}{8F_V^2} + \frac{4\sqrt{2}d_m\kappa_3^{PV}}{F_V}\,, \qquad d_{123} = \frac{F^2}{8F_V^2}\,, \qquad \kappa_5^P = 0\,,$$

$$C_7^W = C_8^W = C_{22}^W = 0\,.$$

Both sets of constraints are compatible, in such a way that

SD constraints $Q^2 = -q^2$

$$\mathcal{F}_{P\gamma^*\gamma^*}(q^2,q^2) \overset{Q^2\to\infty}{\longrightarrow} 0\,, \qquad \mathcal{F}_{P\gamma^*\gamma^*}(q^2,0) \overset{Q^2\to\infty}{\longrightarrow} 0\,,$$

Order by order in the m_P^2 expansion

VVP Green's function, $\mathcal{O}(m_P^0)$

(Compatible with those found previously in τ decays with odd-intrinsic parity contributions)

U(3) explicitly

$$c_1 = c_2 - c_5 = c_3 = c_{125} = c_{1235} = 0,$$

$$c^\star_{1235} = 8c^\star_3 = \frac{16 d_m \kappa_3^{PV} M_V}{M_P^2} = \frac{N_C e_m^V M_V}{8\sqrt{2}\pi^2 F_V},$$

$$c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V} \qquad d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2},$$

$$d^\star_{123} = \frac{F^2}{8F_V^2} + \frac{4 d_m \kappa^{PVV}}{M_P^2},$$

$$C_7^{W\star} = \lambda_V = 0\,.$$

$$C_8^W = C_{22}^W = 0$$

U(3) explicitly

P pole contribution with RχL including flavor breaking

Pablo Roig

TFFs after using SD constraints

$$\mathcal{F}_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{32\pi^2 m_\pi^2 F_V^2 d_{123}^\star - N_C M_V^2 M_\rho^2}{12\pi^2 F_\pi D_\rho(q_1^2) D_\rho(q_2^2)}$$

TFFs after using SD constraints

$$\mathcal{F}_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{32\pi^2 m_\pi^2 F_V^2 d_{123}^\star - N_C M_V^2 M_\rho^2}{12\pi^2 F_\pi D_\rho(q_1^2) D_\rho(q_2^2)}$$

$C_q \to C_q'$, $C_s \to -C_s'$ and $m_\eta \to m_{\eta'}$ for η'

$$\Delta^2_{2K\pi\eta} = 2m_K^2 - m_\pi^2 - m_\eta^2$$
$$\Delta^2_{\eta\pi} = m_\eta^2 - m_\pi^2$$

$$\begin{aligned}\mathcal{F}_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2) \quad = \quad & \frac{1}{12\pi^2 F D_\rho(q_1^2) D_\rho(q_2^2) D_\phi(q_1^2) D_\phi(q_2^2)} \times \\ & \left\{ -\frac{N_C M_V^2}{3}\left[5C_q M_\rho^2 D_\phi(q_1^2) D_\phi(q_2^2) - \sqrt{2} C_s M_\phi^2 D_\rho(q_1^2) D_\rho(q_2^2)\right] \right. \\ & + \frac{32\pi^2 \boxed{F_V^2} d_{123}^\star m_\eta^2}{3}\left[(5C_q D_\phi(q_1^2) D_\phi(q_2^2) - \sqrt{2} C_s D_\rho(q_1^2) D_\rho(q_2^2)\right] \\ & \left. - \frac{256\pi^2 \boxed{F_V^2} d_2^\star}{3}\left[(5C_q \Delta^2_{\eta\pi} D_\phi(q_1^2) D_\phi(q_2^2) + \sqrt{2} C_s \Delta^2_{2K\pi\eta} D_\rho(q_1^2) D_\rho(q_2^2)\right] \right\}\end{aligned}$$

P pole contribution with RχL including flavor breaking

Fits to (space-like) data

No time-like data is fitted:

- For q2>0, resonance widths need to be included → Other NLO effects in $1/N_C$ need to be accounted for.
- Observables in this region get large radiative corrections (Kampf, Novotny & Sànchez-Puertas, arXiv: 1801.02027)

Fits to (space-like) data

In order to stabilize the fit, the decay constants and mixing angles describing the η-η′ system were added as data points

$$\theta_8 = (-21.2 \pm 1.6)^\circ,$$

$$\theta_0 = (-9.2 \pm 1.7)^\circ,$$

R. Kaiser and H. Leutwyler, Eur. Phys. J. C **17** (2000) 623.

T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58** (1998) 114006; Phys. Lett. B **449** (1999) 339.

J. Schechter, A. Subbaraman and H. Weigel, Phys. Rev. D **48** (1993) 339.
T. Feldmann, Int. J. Mod. Phys. A **15** (2000) 159.

$$f_8 = (1.26 \pm 0.04) F_\pi = (116.2 \pm 3.7)\mathrm{MeV},$$

$$f_0 = (1.17 \pm 0.03) F_\pi = (107.9 \pm 2.8)\mathrm{MeV}.$$

$\Gamma(P \to \gamma\gamma)$ also included in the fit

$$|\mathcal{F}_{P\gamma^*\gamma^*}(0,0)|^2 = \frac{64\pi}{(4\pi\alpha)^2}\frac{\Gamma(P \to \gamma\gamma)}{m_P^3}$$

$$\bar{d}_2 \equiv \frac{F_V^2 d_2^\star}{3F_\pi^2}\,, \qquad \bar{d}_{123} \equiv \frac{F_V^2 d_{123}^\star}{3F_\pi^2}$$

Fits to (space-like) data

$M_\rho = M_\omega \approx 807$ MeV and $M_\phi \approx 1126$ MeV

$M_\rho = M_\omega \sim 775$ MeV & $M_\phi \sim 1007$ MeV

	With π^0-BaBar	(Best Fit) Without π^0-BaBar	Fixing M_V and e_m^V
$\bar{d}_2$	$(-2.9 \pm 1.7) \cdot 10^{-2}$	$(-2.7 \pm 1.7) \cdot 10^{-2}$	$(-3.0 \pm 2.3) \cdot 10^{-2}$
$\bar{d}_{123}$	$(-25 \pm 15) \cdot 10^{-2}$	$(-23 \pm 15) \cdot 10^{-2}$	$(-25 \pm 20) \cdot 10^{-2}$
U(3) → M_V	(799 ± 5) MeV	(791 ± 6) MeV	764.3 MeV †
e_m^V	-0.35 ± 0.10	-0.34 ± 0.10	-0.228 †
θ_8	$(-19.5 \pm 0.9)^\circ$	$(-19.5 \pm 0.9)^\circ$	$(-21.7 \pm 0.9)^\circ$
θ_0	$(-9.5 \pm 1.6)^\circ$	$(-9.3 \pm 1.6)^\circ$	$(-10.4 \pm 1.6)^\circ$
f_8	(118 ± 4) MeV	(118 ± 3) MeV	(118 ± 3) MeV
f_0	(107.7 ± 2.7) MeV	(107.5 ± 0.9) MeV	(106.9 ± 2.8) MeV
χ^2/dof	150./106	69./89	101./89

† Taken from Guo & Sanz-Cillero PRD89 (2014) no.9, 094024

Fits to (space-like) data

In order to minimize the fit correlations among the first two parameters:

$$\bar{d}_2 = \frac{\sigma_{d_2}}{\sqrt{2}} \left(\sqrt{1+r}\ \mathcal{P}_1 - \sqrt{1-r}\ \mathcal{P}_2 \right) ,$$

$$\bar{d}_{123} = \frac{\sigma_{d_{123}}}{\sqrt{2}} \left(\sqrt{1+r}\ \mathcal{P}_1 + \sqrt{1-r}\ \mathcal{P}_2 \right) ,$$

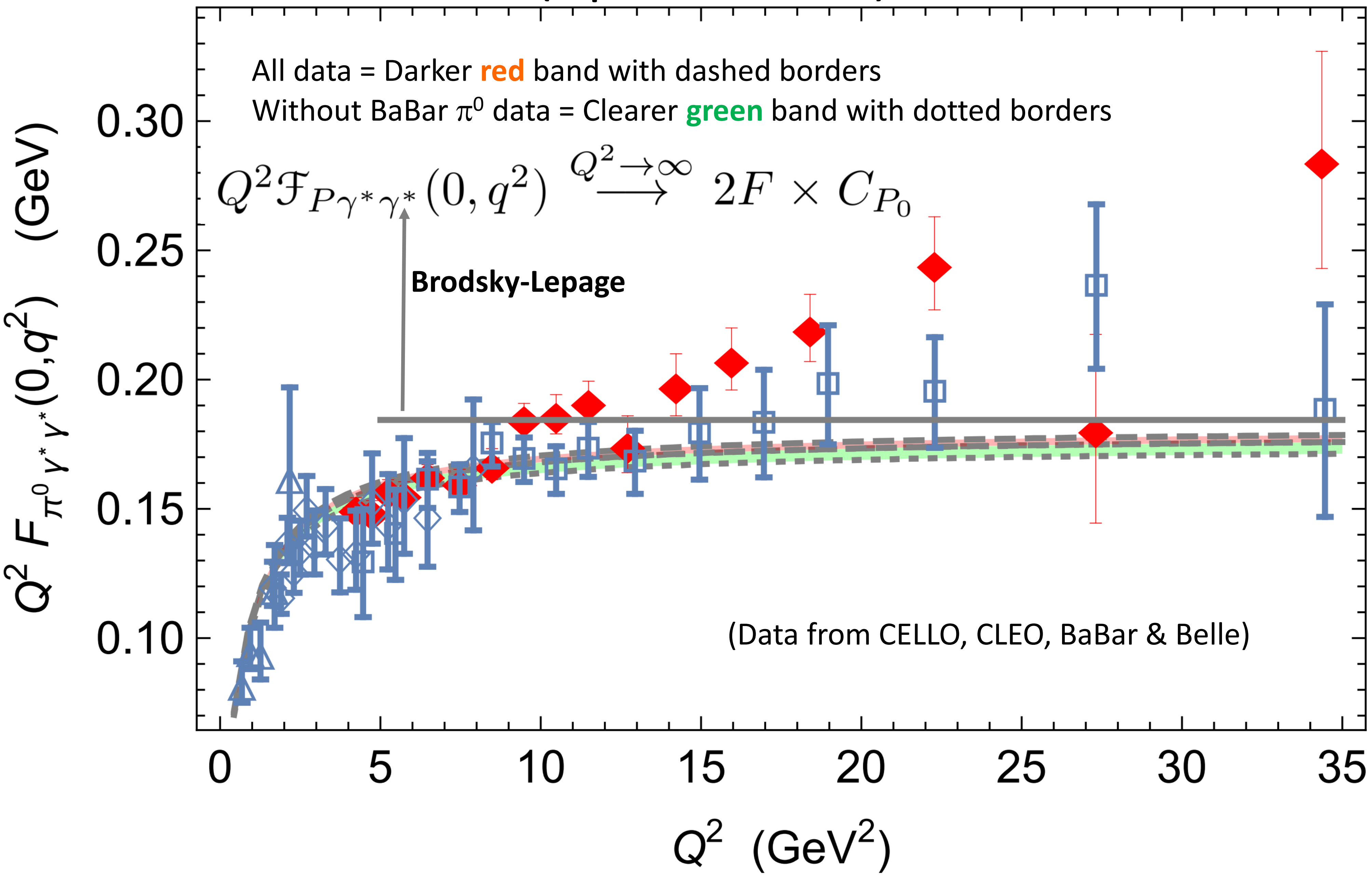
with $\sigma_{d_2} = 10.49 \cdot 10^{-3}$, $\sigma_{d_{123}} = 81.28 \cdot 10^{-3}$ and $r = 0.995$.

Fits to (space-like) data

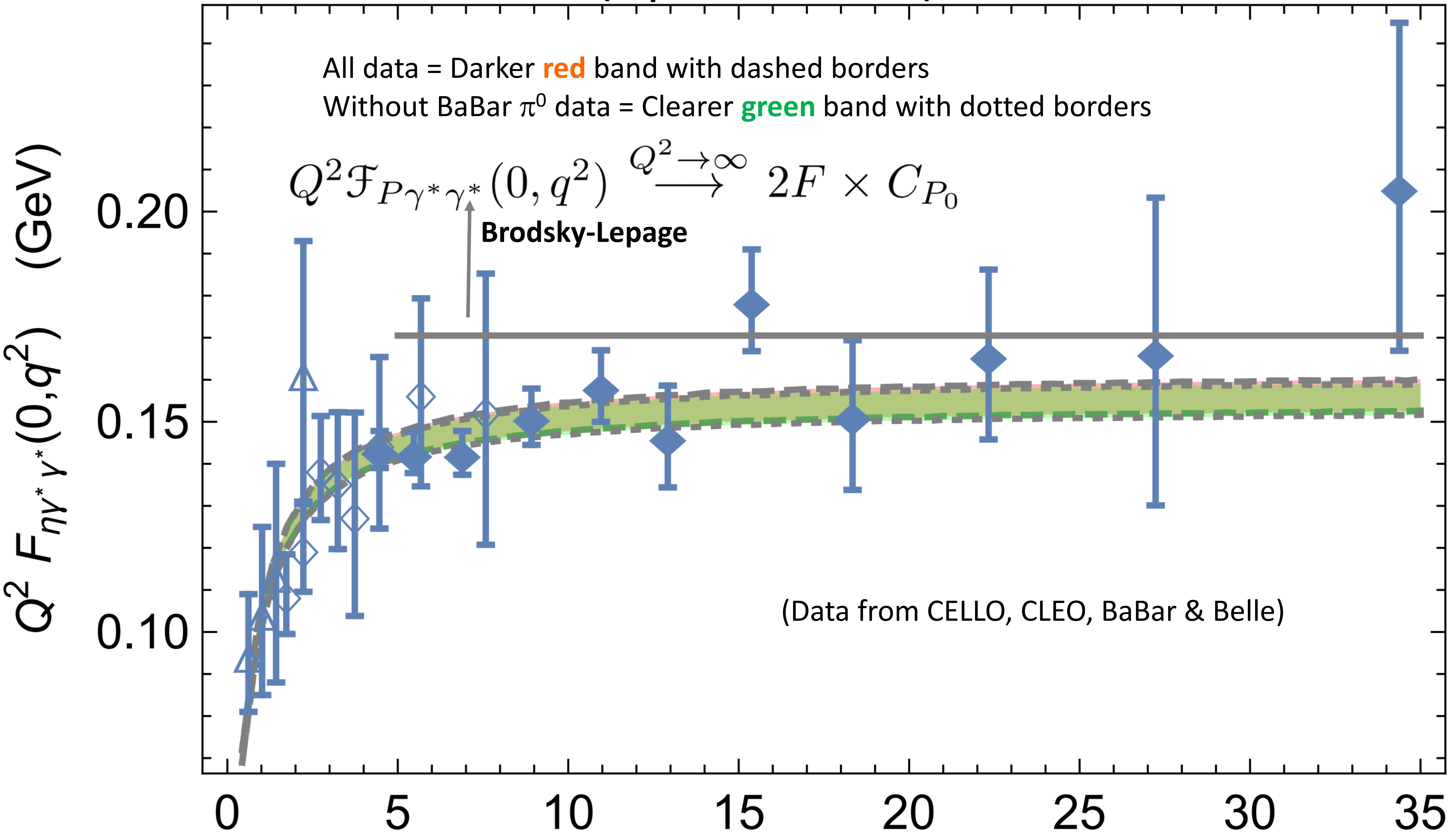
	$\mathcal{P}_1$	$\mathcal{P}_2$	M_V	e_m^V	θ_8	θ_0	f_8	f_0
$\mathcal{P}_1$	1	0.085	0.511	0.638	0.495	0.017	-0.107	-0.025
$\mathcal{P}_2$	0.085	1	0.434	0.439	-0.157	-0.616	-0.058	0.434
M_V	0.511	0.434	1	0.444	0.321	-0.054	-0.129	0.351
e_m^V	0.638	0.439	0.444	1	-0.081	0.059	-0.179	0.319
θ_8	0.495	-0.157	0.321	-0.081	1	-0.046	-0.486	-0.184
θ_0	0.017	-0.616	-0.054	0.059	-0.046	1	-0.020	-0.424
f_8	-0.107	-0.058	-0.129	-0.179	-0.486	-0.020	1	-0.156
f_0	-0.025	0.434	0.351	0.319	-0.184	-0.424	-0.156	1

Table 8: Correlation matrix for our best fit (excluding BaBar π^0–TFF data).

Fits to (space-like) data

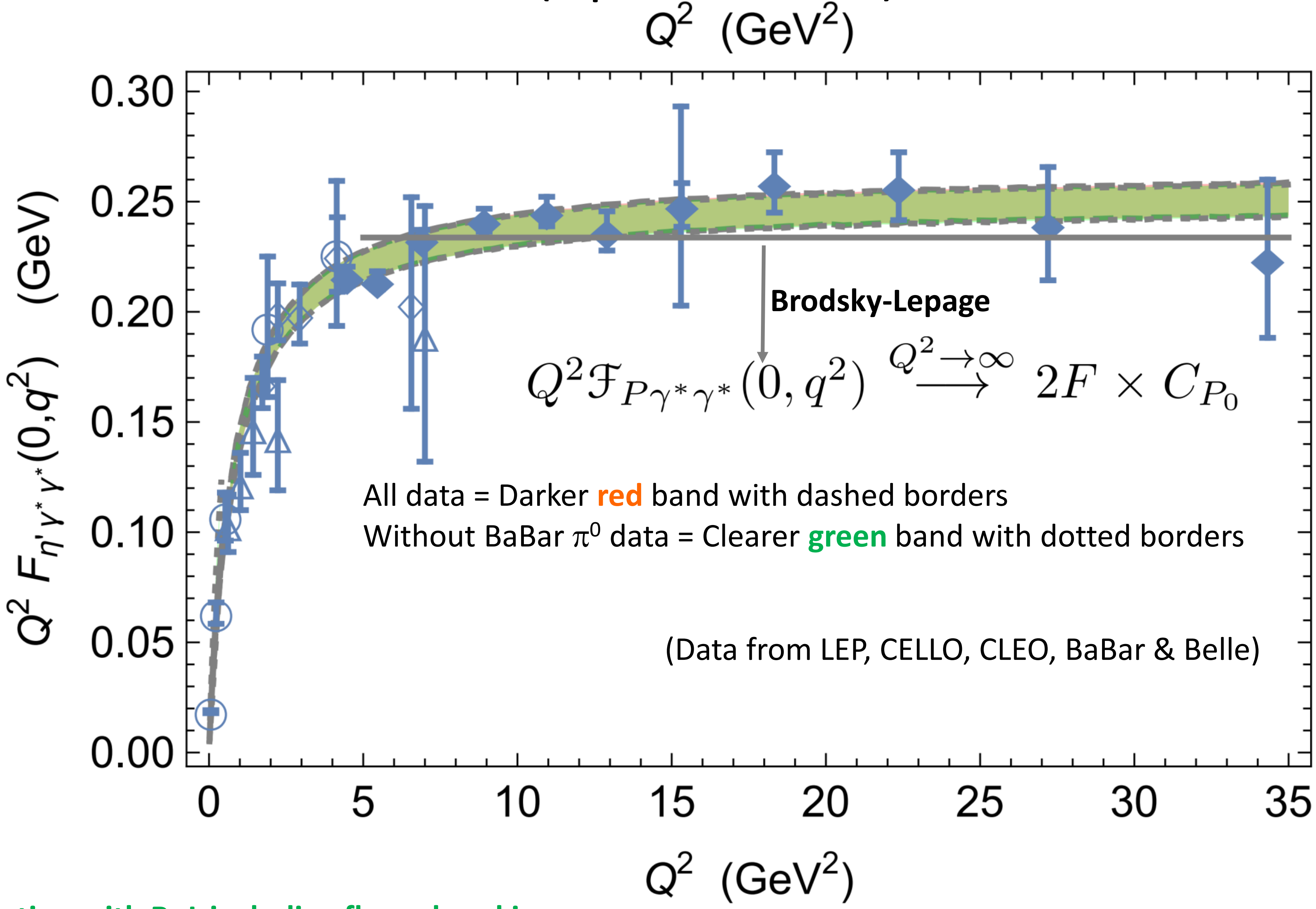


Fits to (space-like) data

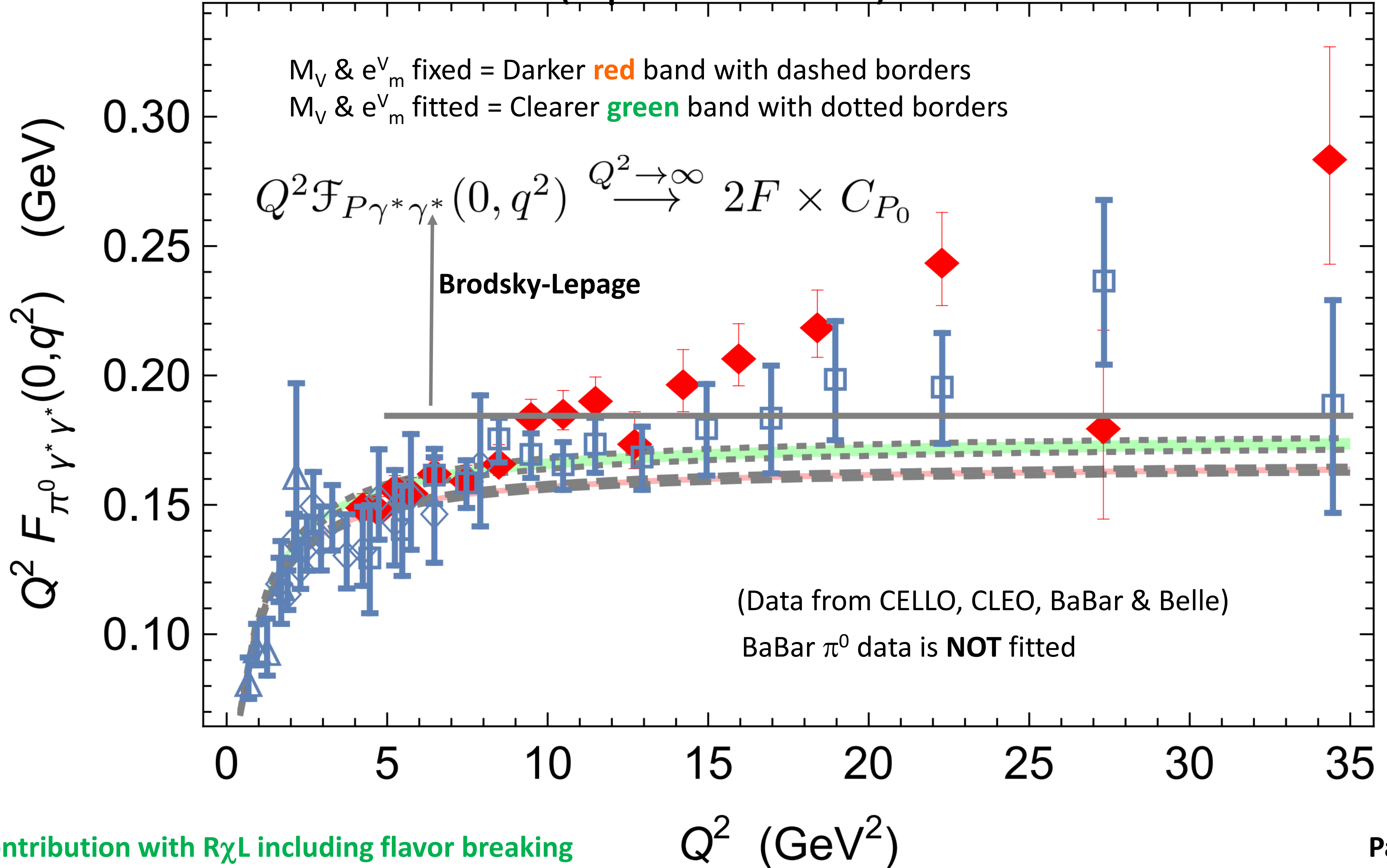


P pole contribution with RχL including flavor breaking

Fits to (space-like) data



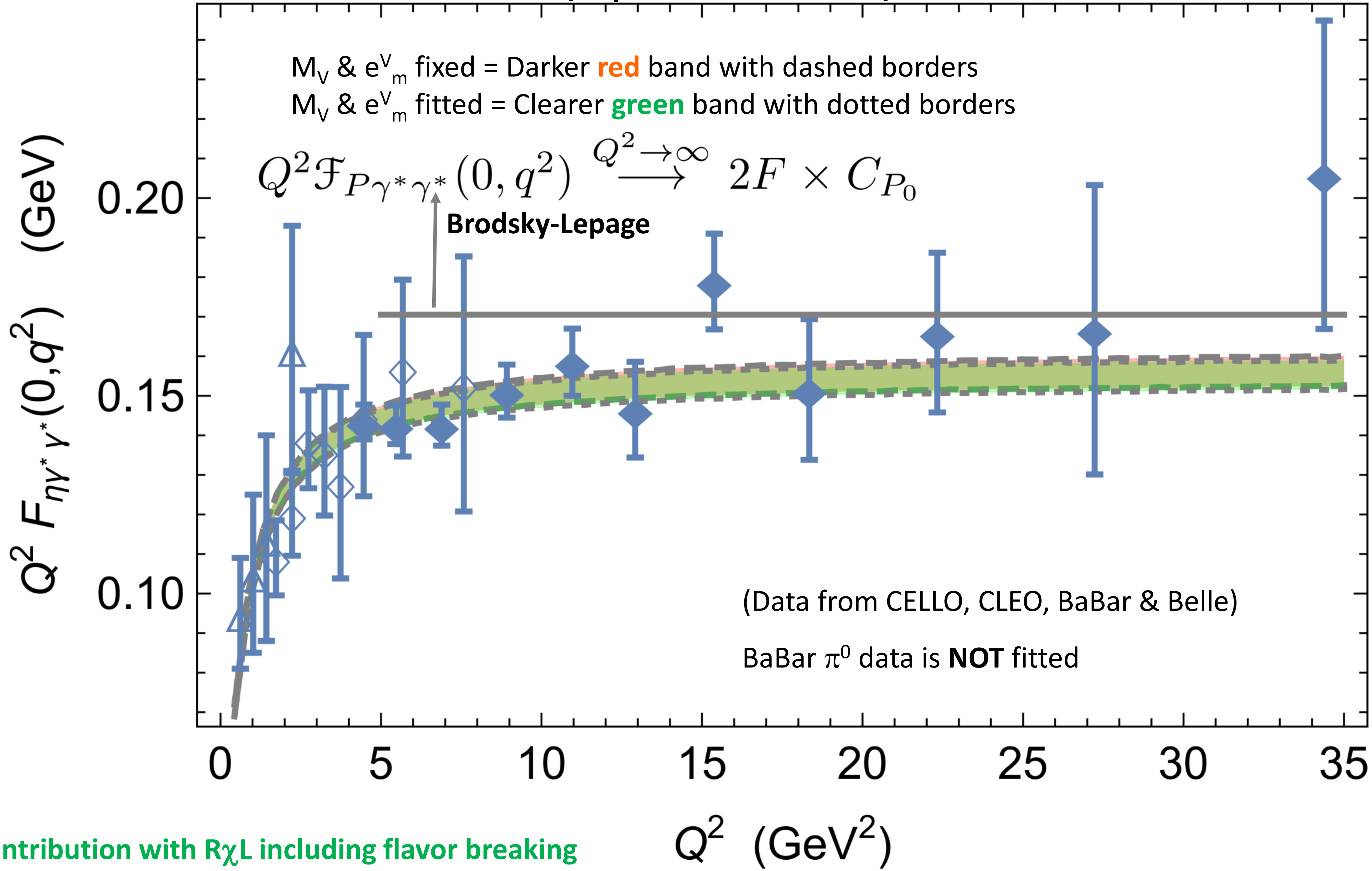
Fits to (space-like) data



P pole contribution with RχL including flavor breaking

Pablo Roig

Fits to (space-like) data



P pole contribution with RχL including flavor breaking

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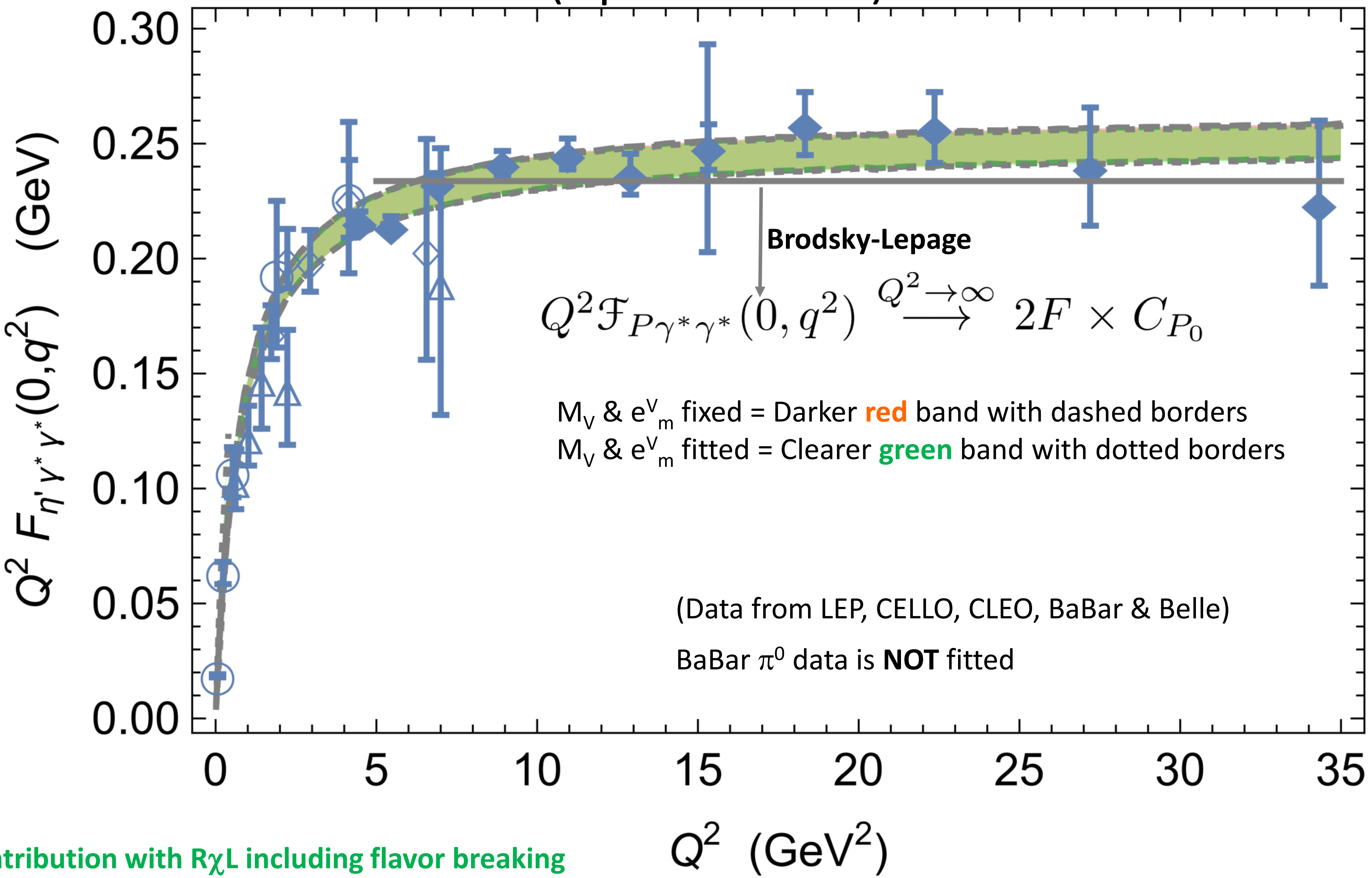
Fits to (space-like) data



P pole contribution with RχL including flavor breaking

Comparison with Czyz *et. al.*

Czyz *et al.* [35]	This work

A WG exercise

Comparison with Czyz *et. al.*

Differences

- 3 V multiplets
- Both q2≤0 & q2>0 data
- Only masses break U(3)
- Lack of chiral covariance
- SD only for TFF
- All data on same footing
- No stabilization for η-η′ mixing.

Czyz *et al.* [35]	This work
f_{V_1}	$\frac{1}{M_\rho}(F_V + 8m_\pi^2\lambda_V)$
F_{ω_1}	1
F_{ϕ_1}	$\frac{M_\rho}{M_\phi}\frac{(F_V+8\Delta_{2K\pi}^2\lambda_V)}{(F_V+8m_\pi^2\lambda_V)}$
$h_{V_1}^{\pi_0}$	$\frac{1}{M_V M_\rho}(-c_{1256}M_\rho^2 + c_{1235}^\star m_\pi^2)$
$h_{V_1}^{\eta}$	$\frac{1}{M_V M_\rho}(-c_{1256}M_\rho^2 + c_{1235}^\star m_\eta^2 - 8c_3^\star\Delta_{\eta\pi}^2)$
H_{ω_1}	1
$A_1^{\pi_0}$	0
$h_{V_1}^{\eta}\left[2C_s - \left(\frac{5}{\sqrt{2}}C_q - C_s\right)A_1^\eta\right]$	$\frac{C_s}{M_V M_\phi}(-c_{1256}M_\phi^2 + c_{1235}^\star m_\eta^2 - 8c_3^\star\Delta_{2K\pi\eta}^2)$
$\sigma_{V_1}^{\pi_0}$	$-\frac{1}{M_\rho M_\omega}\left[d_{123}^\star m_\pi^2 + d_3^\star(M_\rho^2 + M_\omega^2)\right]$
$\sigma_{V_1}^{\eta}$	$-\frac{1}{M_\rho^2}\left[d_{123}^\star m_\eta^2 - d_2^\star\Delta_{\eta\pi}^2 + 2d_3^\star M_\rho^2\right]$
$A_{\phi\omega,1}^{\pi^0}$	0
$A_{\phi\omega,1}^{\eta}$	0
$\frac{\sigma_{V_1}^\eta}{F_{\phi_1}}\left[5C_q A_1^\eta - \sqrt{2}C_s(A_1^\eta + 2)\right]$	$\frac{2C_s}{M_\phi^2}\left[d_{123}^\star m_\eta^2 - d_2^\star\Delta_{2K\pi\eta}^2 + 2d_3^\star M_\phi^2\right]$

Comparison with Czyz *et. al.*

Differences

- 3 V multiplets
- Both q2≤0 & q2>0 data
- Only masses break U(3)
- Lack of chiral covariance
- SD only for TFF
- All data on same footing
- No stabilization for η-η' mixing.

	Czyz *et al.* [35]	This work
f_{V_1}	0.2020 ± 0.0008	0.1979 ± 0.0026
F_{ω_1}	0.88 ± 0.01	1
F_{ϕ_1}	0.783 ± 0.005	0.717 ± 0.044
h_{V_1}	0.0377 ± 0.0008	0.0326 ± 0.0005
H_{ω_1}	1.02 ± 0.03	1
$A_1^{\pi_0}$	-0.083 ± 0.002	0
$h_{V_1}^{\eta}\left[2C_s - \left(\frac{5}{\sqrt{2}}C_q - C_s\right)A_1^{\eta}\right]$	0.39 ± 0.09	0.36 ± 0.07
$\sigma_{V_1}^{\eta}$	0.264 ± 0.007	0.240 ± 0.008
$-h_{V_1}^{\eta'}\left[2C_s' + \left(\frac{5}{\sqrt{2}}C_q' + C_s'\right)A_1^{\eta'}\right]$	-0.37 ± 0.11	-0.45 ± 0.07
$\sigma_{V_1}^{\eta'}$	0.263 ± 0.007	0.240 ± 0.008
$A_{\phi\omega,1}^{\pi^0}$	-0.21 ± 0.04	0
$A_{\phi\omega,1}^{\eta}$	-0.027 ± 0.007	0
$\frac{\sigma_{V_1}^{\eta}}{F_{\phi_1}}\left[5C_qA_1^{\eta} - \sqrt{2}C_s(A_1^{\eta}+2)\right]$	-0.42 ± 0.03	-0.27 ± 0.02
$\frac{\sigma_{V_1}^{\eta'}}{F_{\phi_1}}\left[5C_q'A_1^{\eta'} + \sqrt{2}C_s'(A_1^{\eta'}+2)\right]$	0.43 ± 0.16	0.33 ± 0.05

Consistency check with related processes

Process	Predicted branching fraction	PDG value
$\pi^0 \to e^+e^-\gamma$	$1.15 \cdot 10^{-2}$	$1.174(35) \cdot 10^{-2}$
$\eta \to e^+e^-\gamma$	$7.08 \cdot 10^{-3}$	$6.9(4) \cdot 10^{-3}$
$\eta \to \mu^+\mu^-\gamma$	$3.36 \cdot 10^{-4}$	$3.1(4) \cdot 10^{-4}$
$\eta' \to e^+e^-\gamma$	$5.22 \cdot 10^{-4}$	$4.73(30) \cdot 10^{-4}$
$\eta' \to \mu^+\mu^-\gamma$	$1.54 \cdot 10^{-4}$	$1.09(27) \cdot 10^{-4}$
$\pi^0 \to 2e^+2e^-$	$3.37 \cdot 10^{-5}$	$3.34(16) \cdot 10^{-5}$
$\eta \to 2e^+2e^-$	$2.73 \cdot 10^{-5}$	$2.40(22) \cdot 10^{-5}$
$\eta \to 2\mu^+2\mu^-$	$4.03 \cdot 10^{-9}$	$< 3.6 \cdot 10^{-4}$
$\eta \to e^+e^-\mu^+\mu^-$	$2.43 \cdot 10^{-6}$	$< 1.6 \cdot 10^{-4}$
$\eta' \to 2e^+2e^-$	$2.45 \cdot 10^{-6}$	No bounds
$\eta' \to 2\mu^+2\mu^-$	$2.18 \cdot 10^{-8}$	No bounds
$\eta' \to e^+e^-\mu^+\mu^-$	$1.20 \cdot 10^{-6}$	No bounds

P pole contribution with RχL including flavor breaking

Pablo Roig

η-η' mixing

$$\overline{C}^{(\prime)}_{q/s} \equiv C^{(\prime)}_{q/s} \times (F_\pi/F)$$

	$\overline{C}_q$	$\overline{C}_s$	$\overline{C}'_q$	$\overline{C}'_s$	Mean value $\pm 1\sigma$	Czyż *et al.* [35]
$\overline{C}_q$	1	0.404	0.334	-0.469	0.688 ± 0.033	0.613 ± 0.047
$\overline{C}_s$	0.404	1	0.127	0.00754	0.469 ± 0.031	0.139 ± 0.063
$\overline{C}'_q$	0.334	0.127	1	-0.820	0.595 ± 0.026	0.556 ± 0.058
$\overline{C}'_s$	-0.469	0.00754	-0.820	1	0.581 ± 0.023	0.736 ± 0.084

P pole contributions to a_μ

Data accuracy increases with time

Reference	$10^{10} \cdot a_\mu^{P,LbL}$
Knecht and Nyffeler [73]	8.3 (1.2)
Hayakawa and Kinoshita [78]	7.9 (1.5)
Bijnens, Pallante and Prades [79]	8.3 (3.2)
Goecke, Fischer and Williams [86]	8.1 (2)
Roig, Guevara and López Castro [34]	8.60 (25)
Masjuan and Sánchez-Puertas [72]	9.4 (5)
Czyz, Kisza and Tracz [35]	8.28 (34)
This work	8.47 (16)

Evaluation with a 5000 event MC accounting for means, errors & correlations

Alternative fits

Including BaBar π^0 data: $a_\mu^{P,LbL} = (8.58 \pm 0.16) \cdot 10^{-10}$

Fixing M_V and e_m^V: $a_\mu^{P,LbL} = (8.50 \pm 0.13) \cdot 10^{-10}$

P pole contributions to a_μ

Our best fit (excluding BaBar π^0 data) $a_\mu^{P,LbL} = (8.47 \pm 0.16) \cdot 10^{-10}$

Evaluation with a 5000 event MC accounting for means, errors & correlations

Individual contributions

$$a_\mu^{\pi^0,LbL} = (5.81 \pm 0.09) \cdot 10^{-10}$$

$$a_\mu^{\eta,LbL} = (1.51 \pm 0.06) \cdot 10^{-10}$$

$$a_\mu^{\eta',LbL} = (1.15 \pm 0.07) \cdot 10^{-10}$$

Alternative fits

Including BaBar π^0 data: $a_\mu^{P,LbL} = (8.58 \pm 0.16) \cdot 10^{-10}$

Fixing M_V and e_m^V: $a_\mu^{P,LbL} = (8.50 \pm 0.13) \cdot 10^{-10}$

Conclusions: P pole contributions to a_μ

Our best fit (excluding BaBar π^0 data) $$a_\mu^{P,LbL} = (8.47 \pm 0.16) \cdot 10^{-10}$$

All evaluations in basic agreement, except those using Melnikov-Vainshtein SD constraints & Dorokhov et. al. results with NLχQM (to be checked against Lattice final results)

Individual contributions

$$a_\mu^{\pi^0,LbL} = (5.81 \pm 0.09) \cdot 10^{-10}$$
$$a_\mu^{\eta,LbL} = (1.51 \pm 0.06) \cdot 10^{-10}$$
$$a_\mu^{\eta',LbL} = (1.15 \pm 0.07) \cdot 10^{-10}$$

Alternative fits

Including BaBar π^0 data: $$a_\mu^{P,LbL} = (8.58 \pm 0.16) \cdot 10^{-10}$$

Fixing M_V and e_m^V: $$a_\mu^{P,LbL} = (8.50 \pm 0.13) \cdot 10^{-10}$$

ADDITIONAL MATERIAL

χ Building blocks

$$\ell_\mu = v_\mu - a_\mu = eQA_\mu + \ldots$$
$$r_\mu = v_\mu + a_\mu = eQA_\mu + \ldots$$
$$Q = \mathrm{diag}\left(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}\right)$$

$$u_\mu = i\left(u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right),$$

$$F_L^{\mu\nu} = \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i[\ell^\mu, \ell^\nu] = eQF^{\mu\nu} + \ldots \text{ and } F_R^{\mu\nu} = F_L^{\mu\nu}|_{\ell\to r} = eQF^{\mu\nu} + \ldots$$

$$\Gamma_\mu = \frac{1}{2}\left(u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right),$$

$$\nabla_\mu \cdot = \partial_\mu \cdot + [\Gamma_\mu, \cdot], \qquad f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u,$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u, \qquad \chi = 2B_0(s + ip),$$

$$s + ip = \mathrm{diag}(m_u, m_d, m_s)$$
$$m_u = m_d$$

$$u = \exp\left(\frac{i\Phi}{\sqrt{2}F}\right), \qquad \Phi = \sum_{a=0}^{8} \frac{\lambda_a \phi^a}{\sqrt{2}},$$

TFFs in RχT

$$\mathcal{F}_{\eta\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{2}{3F}\left\{-\frac{(5C_q-\sqrt{2}C_s)}{3}\left[8(q_1^2+q_2^2)C_{22}^W+\frac{N_C}{8\pi^2}\right]\right.$$

$$+32C_7^W\frac{5C_q m_\pi^2-\sqrt{2}C_s\Delta_{2K\pi}^2}{3}+64C_8^W\left(2C_q m_\pi^2-\sqrt{2}C_s\Delta_{2K\pi}^2\right)$$

$$+\left[-\frac{\sqrt{2}C_q}{3M_V}\frac{(F_V+8m_\pi^2\lambda_V)\left(-8c_3\Delta_{\eta\pi}^2+c_{1235}m_\eta^2-c_{1256}q_1^2+c_{125}q_2^2\right)}{D_\rho(q_1^2)}\right.$$

$$-\frac{3\sqrt{2}C_q}{M_V}\frac{(F_V+8m_\pi^2\lambda_V)\left(-8c_3\Delta_{\eta\pi}^2+c_{1235}m_\eta^2-c_{1256}q_1^2+c_{125}q_2^2\right)}{D_\omega(q_1^2)}$$

$$+\frac{4C_s}{3M_V}\frac{(F_V+8\Delta_{2K\pi}^2\lambda_V)\left(c_{1235}m_\eta^2-c_{1256}q_1^2+c_{125}q_2^2+8c_3\Delta_{2K\pi\eta}^2\right)}{D_\phi(q_1^2)}$$

$$+\frac{3C_q(F_V+8m_\pi^2\lambda_V)^2\left(-8d_2\Delta_{\eta\pi}^2+d_{123}m_\eta^2+d_3(q_1^2+q_2^2)\right)}{D_\rho\left(q_1^2\right)D_\rho\left(q_2^2\right)}$$

$$+\frac{C_q(F_V+8m_\pi^2\lambda_V)^2\left(-8d_2\Delta_{\eta\pi}^2+d_{123}m_\eta^2+d_3(q_1^2+q_2^2)\right)}{3D_\omega\left(q_1^2\right)D_\omega\left(q_2^2\right)}$$

$$-\frac{2\sqrt{2}C_s(F_V+8\Delta_{2K\pi}^2\lambda_V)^2\left(d_{123}m_\eta^2+d_3(q_1^2+q_2^2)+8d_2\Delta_{2K\pi\eta}^2\right)}{3D_\phi\left(q_1^2\right)D_\phi\left(q_2^2\right)}$$

$$\left.\left.+\left(q_1\leftrightarrow q_2\right)\right]\right\},$$

P pole contribution with RχL including flavor breaking

Pablo Roig

η-η′ mixing

We note that we did not need to perform the m_P^2 expansion of the mixing coefficients $C_{q/s}^{(\prime)}$. One obtains the previous set of consistent relations for any value of $C_{q/s}^{(\prime)}$: our conditions are actually requiring a good high-energy behaviour to the bare $\phi^a \to \gamma^*\gamma^*$ TFFs, which is inherited by the physical π^0, η and η' TFFs after taking into account the field renormalizations and mixings provided by C_π and $C_{q/s}^{(\prime)}$. This allows us to by-pass the cumbersome problem of the large–N_C limit in the $\eta-\eta'$ mixing [44], where $1/N_C$ and quark mass corrections are found to be of a similar numerical order [44, 45, 51].

[44] R. Kaiser and H. Leutwyler, Eur. Phys. J. C **17** (2000) 623.

[45] T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58** (1998) 114006; Phys. Lett. B **449** (1999) 339.

[51] X. K. Guo, Z. H. Guo, J. A. Oller and J. J. Sanz-Cillero, JHEP **1506** (2015) 175.

Flavor symmetry relations between couplings

	C_{P_0}	C_{P_7}	C_{P_8}
π	1	m_π^2	0
η	$\left(5C_q - \sqrt{2}C_s\right)/3$	$\left(5C_q m_\pi^2 - \sqrt{2}C_s\Delta^2_{2K\pi}\right)/3$	$2C_q m_\pi^2 - \sqrt{2}C_s\Delta^2_{2K\pi}$
η'	$\left(5C'_q + \sqrt{2}C'_s\right)/3$	$\left(5C'_q m_\pi^2 + \sqrt{2}C'_s\Delta^2_{2K\pi}\right)/3$	$2C'_q m_\pi^2 + \sqrt{2}C'_s\Delta^2_{2K\pi}$

Table 4: Values of C_{P_i} for the local interaction.

$$C_{V_i} = \begin{cases} \frac{1}{3}(F_V + 8m_\pi^2\lambda_V) & \text{for } \omega, \\ (F_V + 8m_\pi^2\lambda_V) & \text{for } \rho^0, \\ \frac{\sqrt{2}}{3}\left(F_V + 8\Delta^2_{2K\pi}\lambda_V\right) & \text{for } \phi, \end{cases}$$

C^d_{1R}	ω	ρ^0	ϕ
π^0	3	1	0
η	C_q	$3C_q$	$-2C_s$
η'	C'_q	$3C'_q$	$2C'_s$

C^m_{1R}	ω	ρ^0	ϕ
π^0	$3m_\pi^2$	m_π^2	0
η	$m_\pi^2 C_q$	$3m_\pi^2 C_q$	$-2\Delta^2_{2K\pi}C$
η'	$m_\pi^2 C'_q$	$3m_\pi^2 C'_q$	$2\Delta^2_{2K\pi}C'_s$

P pole contribution with RχL including flavor breaking

Flavor symmetry relations between couplings

C_{2R}^d	$\omega\omega$	$\rho^0\rho^0$	$\omega\rho^0$	$\phi\phi$
π^0	0	0	$\sqrt{2}$	0
η	$\sqrt{2}C_q$	$\sqrt{2}C_q$	0	$-2C_s$
η'	$\sqrt{2}C'_q$	$\sqrt{2}C'_q$	0	$2C'_s$

C_{2R}^m	$\omega\omega$	$\rho^0\rho^0$	$\omega\rho^0$	$\phi\phi$
π^0	0	0	$\sqrt{2}m_\pi^2$	0
η	$\sqrt{2}m_\pi^2 C_q$	$\sqrt{2}m_\pi^2 C_q$	0	$-2\Delta^2_{2K\pi}C_s$
η'	$\sqrt{2}m_\pi^2 C'_q$	$\sqrt{2}m_\pi^2 C'_q$	0	$2\Delta^2_{2K\pi}C'_s$