Physics 1502: Lecture 3
Today’s Agenda

• Announcements:
  – Lectures posted on: www.phys.uconn.edu/~rcote/
  – HW assignments, solutions etc.

• Homework #1:
  – On Masterphysics today: due next Friday
  – Go to masteringphysics.com and register
  – Course ID: MPCOTE33308

• Labs: Begin in two weeks

• No class Monday: Labor Day

Today’s Topic:

• End of Chapter 20
  – Continuous charge distributions => integrate
  – Moving charges: Use Newton’s law

• Chapter 21: Gauss’s Law
  – Motivation & Definition
  – Coulomb’s Law as a consequence of Gauss’ Law
  – Charges on Insulators:
    » Where are they?
Infinite Line of Charge

- Solution:

The Electric Field produced by an infinite line of charge is:
- everywhere perpendicular to the line
- is proportional to the charge density
- decreases as 1/r.

Lecture 3, ACT 1

- Consider a circular ring with a uniform charge distribution (\( \lambda \) charge per unit length) as shown. The total charge of this ring is +Q.

- The electric field at the origin is

(a) zero  (b) \[ \text{Diagram} \]  (c) \[ \text{Diagram} \]
Lecture 4

Summary
Electric Field Distributions

Dipole ~ \(1 / R^3\)

Point Charge ~ \(1 / R^2\)

Infinite Line of Charge ~ \(1 / R\)

Motion of Charged Particles in Electric Fields

• Remember our definition of the Electric Field,

• And remembering Physics 1501,

Now consider particles moving in fields. Note that for a charge moving in a constant field this is just like a particle moving near the earth’s surface.

\[
\begin{align*}
a_x &= 0 \\
v_x &= v_{ox} \\
x &= x_0 + v_{ox}t
\end{align*}
\]

\[
\begin{align*}
a_y &= \text{constant} \\
v_y &= v_{oy} + at \\
y &= y_0 + v_{oy}t + \frac{1}{2}at^2
\end{align*}
\]
For an electron beginning at rest at the bottom plate, what will be its speed when it crashes into the top plate?

Spacing = 10 cm, \( E = 100 \text{ N/C} \), \( e = 1.6 \times 10^{-19} \text{ C} \), \( m = 9.1 \times 10^{-31} \text{ kg} \)

\[
v_f^2 - v_o^2 = 2a\Delta x
\]

Or,

\[
v_f = \sqrt{v_o^2 + 2a\Delta x}
\]

\[
v_f = 1.86 \times 10^8 \text{ m/s}
\]
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Torque on a dipole

- Force on both charges
  - 2 different directions
  - Create a torque

Recall:

And we have:

\[ \varepsilon_0 \oint \vec{E} \cdot d\vec{S} = q \]
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Calculating Electric Fields

- Coulomb's Law
  Force between two point charges
  Can also be used to calculate E fields
  OR

- Gauss' Law
  Relationship between Electric Fields and charges
  Uses the concept of Electric flux

Flux of a Vector Field

\[
\text{Flux} = (\text{normal component}) \times (\text{surface area})
\]
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Electric Dipole
Lines of Force

Consider imaginary spheres centered on:

a) +q (green)
b) -q (red)
c) midpoint (yellow)

• All lines leave a
• All lines enter b
• Equal amounts of leaving and entering lines for c

Electric Flux

• Flux:

Let's quantify previous discussion about field-line “counting”

Define: electric flux $\Phi_E$ through the closed surface $S$

$$\Phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S}$$

• What does this new quantity mean?
  – The integral is an integral over a CLOSED SURFACE
  – The result (the net electric flux) is a SCALAR quantity
  – $d\mathbf{S}$ is normal to the surface and points OUT
  – $\mathbf{E} \cdot d\mathbf{S}$ uses the component of $\mathbf{E}$ which is NORMAL to the SURFACE
  – Therefore, the electric flux through a closed surface is the sum of the normal components of the electric field all over the surface.
  – Pay attention to the direction of the normal component as it penetrates the surface...is it “out of” or “into” the surface?
  – “Out of is “+” “Into” is “-”
Lecture 3, ACT 2

Imagine a cube of side a positioned in a region of constant electric field, strength \( E \), as shown.

Which of the following statements about the net electric flux \( \Phi_E \) through the surface of this cube is true?

(a) \( \Phi_E = 0 \)  (b) \( \Phi_E = 2Ea^2 \)  (c) \( \Phi_E = 6Ea^2 \)

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Gauss' Law

Karl Friedrich Gauss
(1777-1855)
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Gauss' Law

• Gauss' Law (a FUNDAMENTAL Law):

  The net electric flux through any closed surface is proportional to the charge enclosed by that surface.

\[ \varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \varepsilon_0 \Phi = q_{\text{enclosed}} \]

• How to Apply??
  – The above eqn is TRUE always, but it doesn’t look easy to use
  – It is very useful in finding \( E \) when the physical situation exhibits massive SYMMETRY
  – To solve the above eqn for \( E \), you have to be able to CHOOSE a closed surface such that the integral is TRIVIAL
    » Direction: surface must be chosen such that \( E \) is known to be either parallel or perpendicular to each piece of the surface
    » Magnitude: surface must be chosen such that \( E \) has the same value at all points on the surface when \( E \) is perpendicular to the surface.
    » Therefore: that allows you to bring \( E \) outside of the integral

Geometry and Surface Integrals

If \( E \) is constant over a surface, and normal to it everywhere, we can take \( E \) outside the integral, leaving only a surface area

\[ \oint \vec{E} \cdot d\vec{S} = \int E dS \]

\[
\begin{align*}
\oint dS &= 2ac + 2bc + 2ab \\
\int dS &= 4\pi R^2 \\
\oint dS &= 2\pi R^2 + 2\pi RL
\end{align*}
\]
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Gauss ⇒ Coulomb

- We now illustrate this for the field of the point charge and prove that Gauss' Law implies Coulomb's Law.
- Symmetry ⇒ E field of point charge is radial and spherically symmetric
- Draw a sphere of radius R centered on the charge.
  - Why?
    - E normal to every point on surface
    - $\mathbf{E} \cdot d\mathbf{S} = EdS$
    - E has same value at every point on surface
    - can take E outside of the integral!
- Therefore, $\int \mathbf{E} \cdot d\mathbf{S} = \int EdS = \epsilon_0 4\pi R^2 E$
  - Gauss' Law ⇒
    - We are free to choose the surface in such problems...we call this a “Gaussian” surface

Infinite sheet of charge

- Symmetry:
  - direction of $\mathbf{E} = x$-axis
- Therefore, CHOOSE Gaussian surface to be a cylinder whose axis is aligned with the x-axis.
- Apply Gauss' Law:
  - On the barrel, $\oint \mathbf{E} \cdot d\mathbf{S} = 0$
  - On the ends, $\oint \mathbf{E} \cdot d\mathbf{S} = 2AE$
  - The charge enclosed = $\sigma A$

Therefore, Gauss' Law ⇒ $\epsilon_0 (2EA) = \sigma A$

Conclusion: An infinite plane sheet of charge creates a CONSTANT electric field.
**Two Infinite Sheets**

- Field outside the sheets must be zero. Two ways to see:
  - Superposition
  - Gaussian surface encloses zero charge
- Field inside sheets is NOT zero:
  - Superposition
  - Gaussian surface encloses non-zero charge

**Uniformly charged sphere**

What is the magnitude of the electric field due to a solid sphere of radius $a$ with uniform charge density $\rho$ (C/m$^3$)?

- Outside sphere: ($r > a$)
  - We have spherical symmetry centered on the center of the sphere of charge
  - Therefore, choose Gaussian surface = hollow sphere of radius $r$
Uniformly charged sphere

- Inside sphere: \( r < a \)
  - We still have spherical symmetry centered on the center of the sphere of charge.
  - Therefore, choose Gaussian surface = sphere of radius \( r \).

Infinite Line of Charge

- Symmetry \( \Rightarrow \) E field must be \( \perp \) to line and can only depend on distance from line
  - Therefore, CHOOSE Gaussian surface to be a cylinder of radius \( r \) and length \( h \) aligned with the x-axis.

- Apply Gauss’ Law:
  - On the ends, \( \vec{E} \cdot d\vec{S} = 0 \)
  - On the barrel, \( \int \vec{E} \cdot d\vec{S} = 2\pi rhE \) \( \text{AND} q = \lambda h \) \( \Rightarrow \)

NOTE: we have obtained here the same result as we did last lecture using Coulomb’s Law. The symmetry makes today’s derivation easier!
Conductors & Insulators

- Consider how charge is carried on macroscopic objects.
- We will make the simplifying assumption that there are only two kinds of objects in the world:
  - Insulators.. In these materials, once they are charged, the charges ARE NOT FREE TO MOVE. Plastics, glass, and other “bad conductors of electricity” are good examples of insulators.
  - Conductors.. In these materials, the charges ARE FREE TO MOVE. Metals are good examples of conductors.

- How do the charges move in a conductor??
  - Hollow conducting sphere
    Charge the inside, all of this charge moves to the outside.

Conductors vs. Insulators
Charges on a Conductor

• Why do the charges always move to the surface of a conductor?
  – Gauss' Law tells us!!
  – $E = 0$ inside a conductor when in equilibrium (electrostatics)!!
    » Why?
      If $E \neq 0$, then charges would have forces on them and they would move!!

• Therefore from Gauss' Law, the charge on a conductor must only reside on the surface(s)!!

- Infinite conducting plane
- Conducting sphere