Physics 5201 - Theoretical Mechanics
Fall 2009
Assignment 6
Due Monday, Dec 14, 10am (my mailbox)

1. Prove (a) Leibniz’s Rule \([uv, w] = u [v, w] + [u, w] v\) and (b) Jacobi’s identity for Poisson Brackets.

2. Obtain the motion in time of a linear harmonic oscillator from the series expansion of the Poisson Bracket equations of motion. Assume \(x(0) = x_0\) and \(p(0) = p_0\).

3. Show that the following transformations are canonical
   a) \(Q = \ln \left(\frac{1}{q} \sin p\right), \quad P = q \cot p\)
   b) \(P = \frac{1}{2} (p^2 + q^2), \quad Q = \tan^{-1} \frac{q}{p}\)
   c) \(P = q^{-1}, \quad Q = pq^2\).

4. Show that the transformation
   \[ Q = p + i\alpha q, \quad P = \frac{1}{2i\alpha} (p - i\alpha q) \]
   is canonical and find a generating function. Use the transformation to solve the linear harmonic oscillator problem.

5. Find the Hamiltonian for the spherical pendulum in spherical coordinates. Show explicitly, in these coordinates, that the fundamental angular momentum Poisson Brackets are satisfied:
   \[ [L_i, L_j] = \epsilon_{ijk} L_k \]

6. Show that the Runge-Lenz vector is a constant of the motion for the Kepler problem by explicitly evaluation the Poisson Bracket \([A, H]\).