Calculus of variations

1. (30 points) In optics, Fermat’s principle or the principle of least time is the principle that the path taken between two points by a ray of light is the path that can be traversed in the least time.

Use Fermat’s principle to determine the optical path in an atmosphere in which the refractive index is inversely proportional to the height, \( y \):

\[
n(y) \equiv \frac{c}{v(y)} = \frac{\alpha}{y}
\]

where \( c \) is the speed of light in vacuum, \( \alpha > 0 \) is a parameter describing the properties of the atmosphere.

Hint: write the equation of the ray as \( x(y) \).

2. (30 points) Two rings, each of radius \( a \), are placed parallel with their centers on a common normal (which we take as \( x \) axis) distance \( 2b \) apart. An open-ended axially symmetric soap film is formed between the rings. Find the shape of the film, i.e. the revolving shape \( y(x) \).